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XIX. On some elementary applications of Abel's theorem

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lute solution of muriate of ammonia. This precipitate is almost black in the mass, but when spread over the surface of white porcelain or paper, it appears of a fine sap-green; by exposure to the air it becomes nearly white on the surface, which discoloration speedily extends to some depth, (if all the hydrosulphuret has not been washed away, this change does not take place until after some time); it is then to be dried on a sand-bath and digested in weak hydrochloric acid, by which almost all the sulphuret of iron is removed; the insoluble portion is then to be again dried and ignited, in a platinum capsule over the circular-wick spirit-lamp (or if in considerable quantity, on a platinum tray in a small muffle); a cream-coloured powder is thus obtained, still containing a minute portion of iron, which may be got rid of by mixing it with muriate of ammonia, and exposing it for some time to a temperature below ignition. The titanic acid thus procured is tolerably pure.

In conclusion we would wish to remark that titanium appears to be more generally diffused through the mineral kingdom than is generally stated (in chemical works), as appears particularly from the following passage in Thénard's *Traité de Chimie*: "Le deutoxide de fer se rencontre sous forme de sables. Ces sables contiennent ordinairement de l'oxide de titane ou de l'oxide de chrome en combinaison avec l'oxide de fer. M. Descotils a retiré jusqu'à 30 parties de titane de 100 parties de sables ferrugineux de Saint-Quay, département des Côtes du Nord. M. Robiquet l'a rencontré dans le deutoxide de fer des roches steatiteuses de la Corse."

Guy's Hospital, Dec. 26th, 1834.

XIX. *On some Elementary Applications of* Abel's *Theorem.* *By* J. W. Lubbock, *Esq., V.P. & Treas. R.S.**

Abel, in the third volume of Crelle's Journal, gave a theorem, which constitutes one of the most remarkable discoveries ever made in analysis, by which the methods of finding the sum of certain definite integrals were greatly extended. Cut off in the prime of life†, it was not given to the mathematician of Christiania to pursue the career which is opened to analysts by the theorem in question, or to illustrate its application by examples. This has been done to a certain extent by Legendre in the third Supplement of his work entitled *Théorie des Fonctions Elliptiques* ‡. As, however, no notice of this theorem has yet appeared in any work in the En-

* Communicated by the Author.

† [See Phil. Mag. and Annals, N.S. vol. vii. p. 77.—Edit.]

‡ [See Lond. and Edinb. Phil. Mag., vol. iv. p. 143.—Edit.]

glish language, to my knowledge, the following examples of its application to some of the simplest cases which can be proposed are here offered, with a view of attracting attention to an important analytical discovery*. Legendre has applied Abel's theorem to the integral

$$\int \frac{dx}{\sqrt{1-x^5}}.$$

The subject of the first example here detailed is the integral

$$\int \frac{dx}{\sqrt{1-x^3}},$$

which may, in fact, be reduced to an elliptic integral of the first kind; and thus, through the well-known integral due to Euler, a confirmation may be easily obtained, in this instance, of the result found by the method of Abel.

The subject of the second example is

$$\int \frac{dx}{\sqrt{1+x^n}},$$

which cannot in general be reduced to an elliptic integral. Here I have chosen for the equations of condition between the limits x_1, x_2, x_3, &c., equations similar to those employed by Mr. Talbot in his paper on the sum of three arcs of the equilateral hyperbola†. I have also applied the method of Abel to the integral

$$\int \frac{x^2 dx}{\sqrt{1+x^4}},$$

and deduced therefrom the theorem given by Mr. Talbot in the paper to which I have referred.

The theorem of Abel is as follows:

"Soit ϕx une fonction entière de x, décomposée d'une manière quelconque en deux facteurs entiers $\phi_1 x$ et $\phi_2 x$, ensorte que $\phi x = \phi_1 x . \phi_2 x$. Soit fx une autre fonction entière quelconque et

$$\psi x = \int \frac{fx\, dx}{(x-\alpha)\sqrt{(\phi x)}},$$

où α est une quantité constante quelconque. Désignons par $a_0, a_1, a_2, \ldots ; c_0, c_1, c_2, \ldots$ des quantités quelconques dont l'une au moins soit variable. Celà posé, si l'on fait

* Professor Jacobi says, "Wir halten es, wie es in einfacher Gestalt ohne Apparat von Calcul den tiefsten und umfassendsten mathematischen Gedanken ausspricht, für die grösste mathematische Entdeckung unsrer Zeit, obgleich erst eine künftige, vielleicht späte, grosse Arbeit ihre ganze Bedeutung aufweisen kann.—*Crelle's Journal*, vol. viii. p. 415.

† [A notice of Mr. Talbot's paper will be found in Lond. and Edinb. Phil. Mag. vol. iv. p. 225.—Edit.]

$$(a_0+a_1 x+a_2 x^2 + \ldots\ldots + a_n x^n)^2 . \phi_1 x$$
$$-(c_0+c_1 x+c_2 x^2 + \ldots\ldots + c_m x^m)^2 . \phi_2 x$$
$$= A\,(x-x_1)\,(x-x_2)(x-x_3)\ldots\ldots(x-x_\mu),$$

où A ne dépend pas de x, je dis qu'on aura

$$\varepsilon_1 \psi x_1 + \varepsilon_2 \psi x_2 + \varepsilon_3 \psi x_3 + \ldots\ldots + \varepsilon_\mu \psi x_\mu$$

$$= \frac{f\alpha}{\sqrt{\phi\alpha}} . \log \left\{ \frac{(a_0+a_1\alpha+\ldots+a_n\alpha^n)\sqrt{(\phi_1\alpha)}+(c_0+c_1\alpha+\ldots+c_m\alpha^m)\sqrt{(\phi_2\alpha)}}{(a_0+a_1 x+\ldots+a_n\alpha^n)\sqrt{(\phi_1\alpha)}-(c_0+c_1\alpha+\ldots+c_m\alpha^m)\sqrt{(\phi_2\alpha)}} \right\}$$
$$+ r + C$$

où C est une quantité constante et r le coëfficient de $\frac{1}{x}$ dans le développement de la fonction

$$\frac{fx}{(x-\alpha)\sqrt{\phi x}} . \log \left\{ \frac{(a_0+a_1 x+\ldots+a_n x^n)\sqrt{(\phi_1 x)}+(c_1+c_1 x+\ldots+c_m x^m)\sqrt{(\phi_2 x)}}{(a_0+a_1 x+\ldots+a_n x^n)\sqrt{(\phi_1 x)}-(c_0+c_1 x+\ldots+c_m x^m)\sqrt{(\phi_2 x)}} \right\},$$

suivant les puissances descendantes de x. Les quantités $\varepsilon_1, \varepsilon_2, \ldots\ldots \varepsilon_\mu$ sont égales à $+1$ ou à -1, et leurs valeurs dépendent de celles des quantités $x_1, x_2 \ldots\ldots x_\mu$." ....

Suppose generally,

$$\theta x = a_0 + a_1 x + a_2 x^2 \ldots\ldots + a_n x^n$$
$$\theta_1 x = c_0 + c_1 x + c_2 x^2 \ldots\ldots + c_m x^m.$$

The values of $\varepsilon_1, \varepsilon_2, \varepsilon_3 \ldots\ldots \varepsilon_\mu$ are not arbitrary, they depend upon the magnitude of $x_1, x_2 \ldots\ldots x_\mu$, and this is determined by the equation

$$\theta x \sqrt{\phi_1 x} = \varepsilon \theta_1 x \sqrt{\phi_2 x}.$$

Moreover, if fx is divisible by $x-\alpha$, $f\alpha = 0$; and if $(fx)^2$ is of less dimensions than ϕx, r vanishes, and

$$\varepsilon_1 \psi x_1 + \varepsilon_2 \psi x_2 \ldots\ldots\ldots + \varepsilon_\mu \psi x_\mu = C.$$

In the following examples A is made equal to unity, which is allowable.

It is intended here first to apply the theorem to the integral

$$\int \frac{dx}{\sqrt{1-x^3}} = \psi x \qquad 1-x^3 = (1+x+x^2)(1-x).$$

In this case $\phi_1 x = 1+x+x^2$, $\phi_2 x = 1-x$

$$1+x+x^2-c^2(1-x) = (x-x_1)(x-x_2) \quad \theta x = 1 \qquad \theta_2 x = c.$$

Equating the coefficients of powers of x,

$$1 - c^2 = x_1 x_2$$
$$1 + c^2 = -x_1 - x_2$$
$$2 = x_1 x_2 - x_1 - x_2 \qquad 3 = (1-x_1)(1-x_2)$$
$$x_1 = \frac{2+x_2}{x_2-1}$$

When $x_2 = 1 \quad x_1 = -$ infinity

$$x_2 = \frac{1}{2} \quad x_1 = -5$$

$$x_2 = \frac{1}{3} \quad x_1 = -\frac{7}{2}.$$

$$\int \frac{dx}{\sqrt{1-x^3}} = x + \frac{x^4}{2.4} + \frac{1.3\,x^7}{2.4.7} + \frac{1.3.5\,x^{10}}{2.4.6.10} +, \&c.$$

Making $x = \frac{1}{2}$ this series gives $\psi x = \cdot 508264$

$x = \frac{1}{3}$ .... $\psi x = \cdot 334901.$

Making $x = -y$

$$\int \frac{dx}{\sqrt{1-x^3}} = -\int \frac{dy}{\sqrt{1+y^3}}.$$

Now, let $\frac{1}{1+y^3} = u$

$$-\int \frac{dy}{\sqrt{1+y^3}} = 2\left\{ u^{\frac{1}{6}} + \frac{2}{3.7}u^{\frac{7}{6}} + \frac{2.5}{3.6.13}u^{\frac{13}{6}} +, \&c. \right\}$$

If $\psi' \frac{1}{0}$ denote the integral

$$\int \frac{dy}{\sqrt{1+y^3}}$$

taken from $y = 0$ to $y = \infty$, the preceding series gives when

$$x = -5, \; u = \frac{1}{126}, \; \psi x = \cdot 893917 - \psi' \frac{1}{0}$$

$$x = -\frac{7}{2}, \; u = \frac{8}{351}, \; \psi x = 1\cdot 06728 - \psi' \frac{1}{0}.$$

By the theorem of Abel, if x_1 and x_2 are connected by the equation

$$x_1 = \frac{2+x_2}{x_2-1}. \quad \text{(See line 4 above.)}$$

$$\epsilon_1 \psi x_1 + \epsilon_2 \psi_2 = \text{constant, or}$$

$$\epsilon_1 \int_0^{x_1} \frac{dx}{\sqrt{1-x^3}} + \epsilon_2 \int_0^{x_2} \frac{dx}{\sqrt{1-x^3}} = \text{constant.}$$

In this example $\varepsilon_1 = +1$, $\varepsilon_2 = +1$ $\psi x_1 + \psi x_2 = \psi 1 - \psi'_{\frac{1}{0}}$.

By the known properties of the function Γ, (see *Théorie des Fonctions Elliptiques*, vol. ii. p. 417,)

$$\int x^{p-1} dx\, (1-x)^{q-1} = \frac{\Gamma p\, \Gamma q}{\Gamma(p+q)}$$

$$\psi_1 = \frac{1}{3} \frac{\Gamma\frac{1}{3}\, \Gamma\frac{1}{2}}{\Gamma\frac{5}{6}} = \frac{5}{6}\, \pi^{\frac{1}{2}}\, \frac{\Gamma(\frac{4}{3})}{\Gamma(\frac{11}{6})}$$

$$\log \pi^{\frac{1}{2}} = 0{\cdot}2485749 \qquad \log \Gamma\left(\frac{11}{6}\right) = 9{\cdot}9734262$$

$$\log \Gamma\left(\frac{4}{3}\right)^* = 9{\cdot}9508435 \qquad \log 6 = 0{\cdot}7781513$$

$$\log 5 = 0{\cdot}6989700 \qquad\qquad 0{\cdot}755775$$

$$0{\cdot}8983884$$
$$0{\cdot}7515775$$

$$\log \psi_1 = 0{\cdot}1468109$$
$$\psi_1 = 1{\cdot}402202$$

$$\psi'_{\frac{1}{0}} = \frac{1}{\cos 60^\circ} \psi_1 = 2\psi_1 = 2{\cdot}804405.$$

It will now be seen that the numerical values of the integrals ϕx_1, ϕx_2 verify the equation

$$\psi x_1 + \psi x_2 = \psi_1 - \psi'_{\frac{1}{0}},$$

for

$$\cdot508264 + \cdot893917 - \psi'_{\frac{1}{0}} = \psi_1 - \psi'_{\frac{1}{0}}$$
$$\cdot508264 + \cdot893917 = \psi_1$$
$$\cdot334901 + 1{\cdot}06728 = \psi_1.$$

The preceding results may easily be shown to be in accordance with the known theory of elliptic integrals by putting for x

$$1 - \frac{\sqrt{3}}{\tan^2 \frac{1}{2}\phi}. \quad \text{(See Legendre, vol. ii. p. 381.)}$$

$$\int \frac{dx}{\sqrt{1-x^3}} = (3)^{\frac{1}{4}} \int \frac{d\phi}{\sqrt{1-\sqrt{3}\sin^2\phi}}.$$

The equation

$$3 = (1-x_1)(1-x_2) \text{ gives}$$

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* $\frac{4}{3} = 1{\cdot}33333$ and $\frac{11}{8} = 1{\cdot}83333$, &c. (See the Table of the values of log Γa, *Théorie des Fonctions Elliptiques*, vol. ii. p. 493.)

$$\tan^2 \frac{\phi_1}{2} \tan^2 \frac{\phi_2}{2} = 1$$

$$\phi_1 + \phi_2 = \pi.$$

I shall now consider the integral

$$\int \frac{dx}{\sqrt{1+x^n}},$$

which cannot, except in certain cases, be reduced to an elliptic integral.

Suppose

$$\sqrt{1+x^n} = \sqrt{\phi x}, \quad 1+x^n = \phi_1 x, \quad \phi_2 x = 1$$

$$\theta x = 1, \quad \theta_2 x = 1 + c_1 x$$

$$1+x^n - (1+c_1 x)^2 = x\,(x^{n-1} - \frac{x_1^2 x_2^2 x_3^2 \dots x^2_{n-1} x}{4}$$

$$\pm x_1 x_2 x_3 \dots x_{n-1}),$$

the upper sign to be taken if n is an uneven number, and the lower if x is even, $x_1, x_2, x_3 \dots x_{n-1}$ being the roots of the equation $\quad x^{n-1} - Px + Q = 0$

$$P = \frac{x_1^2 x_2^2 x_3^2 \dots x^2_{n-1}}{4},$$

$$x_1 + x_2 + x_3 \dots\dots\dots + x_{n-1} = 0$$

together with the other conditions implied by the nature of the equation $\quad x^{n-1} - Px + Q = 0$

$$c_1 = \mp \frac{x_1 x_2 x_3 \dots x_{n-1}}{2}.$$

$x_1, x_2, x_3 \dots x_{n-1}$, being subject to the above condition

$$\varepsilon_1 \int_0^{x_1} \frac{dx}{\sqrt{1+x^n}} + \varepsilon_2 \int_0^{x_2} \frac{dx}{\sqrt{1+x^n}} \dots\dots + \varepsilon_{n-1} \int_0^{x_{n-1}} \frac{dx}{\sqrt{1+x^n}}$$

$$= \text{constant}.$$

Also,

$$\varepsilon_1 \int_0^{x_1} \frac{x^2\,dx}{\sqrt{1+x^n}} + \varepsilon_2 \int_0^{x_2} \frac{x^2\,dx}{\sqrt{1+x^n}} \dots\dots + \varepsilon_{n-1} \int_0^{x_{n-1}} \frac{x^2\,dx}{\sqrt{1+x^n}}$$

$$= \text{constant}.$$

⊦ coefficient of $\frac{1}{x}$ in the development of

$$\frac{x^2}{\sqrt{1+x^n}} \log. \left\{ \frac{1 + \frac{1+c_1 x}{\sqrt{1+x^n}}}{1 - \frac{1+c_1 x}{\sqrt{1+x^n}}} \right\}$$

according to descending powers of x.

Suppose $n = 4$, then the equation

$x^{n-1} - Px + Q = 0$ has only three roots, x_1, x_2, x_3

$$x_1 + x_2 + x_3 = 0$$

$$x_1 x_2 + x_2 x_3 + x_1 x_3 = -\frac{x_1^2 x_2^2 x_3^2}{4}$$

or, $$\frac{1}{x_3} + \frac{1}{x_2} + \frac{1}{x_1} = -\frac{x_1 x_2 x_3}{4}.$$

These are Mr. Talbot's equations of condition in the paper to which I have referred.

Since $$\varepsilon_1 \sqrt{1+x_1^4} = 1 + c_1 x_1$$

$$c_1 = \varepsilon_1 \sqrt{1+x_1^4} - 1 = \frac{x_1 x_2 x_3}{2}$$

$$\frac{x_2 x_3}{2} = \frac{\varepsilon_1 \sqrt{1+x_1^4} - 1}{x_1^2}$$

$$x_2 + x_3 = -x$$

If $x_2 x_3 = 2A$

x_2 and x_3 are found by solving the quadratic

$$x^2 - x x_1 + 2A = 0$$

when x_1 is given.

Making $x_1 = 2$ $\quad \frac{1}{x_1} = \cdot 5 \quad \varepsilon_1 = -1$

I find $x_2 = -2\cdot 88721 \quad \frac{1}{x_2} = -\cdot 346355 \quad \varepsilon_2 = +1$

$x_3 = \cdot 88721 \quad \frac{1}{x_3} = 1\cdot 12713 \quad \varepsilon_3 = -1$

Making $x_1 = 5 \quad \frac{1}{x_1} = \cdot 2 \quad \varepsilon_1 = -1$

I find $x_2 = -5\cdot 38645 \quad \frac{1}{x_2} = -\cdot 185651 \quad \varepsilon_2 = +1$

$x_3 = \cdot 38645 \quad \frac{1}{x_3} = -2\cdot 58765 \quad \varepsilon_3 = -1$

In order to obtain convergent expressions for the integral required, I make

$$\frac{1}{1+x^4} = u, \text{ then}$$

$$\int \frac{dx}{\sqrt{1+x^4}} = u^{\frac{1}{4}} + \frac{3}{4.5} u^{\frac{5}{4}} + \frac{3.7}{4.8.9} u^{\frac{9}{4}} + \frac{3.7.11}{4.8.12.13} u^{\frac{13}{4}} + \&c.$$

If this series be called U, the integral

$$\int \frac{dx}{\sqrt{1+x^4}} = \psi' \frac{1}{0} - U$$

I find from this series for the values of the integral

from $x = 0$ to $x = 2$, $\psi'\frac{1}{0} - \cdot 49695$

............$x = 0$ to $x = -2\cdot 88721$, $-\psi'\frac{1}{0} + \cdot 34586$

............$x = 0$ to $x = 5$, $\psi'\frac{1}{0} - \cdot 199968$

............$x = 0$ to $x = -5\cdot 38645$, $-\psi'\frac{1}{0} + \cdot 185628$

In order to have a convergent series for the value of the integral from $x = 0$ to $x = \cdot 88721$, I make

$$\frac{x^4}{1+x^4} = u$$

$$\int \frac{dx}{\sqrt{1+x^4}} = u^{\frac{1}{4}} + \frac{3u^{\frac{5}{4}}}{4\,.\,5} + \frac{3\,.\,7u^{\frac{9}{4}}}{4\,.\,8\,.\,9} + \frac{3.7.11\,u^{\frac{13}{4}}}{4.8.12.13} + \&c.$$

and I find for the integral in question, the number $+\cdot 84288$.

The integral from $x = 0$ to $x = \cdot 38645$ may be found at once from the expression

$$\int \frac{dx}{\sqrt{1+x^4}} = x - \frac{x^5}{2\,.\,5} + \frac{1\,.\,3\,x^9}{2\,.\,4\,.\,9} - \frac{1\,.\,3\,.\,5\,x^{13}}{2\,.\,4\,.\,6\,.\,13} + \&c.$$

and I obtain for this integral the number $+ \cdot 38561$.

It appears in this case that the constant on the right hand side of the equation equals $-2\,\psi'\frac{1}{0}$, and

$$\cdot 49695 + \cdot 34586 - \cdot 84288 = 0 \text{ nearly*}$$
$$\cdot 19968 + \cdot 185624 - \cdot 38561 = 0 \text{ ——}$$

according to the general theorem. It may be proper to mention, that the transformations adopted in order to procure convergent series for the integrals required, are all taken from Legendre's work.

The equations of condition between the quantities x_1, x_2, x_3 ...... &c., may be varied to an almost indefinite extent: those which I have adopted in the last example are the same which were used by Mr. Talbot, and through which he obtained a

* The theorem is of course rigorous, but it can only be verified approximately in numerical examples.

theorem relative to the sum of three arcs of an equilateral hyperbola.

The coefficient of $\frac{1}{x}$ in the development of

$$\frac{x^2}{\sqrt{1+x^4}} \log \left\{ \frac{1 + \frac{1+c_1 x}{\sqrt{1+x^4}}}{1 - \frac{1+c_1 x}{\sqrt{1+x^4}}} \right\}$$

$=$ coefficient of $\frac{1}{x}$ in the development of

$$\frac{x^2}{x^2}\left\{1 - \frac{1}{2x^4} + \&c.\right\} \log \left\{ \frac{1 + \left\{1 + \frac{x_1 x_2 x_3 x}{2}\right\}\left\{1 - \frac{1}{2x^4} + \&c.\right\}}{1 - \left\{1 + \frac{x_1 x_2 x_3 x}{2}\right\}\left\{1 - \frac{1}{2x^4} + \&c.\right\}} \right\}$$

$$= x_1 x_2 x_3.$$

Hence (see p. 118, line 5,)

$$\epsilon_1 \int \frac{x^2\,dx}{\sqrt{1+x^4}} + \epsilon_2 \int \frac{x^2\,dx}{\sqrt{1+x^4}} + \epsilon_3 \int \frac{x^2\,dx}{\sqrt{1+x^4}} = \text{constant} + x_1 x_2 x_3$$

$$\int \frac{\sqrt{1+x^4}}{x^2}\,dx = -\frac{\sqrt{1+x^4}}{x} + 2\int \frac{x^2\,dx}{\sqrt{1+x^4}},$$

and, since $\frac{\sqrt{1+x^4}}{x} = \frac{1+c_1 x}{x} = \frac{1}{x} + c_1$

$$\epsilon_1 \int_0^{x_1} \frac{\sqrt{1+x^4}}{x^2}\,dx + \epsilon_2 \int_0^{x_2} \frac{\sqrt{1+x^4}}{x^2}\,dx + \epsilon_3 \int_0^{x_3} \frac{\sqrt{1+x^4}}{x^2}\,dx.$$

$$= -\frac{1}{x_1} - \frac{1}{x_2} - \frac{1}{x_3} - \frac{3x_1 x_2 x_3}{2} + 2\,x_1 x_2 x_3 + \text{constant}$$

$$= \frac{3}{4}\, x_1 x_2 x_3 + \text{constant},$$

which is Mr. Talbot's theorem.

In the numerical examples offered in this paper, I have only carried the approximation as far as could be done conveniently by the common tables of logarithms, that is, to six places of decimals.

If L be the logarithm of the number N, $L + x$ the logarithm of the number $N + y$,

$$N = 1 + p\,L + \frac{p^2}{1\,.\,2}\,L^2 + \frac{p^3}{1\,.\,2\,.\,3}\,L^3 + \&c.$$

p being the Napierian logarithm of the base.

$$N + y = N\left\{1 + p\,x + \frac{p^2}{1\,.\,2}\,x^2 + \&c.\right\}$$

$$y = N\,p\,x\left\{1 + \frac{p\,x}{2} + \frac{p^2}{2\,.\,3}\,x^2 + \&c.\right\}$$

$$\log y = \log\,(N\,p\,x) + \log\left\{1 + \frac{p}{2}\,x + \frac{p^2\,x^2}{2\,.\,3} + \&c.\right\}$$

$$= \log\,(N\,p\,x) + \frac{x}{2} + \&c.$$

Also $$x = \frac{1}{p}\left\{\frac{y}{N} = \frac{y^2}{2\,n^2} + \&c.\right\}$$

$\log p = \cdot 36221\ 56887.$

With the help of these expressions, and the table of Brigg's logarithms to sixty-one places of decimals given in Callet, the approximations may be carried much further if desired.

XX. *Experimental Researches in Electricity.—Eighth Series.* *By* Michael Faraday, *D.C.L. F.R.S. Fullerian Prof. Chem. Royal Institution, Corr. Memb. Royal and Imp. Acadd. of Sciences, Paris, Petersburgh, Florence, Copenhagen, Berlin, &c. &c.*

[Continued from p. 45: with an Engraving.]

915. RETURNING to the consideration of the source of electricity (878, &c.), there is another proof of the most perfect kind that metallic contact has nothing to do with the *production* of electricity in the voltaic circuit, and further, that electricity is only another mode of the exertion of chemical forces. It is, the production of the *electric spark* before any contact of metals is made, and by the exertion of *pure and unmixed chemical forces*. The experiment, which will be described further on (956.), consists in obtaining the spark upon making contact between a plate of zinc and a plate of copper plunged into dilute sulphuric acid. In order to make the arrangement as elementary as possible, mercurial surfaces were dismissed, and the contact made by a copper wire connected with the copper plate, and then brought to touch a clean part of the zinc plate. The electric spark appeared, and it must of necessity have existed and passed *before the zinc and the copper were in contact.*

916. In order to render more distinct the principles which