

ON THE SECOND POSTULATE OF THE THEORY OF RELATIVITY: AN EXPERIMENTAL DEMONSTRATION OF THE CONSTANCY OF THE VELOCITY OF LIGHT REFLECTED BY A MOVING MIRROR.<sup>1</sup>

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THE theory of relativity is based upon two well-known fundamental postulates. The first postulate asserts the impossibility of detecting the movement of a system without referring it to other systems; that is, it denies the physical reality of *absolute movement*. The second declares that  $c$ , the velocity of light in free space, is a universal constant. Both of these postulates are generalizations from facts or principles heretofore accepted by physicists.

In fact, the first postulate may be considered as the extension of a principle of classical mechanics to the optical or electrical phenomena; an extension justified by the negative results of the experiments (Michelson & Morley, Cronton & Noble) designed to discover the absolute movement of the earth or the ether which permeates all terrestrial objects. The second postulate is the generalization of the elementary principle of the electromagnetic or the undulatory theory of ether.

But, if these principles were taken from quite different chapters of physics, and were severally accepted by modern physicists—ignoring their origin, there would result from their union an ingenious construction; the theory of relativity. This theory, even though contested by Einstein and others, is a theoretical conception which led to the formulation of the second postulate (the ether), and serves to explain the failure of the experiments cited.

Now our thought, accustomed, as W. Ritz had said, to “substantialize” the optical phenomena, may easily grasp the essence of the first postulate, but it cannot do so with the second; especially since, as mentioned before, the relativistic theories do not depend necessarily upon the existence of a transmitting medium to explain the constancy of  $c$ . On the other hand, the conclusions which seem artificial and strange to all the relativistic theories<sup>2</sup> are due to the second postulate,

<sup>1</sup> Manuscript rendered from French by Kia-Lok Yen, Ryerson Laboratory, University of Chicago.

<sup>2</sup> Carmichael, *PHYS. REV.*, 1912, XXXV., p. 168.

or, more precisely, to certain parts of it. This second postulate should be understood in the sense that an observer who measures the velocity of light, finds its value the same whether both he and the source are relatively or absolutely (provided he admits the possibility) at rest, or whether either the source or the observer or both of them are in uniform motion. That is, the second postulate affirms the absolute independence of  $c$  of whatever contingent unaccelerated velocity of either the source or the observer.

It is known that an hypothesis of a mechanical character (emissive or ballistic), according to which the velocity of the source should be added to the ordinary velocity of light, could, as the theory of relativity, explain the failure of the experiments cited before. But such an hypothesis would be in radical contrast with the electromagnetic theory and consequently would not find much favor.<sup>1</sup> But in any case laboratory experiments which could decide between said hypothesis, or mechanical theory, and the relativistic theory, are imaginable. Indeed, it is possible to see that some method, even already known, adopted for the verification of Doppler's principle in optics, may be able to furnish a solution to this problem.

In order to see this, let us consider a source of light  $S$ , which emits waves the length of which is  $\lambda$  and the frequency  $n$ , and which moves with a velocity  $v$  toward the observer remaining at rest at  $O$  (Fig. 1).

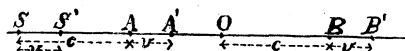


Fig. 1.

If we suppose that the waves are transmitted through a fixed ether, the  $n$  waves emitted from  $S$  in one second will be distributed over the segment  $S'A = c - v$ . In the same interval there will pass by  $O$  all the waves  $n$  distributed over the segment,  $OB = c$ . Consequently we have:

$$\frac{c - v}{n} = \frac{c}{n'}, \quad \text{or} \quad n' = n \frac{c}{c - v}.$$

If  $v/c = \beta$ , we have  $n' = n(1 + \beta)$ , neglecting the terms containing  $\beta$  of higher than the first order. And since  $c = n\lambda = n'\lambda'$ . Therefore  $\lambda' = \lambda(1 - \beta)$ , which is the length of the new waves.

If instead of the hypothesis of the fixed medium we employ the previously mentioned ballistic or emissive hypothesis, we will find that in one second the  $n$  waves emitted from  $S$  will be distributed over the segment

<sup>1</sup> In this connection attention should be called to the important critical work of W. Ritz (Ouvres, p. 317), which, perhaps, has not received sufficient consideration from physicists.

$S'A' = c$ . In an equal interval there will pass by  $O$ ,  $n'$  waves which will distribute over the segment  $OB' = c + v$ . Thus we will have:

$$\frac{c}{n} = \frac{c + v}{n'}, \quad \text{or} \quad n' = n(1 + \beta).$$

And since in this case  $c = n\lambda$  and  $c + v = n'\lambda'$ , therefore  $\lambda' = \lambda$ .

As far as frequency is concerned the same result—excepting the terms in  $\beta^2$ —is reached by both the ether and the ballistic hypotheses. But, the values for the wave-length resulting from the two hypotheses are different, and these values differ for the first order of  $\beta$ . Thus, if Doppler's effect is measured by the observation of the wave-length, different results will be obtained according to whether one or the other of the hypotheses is accepted.<sup>1</sup> Now the observations of Doppler effect have been made up to the present by measuring the displacement of the spectral lines by means of either prisms or diffraction gratings. In the case of prisms, it may be observed that the theories of dispersion heretofore accepted conduce to the supposition that this phenomenon may depend only upon the frequency of the incident light. Consequently the displacement of the spectral lines may be caused by the simple variation of frequency due to the Doppler effect, and so the ether and the ballistic hypotheses are equally acceptable. From this point of view, therefore, the question whether the velocity of the propagation of light emitted from a source does or does not change with the velocity of the latter cannot be settled.

But besides prisms, Doppler effect has been verified by diffraction gratings for the astronomical as well as the terrestrial sources.<sup>2</sup> If the function of the grating is considered, from the geometric point of view, as dependent entirely upon the incident wave-lengths, the positions of the successive spectral lines will remain exactly definite. But since, according to the ballistic hypothesis, the value of  $\lambda$  does not vary with the velocity of the source, it can be easily seen that gratings will not give appreciable results in the examination of the Doppler effect, and so, as has been said, will not confirm the experiment. Therefore it may be concluded from the observations of the Doppler phenomenon in the stars and the sun, with moving mirrors (Galitzin and Wilip), or again in the canal rays (Stark, Paschen), that the velocity of light is constant and entirely independent of the movement of the source; which is equivalent to the rejection of the ballistic or emissive theory. Tolman<sup>3</sup>

<sup>1</sup> These conclusions are the same as those already published by others; see, for instance, Tolman, *PHYS. REV.*, 1910, XXI., p. 26.

<sup>2</sup> Galitzin et Wilip, *Communications Acc. Russe*, 1907, p. 213; Stark, *Ann. d. Phys.*, 1909, 28, p. 974.

<sup>3</sup> *PHYS. REV.*, 1912, XXXV., p. 136.

is of this opinion as contrary to that of Stewart.<sup>1</sup> As a matter of fact, it should be remarked that the common theory of grating<sup>2</sup> would no longer be exact in the case of the mechanical (ballistic or emissive) theory of light. In any case it is necessary to note that the astronomical observations of the Doppler effect are not always made with the *a priori* knowledge of the relative velocity between the source and the observer. In the case of the sun, on the contrary, it is necessary to establish a relation between the displacement of the lines and the velocity of the borders by the observation of the sun spots; in fact, the light of the borders may be entirely refracted by the peripheral incandescent gas, and consequently the value of Doppler effect may change considerably.<sup>3</sup> In so far as the terrestrial observations and those of the canal rays (Stark, Paschen) are concerned, they do not give very precise measurement of the phenomenon, and it is not possible to determine the exact velocity of the luminous particles by another method. Finally, the observations made with moving mirrors are not correlative to the moving sources and so may give different and misleading results.<sup>4</sup>

Hence it may be concluded that we have not so far possessed an altogether sure proof of the immutability of  $c$  by the variable velocity of the source—if, of course, we do not wish to admit as definitely accepted, either the simple electromagnetic theory, or Lorentz's theory of moving bodies, or Einstein's theory of relativity. The confirmation of this conclusion may be found in the works of those who strongly support the last theory, and implicitly the second postulate. In these works there is frequently found expressed the desire to discover further facts in order to confirm definitely the said theory; this desire is found in the recent discussions of this theory.

But, on the other hand, as Levi-Civita observed, after Einstein's last investigations which gathered into an admirably comprehensive synthesis all the physical phenomena (gravitation included), it is difficult to avoid the impression that in so far as the theory of relativity is concerned there is present something which is definitely unquestionable. But even if this is taken into account it does not mean that an attempt to obtain a final confirmation, from an experimental point of view, of a theory which has upset even our most simple physical ideas may be neglected. This confirmation may arise from the accurate study of the velocity of propagation of light emitted from a moving source, or, what amounts to the same thing, of the value of the wave-length  $\lambda$  of this light.

<sup>1</sup> PHYS. REV., 1911, XXXII., p. 418.

<sup>2</sup> La Rosa, Nuovo Cimento, 1912, III., p. 356.

<sup>3</sup> Michelson, Astroch. Jour., 1901, 13, p. 192. Harnack, Ann. d. Phys., 1915, 46, p. 558.

<sup>4</sup> See theory proposed by Ritz, Oeuvres, p. 321, 371, 444.

In order to realize such study it will be necessary to imagine a disposition which, free of all external disturbance, would facilitate the examination of the structure of the light wave in its propagation,—or transmission—when the velocity of the source is varied at will. Now even leaving aside the fact that the execution of the experiment under the eventual action of the earth will be inevitable,<sup>1</sup> there will remain still two serious and almost insurmountable difficulties in the way of the realization of such a programme. In the first place, it is difficult to produce artificially a rapid movement in a luminous source,<sup>2</sup> more so if the latter is to remain rigorously monochromatic; however I shall give an account, in a future publication, of a disposition of this nature on which I am experimenting. Secondly, in order to examine the structure of the light-wave emitted from a moving source, no matter with what disposition, it is necessary to subject the same light to reflections, refractions, etc., which are sometimes quite numerous; that is, the light pencil has to encounter ponderable material after leaving the source. Thus, even if  $c$  in free space does vary with the proper velocity of the source, the intensity would not return to the same fixed value after said phenomena of reflection, refraction, etc. It would be better, therefore in an experiment of this kind, to try to eliminate the greatest possible number of cases of complications from the phenomenon, and, in any case, to discuss carefully its result.

However, in order to begin a relatively simpler experiment, the study of the wave-length of a light pencil reflected from a moving mirror may be undertaken. This is like the experiment already performed, several years ago, by Belopolski, and repeated afterwards by Galitzin and Wilip. But if the first of these authors employed prisms in the observation of Doppler effect—and, consequently did not solve the question of the eventual variation of  $\lambda$ —the two others employed diffraction gratings which gave rise to the controversy mentioned before. It will be better, therefore, to examine the pencil reflected from a moving mirror by an interferential method which is simpler than those dependent upon the function of the diffraction grating.

Before stating this method it may be well to point out that considerable theoretical work has been done upon the influence of the motion of the mirror upon the wave of the reflected light. Among these treatments are those of Abraham, Brown, Edser, Harnack, Larmore, and Plank. These

<sup>1</sup> I am not imagining an interferential experiment of the sort proposed jocularly by Rose-Innes. See *Phil. Mag.*, 1914, XXVII., p. 150.

<sup>2</sup> I mean by that a velocity greater than several hundred meters per sec. Such value may perhaps be reached but it is difficult to conceive of a practical disposition for a greater velocity. Naturally I set aside the employment of canal rays which do not give pure and also known velocities.

works have been simply concerned either with geometrical study or with the application of the electromagnetic theory of light. But without discussing the results of these studies we may accept the conclusions of Harnack<sup>1</sup> regarding the frequency of the vibrations reflected from a uniformly moving mirror. If  $v$  be the velocity of the mirror measured normally to its plane, and evaluated positively when towards the source,  $c$  the velocity of the light pencil in free space which makes an angle of incidence  $I$  with the mirror,  $n$ ,  $n'$  the frequencies of the pencil before and after reflection, and if both the source and the observer are at rest, we will have, putting  $\beta = v/c$ , the following formula:

$$n' = n \frac{1 + 2\beta \cos I + \beta^2}{1 - \beta^2},$$

which may be reduced, by neglecting the terms containing  $\beta^2$ , to:

$$n' = n(1 + 2\beta \cos I),$$

which is the same as that of Ketteler,<sup>2</sup> which was employed by Belopolski<sup>3</sup> in his study of Doppler effect, and which was deduced similarly from the consideration that the image of the source moves with a velocity  $2v$  along the normal to the mirror and consequently the component of this velocity along the reflected pencil is  $2v \cos I$ .

Now if, by suitable devices, the pencil is reflected  $k$  times, with the incidence  $I$ , upon several mirrors moving with a velocity  $v$ , we will have

$$n' = n(1 + 2k\beta \cos I).$$

Consequently, according to hypothesis of the constancy of the velocity of light, we will have (neglecting the terms containing  $\beta^2$ ):

$$\lambda' = \lambda(1 - 2k\beta \cos I).$$

If, on the other hand, the velocity of the reflected light is variable, and is equal to  $c = 3 \cdot 10^{10}$  cm. plus the component of the velocity of the image along the pencil, we will have  $c' = c + 2kv \cos I$ . And since  $c' = n'\lambda'$  and  $c = n\lambda$ , we will have  $\lambda' = \lambda$ . The question then is to see experimentally whether or not, besides the Doppler effect, any variation in the value of  $\lambda$  could be detected, and hence whether  $c$  remains constant upon the reflection by the moving mirror. I have not observed the Doppler effect in this investigation since its existence has without doubt been verified experimentally by the authors cited; I have rather investigated whether and how  $\lambda$  does vary with the velocity of the mirror.

<sup>1</sup> Ann. d. Phys., 1912, 39, p. 1053; and 1915, 46, p. 547.

<sup>2</sup> Astronomische Undulationtheorie.

<sup>3</sup> Communications Acc. Russe, 1900, 13, p. 461.

Belopolski's device for the study of Doppler effect had a disadvantage due to the minuteness of the light pencils necessary for obtaining multiple reflections upon the same mirror. For this reason he could not observe the displacement of lines on the photographs. Consequently an arrangement as represented by Fig. 2 is adopted. On the periphery of the horizontal brass wheel  $R$ , 35 cm. in diameter and 6 mm. in thickness, are mounted 10 glass mirrors  $M$  with their planes vertical and their back surfaces silvered. Thus the velocity of the centers of the mirrors at the maximum speed of revolution is more than 100 meters per second. The number of revolutions of the wheel is determined accurately in each experiment. The mirrors, equally spaced on the circumference of the wheel, make an angle  $\alpha$  of  $29^\circ$  with the radius of  $R$  passing through each

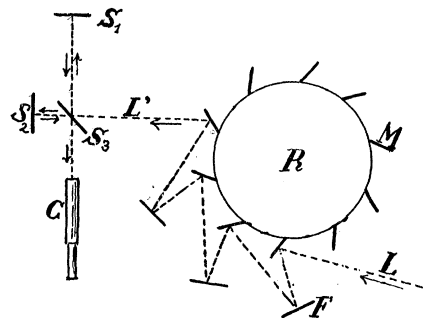


Fig. 2.

of their centers. They are fastened securely to the wheel by screws capable of rigorous adjustment. The support of the axle of  $R$  carries the fixed mirrors  $F$  with their planes vertical as  $M$ . They are three in number but may be decreased or increased at will up to nine. The positions of  $M$  and  $F$  are such that when  $R$  is at a determined angular position a parallel beam of light  $L$  may, after a number of reflections—7 in the figure—travel in the direction  $L'$ . Naturally the intensity of  $L'$  is considerably smaller than that of  $L$ ; and the diminution is much greater when  $R$  is in motion, since in this case the light travels in  $L'$  only in very short instances—10 times per revolution. It was observed that practically the four moving and the three fixed reflections resulted in  $L'$ , a light still sufficiently intense even when  $R$  is in motion. Thus it will be possible to make direct observations—without photographs—in order to verify the light phenomenon.

In order to study the value of  $\lambda$ , the light  $L'$  is examined by the well-known Michelson interferometer indicated schematically in the figure.

It is known that if the distances  $s_1s_3$  and  $s_2s_3$  are exactly equal fringes can be seen in the telescope  $c$  even if the light is not monochromatic: these fringes will have the appearance of the Newton rings. But as soon as there is a difference of path—even of only several microns—the fringes can no longer be produced by white light. It is necessary to employ monochromatic light, and the order of the interference fringes increases with (the path) this difference. Their visibility is greater when the vibrations are simpler. Michelson's<sup>1</sup> studies showed that from this point of view the line which gives the greatest visibility to the fringes with the largest path difference is the green mercury line ( $\lambda = 546 \mu\mu$ ). In this case, the circular fringes at infinity are visible even with a path difference of  $l = 2(s_1s_3 - s_2s_3) = 40$  cm. Consequently, the mercury arc in vacuum is here chosen as the source  $L$ ; the light from the arc is filtered through solutions of potassium chromate and nickel chloride in order to absorb the violet and the yellow radiations. Thus the circular fringes at infinity can be observed with sufficient clearness by means of the telescope  $c$  even when  $l = 32$  cm. But in this investigation the path difference is limited to 13 cm. or even less.

The disposition described is especially suitable for detecting the very small variations in the wave-length of the incident light. In fact, as the path difference is large, there are contained in this distance a very large number of  $\lambda$ —200,000 if  $\lambda = 0.5 \mu$  and  $l = 10$  cm.—and consequently very sensible displacements in the position of one fringe, corresponding to the variations, can be observed.

The apparatus thus arranged, the observation is made by first setting the cross hair of the telescope micrometer in a position identical with that of a certain fringe—for instance the first central bright one—when  $R$  is at rest, or, better still, when it is moving with a negligible speed—say 1 revolution per sec. Now if the speed of  $R$  is increased to about 60 revolutions per sec. *a displacement of the fringe referred to will be seen clearly.* This displacement will indicate the diminution of  $\lambda$  if the mirrors move in the direction opposite to that of the incident ray; and will indicate the augmentation if the movement of the wheel is reversed. To determine the sense of the displacement it may be said that in examining the system of circular fringes with the telescope focused for parallel rays, the diameter of each of them increases when the mirrors are moving against the incident light, and as those of greater diameter displace very little, these fringes crowd together; and at the same time some new fringes come into being out of the center of the system. On the other hand, when the mirrors are moving in the direction of propagation of

<sup>1</sup> Travaux et Memoires, Bur. Int. de poids and mesures, 1895, XI., p. 146.



the incident light, the diameter of each of the fringes decreases, and the fringes become widely separated and some of them remain as if swallowed up by the center.

Before saying what would be the extent of the displacement observed, it may be anticipated by way of a hypothesis that the velocity of the light reflected from a mirror would be the same as that of the incident light. Let  $g$  be the number of revolutions of  $R$  per second, and  $d$ —the distance between the centers of two opposite mirrors—be the diameter of  $R$ , then  $\pi dg$  will be the instantaneous linear velocity of the mirrors. Since these mirrors make angle  $\alpha$  with the radius of the wheel passing through each of their centers the component of this velocity in the direction normal to the plane of each of these mirrors will be:

$$v = \pi dg \cos \alpha.$$

Therefore we have:

$$n' = n \left[ 1 + \frac{2k\pi dg \cos \alpha \cos I}{c} \right];$$

and from the hypothesis of the immutability of  $c$ :

$$\lambda' = \lambda \left[ 1 - \frac{2k\pi dg \cos \alpha \cos I}{c} \right].$$

Therefore when  $\lambda$  changes into  $\lambda'$ —that is when the velocity of the wheel varies from zero to  $g$  revolutions per second—the number of fringes which will pass the cross hair of the telescope micrometer will be

$$f = \frac{l}{\lambda} \cdot \frac{2k\pi dg \cos \alpha \cos I}{c},$$

where  $l$  is the difference of path of the two interfering pencils in the Michelson interferometer.

If the observation is made by locating first the position of the fringes when the wheel is turning in one direction with a speed  $g$  and then that corresponding to an equal and opposite speed, the number of fringes which will pass the crosshair of the micrometer will be  $2f$ .

Now in the present apparatus,  $d = 38$  cm.,  $\alpha = 29^\circ$ ,  $I = 27^\circ$ ,  $k = 4$  (as in the figure). If  $\lambda = 0.546 \mu$  (green Hg line),  $l = 13$  cm.,  $c = 3 \cdot 10^{10}$  cm., and  $g = 60$  rev./sec. we will have by reversing the speed of  $R$ , according to the preceding formula, a displacement of  $2f = 0.71$  fringe.

Actual experiment gives, for the case cited, a displacement of from 0.7 to 0.8 fringe; and it is not possible, on account of the visibility, to push the accuracy of observer any further. But, as it may be seen, the agreement between the predicted and observed results is sufficient. This agreement is confirmed by observations made by choosing other

convenient values for  $l$  and  $g$ ; but the discussion of these may for the sake of brevity be dispensed with here.

In view of this result we are justified in concluding that *the reflection of light by a moving metallic mirror does not modify the velocity of propagation of that light in air, and consequently—with great probability—also in free space; this is at least so under the experimental conditions herein described.* This experimental result, about which there can be no question, is contrary to the hypothesis of some authors, such as Stewart,<sup>1</sup> who, on the ground of Thomson's electromagnetic theory of emission, asserts the possibility that the light, after reflection, may travel with a velocity  $c + v$ ; where  $v$  is the component of the velocity of the image in the direction of the reflected ray.

In order to complete these investigations I intend, as I said before, to study further, with the same interferential disposition, the velocity of propagation of the light from a source set in motion artificially. But this study, as well as the general conclusions which may be drawn from these investigations, I reserve for future publication.

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<sup>1</sup> PHYS. REV., 1911, XXXII., p. 418.