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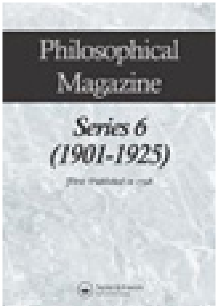

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VII. On electrical vibrations and the constitution of the atom

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The full line curves Q and P show the stress at the bottom and top ends of the rope respectively. The height OP represents 36,000 lb. per sq. inch. The dotted curve P shows what both of those would be according to the approximate solution. In all cases gravitational forces are neglected.

As was to be expected, the approximate solution is quite different from the other when the rope is long, and seems to agree with the other better and better as the rope is shorter. Also it is evident that if internal friction in the rope or any other cause is likely to destroy the discontinuities which we observe at the times of reflexion, then the approximate solution is probably more correct than the other.

VII. *On Electrical Vibrations and the Constitution of the Atom.* *By* Lord Rayleigh, *O.M., F.R.S.**

In illustration of the view, suggested by Lord Kelvin, that an atom may be represented by a number of negative electrons, or negatively charged corpuscles, enclosed in a sphere of uniform positive electrification, Prof. J. J. Thomson has given some valuable calculations† of the stability of a ring of such electrons, uniformly spaced, and either at rest or revolving about a central axis. The corpuscles are supposed to repel one another according to the law of inverse square of distance and to be endowed with inertia, which may, however, be the inertia of æther in the immediate neighbourhood of each corpuscle. The effect of the sphere of positive electrification is merely to produce a field of force directly as the distance from the centre of the sphere. The artificiality of this hypothesis is partly justified by the necessity, in order to meet the facts, of introducing from the beginning some essential difference, other than of mere sign, between positive and negative.

Some of the most interesting of Prof. Thomson's results depend essentially upon the finiteness of the number of electrons; but since the experimental evidence requires that in any case the number should be very large, I have thought it worth while to consider what becomes of the theory when the number is infinite. The cloud of electrons may then be assimilated to a fluid whose properties, however, must differ in many respects from those with which we are most familiar. We suppose that the whole quantities of positive and negative are equal. The difference between

* Communicated by the Author.

† Phil. Mag. vii. p. 237 (1904).

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them is that the positive is constrained to remain undisplaced, while the negative is free to move. In equilibrium the negative distributes itself with uniformity throughout the sphere occupied by the positive, so that the total density is everywhere zero. There is then no force at any point; but if the negative be displaced, a force is usually called into existence. We may denote the density of the negative at any time and place by ρ, that of the positive and of the negative, when in equilibrium, being ρ_0. The repulsion between two elements of negative $\rho d\mathrm{V}$, $\rho' d\mathrm{V}'$ at distance r is denoted by

$$\gamma \,.\, r^{-2} .\, \rho d\mathrm{V} \,.\, \rho' d\mathrm{V}' \quad . \quad . \quad . \quad . \quad . \quad (1).$$

The negative fluid is supposed to move without circulation, so that a velocity-potential (ϕ) exists; and the first question which presents itself, is as to whether there is "condensation." If this be denoted by s, the equation of continuity is, as usual *,

$$\frac{ds}{dt} + \nabla^2\phi = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Again, since there is no outstanding *pressure* to be taken into account, the dynamical equation assumes the form

$$\frac{d\phi}{dt} = \mathrm{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3),$$

where R is the potential of the attractive and repulsive forces. Eliminating ϕ, we get

$$\frac{d^2s}{dt^2} = -\nabla^2\mathrm{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

In equilibrium R is zero, and the actual value depends upon the displacements, which are supposed to be small. By Poisson's formula

$$\nabla^2\mathrm{R} = 4\pi\gamma\rho_0 s \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5),$$

so that

$$\frac{d^2s}{dt^2} + 4\pi\gamma\rho_0 s = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

This applies to the interior of the sphere; and it appears that any departure from a uniform distribution brings into play forces giving stability, and further that the times of oscillation are the same whatever be the character of the

* 'Theory of Sound,' § 244.

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disturbance. It is worthy of note that the constant $(\gamma\rho_0)$ of itself determines a *time*.

In considering the significance of the vibrations expressed by (6), we must remember that when s is uniform no external forces having a potential are capable of disturbing the uniformity.

We now pass on to vibrations not involving a variable s, that is of such a kind that the fluid behaves as if incompressible. An irrotational displacement now requires that some of the negative fluid should traverse the surface of the positive sphere (a). In the interior $\nabla^2 R=0$.

To represent simple vibrations we suppose that ϕ, &c. are proportional to e^{ipt}. By (3) $\nabla^2\phi=0$; and we take (at any rate for trial)

$$\phi=e^{ipt}r^n S_n \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7),$$

where S_n is a spherical surface harmonic of the nth order. The velocity across the surface of the sphere at $r=a$ is

$$d\phi/dr=n\,a^{n-1}e^{ipt}S_n\,;$$

and thus the quantity of fluid which has passed the element of area $d\sigma$ at time t is

$$\rho\int\frac{d\phi}{dr}dt\,.\,d\sigma=\frac{\rho_0 n a^{n-1}}{ip}e^{ipt}S_n\,d\sigma \quad . \quad . \quad . \quad (8).$$

The next step is to form the expression for R, the potential of all the forces. In equilibrium the positive and negative densities everywhere neutralize one another, and thus in the displaced condition R may be regarded as due to the surface distribution (8). By a well-known theorem in Attractions we have

$$R=-\frac{4\pi\gamma\rho_0 n r^n S_n e^{ipt}}{ip(2n+1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

But by (3) this is equal to $d\phi/dt$, or $ipe^{ipt}r^n S_n$. The recovery of $r^n S_n$ proves that the form assumed is correct; and we find further that

$$p^2=\frac{4\pi\gamma\rho_0\,.\,n}{2n+1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

This formula for the frequencies of vibration gives rise to two remarks. The frequency depends upon the density ρ_0, but not upon the radius (a) of the sphere. Again, as n increases, the pitch rises indeed, but approaches a finite limit given by $p^2=2\pi\gamma\rho_0$. The approach to a finite limit as we

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advance along the series is characteristic of the series of spectrum-lines found for hydrogen and the alkali metals, but in other respects the analogy fails. It is p^2, rather than p, which is simply expressed; and if we ignore this consideration and take the square root, supposing n large, we find

$$p \propto 1 - 1/2n,$$

whereas according to observation n^2 should replace n. Further, it is to be remarked that we have found only one series of frequencies. The different kinds of harmonics which are all of one order n do not give rise to different frequencies. Probably the simplicity of this result would be departed from if the number of electrons was treated merely as great but not infinite.

The principles which have led us to (10) seem to have affinity rather with the older views as to the behaviour of electricity upon a conductor than with those which we associate with the name of Maxwell. It is true that the vibrations above considered would be subject to dissipation in consequence of radiation, and that this dissipation would be very rapid, at any rate in the case of n equal to unity*. But this hardly explains the difference between the two views .

The problem of the vibration of electricity upon a conducting sphere has been considered by Prof. Thomson†, but his solution does not appear to me to have the significance usually attributed to it. For the vibration of order 1, the value of p (with the same meaning as above) is

$$p = \frac{V}{a}\left\{\frac{i}{2} + \frac{\sqrt{3}}{2}\right\} \quad . \quad . \quad . \quad . \quad . \quad (11).$$

But the solution corresponding thereto is

$$\frac{e^{ipt}\,e^{i\lambda r}}{\lambda r}\left(i + \frac{1}{\lambda r}\right),$$

where $\lambda = p/V$, V is the velocity of light, and a the radius of the sphere. Considering only the exponential factors, we have

$$e^{ipt}\,e^{i\lambda r} = e^{-\frac{1}{2}(1+i\sqrt{3})(Vt-r)/a} \quad . \quad . \quad . \quad . \quad (12),$$

including the non-periodic factor $e^{\frac{1}{2}r/a}$. Thus, although (12) diminishes exponentially with the time and represents a

* In this case we should have to consider how the positive sphere is to be held at rest.

† Proc. Lond. Math. Soc. xv. p. 197, 1884; 'Recent Researches,' § 312, 1893.

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motion in a sense divergent, the disturbance *increases* exponentially with r; and thus (12) cannot apply to a problem where the disturbance is supposed to originate in the neighbourhood of the sphere.

The analysis of the electrical problem is necessarily rather elaborate, and in illustration it may be well to consider the analogous question for sound. For the term of order zero, the velocity potential ψ_0 of a divergent wave takes the form*

$$\psi_0 = \frac{S_0}{r} e^{ik(at-r)} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13),$$

where a denotes the velocity of sound. In the usual theory the divergent vibration is supposed to be maintained by forces operative at $r=0$ with a prescribed frequency. At present we regard (13) as applicable to the space outside a certain sphere of radius c, whose surface remains at rest, so that the case is that of air vibrating round a solid ball of radius c. The condition to be satisfied at $r=c$ is $d\psi_0/dr=0$; so that

$$1+ikc=0 \quad . \quad . \quad . \quad . \quad . \quad . \quad (14),$$

and

$$\psi_0 = \frac{S_0}{r} e^{-(at-r)/c} \quad . \quad . \quad . \quad . \quad . \quad (15).$$

In like manner for the term of order unity we have

$$r\psi_1 = A \cos\theta \, e^{-ikr}\left(1+\frac{1}{ikr}\right) e^{ikat} \quad . \quad . \quad . \quad (16),$$

and the surface condition gives

$$ikc+2+\frac{2}{ikc}=0 \quad . \quad . \quad . \quad . \quad . \quad (17),$$

whence

$$ikc=-1\pm i \quad . \quad . \quad . \quad . \quad . \quad . \quad (18).$$

When r is great, (16) becomes accordingly

$$\psi_1 = \frac{A\cos\theta}{r} e^{(-1\pm i)(at-r)/c} \quad . \quad . \quad . \quad . \quad (19).$$

Both in (15) and (19) ψ diminishes exponentially with the time but increases exponentially with the distance r. The case is not mended if we start with $e^{ik(at+r)}$. Instead of (15) we then find

$$\psi_0 = \frac{S_0}{r} e^{(at+r)/c} \quad . \quad . \quad . \quad . \quad (20),$$

increasing exponentially with r as before and now also with t.

It does not appear that any solution exists of the kind

* 'Theory of Sound,' § 325.

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sought, unless we introduce another reflecting surface. For the enclosed space thus defined, we find of course undissipated vibrations, and p becomes wholly real.

In the calculation of frequencies given above for a cloud of electrons the undisturbed condition is one of equilibrium, and the frequencies of radiation are those of vibration about this condition of equilibrium. Almost every theory of this kind is open to the objection that I put forward some years ago*, viz. that p^2, and not p, is given in the first instance. It is difficult to explain on this basis the simple expressions found for p, and the constant differences manifested in the formulæ of Rydberg and of Kayser and Runge. There are, of course, particular cases where the square root can be taken without complication, and Ritz † has derived a differential equation leading to a formula of this description and capable of being identified with that of Rydberg. Apart from the question whether it corresponds with anything mechanically possible, this theory has too artificial an appearance to inspire much confidence.

A partial escape from these difficulties might be found in regarding actual spectrum lines as due to *difference tones* arising from primaries of much higher pitch,—a suggestion already put forward in a somewhat different form by Julius.

In recent years theories of atomic structure have found favour in which the electrons are regarded as describing orbits, probably with great rapidity. If the electrons are sufficiently numerous, there may be an approach to steady motion. In case of disturbance, oscillations about this steady motion may ensue, and these oscillations are regarded as the origin of luminous waves of the same frequency. But in view of the discrete character of electrons such a motion can never be fully steady, and the system must tend to radiate even when undisturbed ‡. In particular cases, such as some considered by Prof. Thomson, the radiation in the undisturbed state may be very feeble. After disturbance oscillations about the normal motion will ensue, but it does not follow that the frequencies of these oscillations will be manifested in the spectrum of the radiation. The spectrum may rather be due to the upsetting of the balance by which before disturbance radiation was prevented, and the frequencies will correspond (with modification) rather to the original distribution of electrons than to the oscillations. For example,

* Phil. Mag. xliv. p. 362, 1897; 'Scientific Papers,' iv. p. 345.

† Drude, *Ann.* Bd. xii. p. 264, 1903.

‡ Confer Larmor, 'Matter and Æther.'

if four equally spaced electrons revolve in a ring, the radiation is feeble and its frequency is four times that of revolution. If the disposition of equal spacing be disturbed, there must be a tendency to recovery and to oscillations about this disposition. These oscillations may be extremely slow; but nevertheless frequencies will enter into the radiation once, twice, and thrice as great as that of revolution, and with intensities which may be much greater than the original radiation of fourfold frequency.

An apparently formidable difficulty, emphasised by Jeans, stands in the way of all theories of this character. How can the atom have the definiteness which the spectroscope demands? It would seem that variations must exist in (say) hydrogen atoms which would be fatal to the sharpness of the observed radiation; and indeed the gradual change of an atom is directly contemplated in view of the phenomena of radioactivity. It seems an absolute necessity that the large majority of hydrogen atoms should be alike in a very high degree. Either the number undergoing change must be very small or else the changes must be sudden, so that at any time only a few deviate from one or more definite conditions.

It is possible, however, that the conditions of stability or of exemption from radiation may after all really demand this definiteness, notwithstanding that in the comparatively simple cases treated by Thomson the angular velocity is open to variation. According to this view the frequencies observed in the spectrum may not be frequencies of disturbance or of oscillations in the ordinary sense at all, but rather form an essential part of the original constitution of the atom as determined by conditions of stability.

Terling Place, Witham, Nov. 3.

VIII. *On the Constitution of Natural Radiation.* *By* Lord Rayleigh, *O.M.*, *F.R.S.**

THE expression of Prof. Larmor's views in his paper † "On the Constitution of Natural Radiation" is very welcome. Although it may be true that there has been no direct contradiction, public and private communications have given me an uneasy feeling that our views are not wholly in harmony; nor is this impression even now removed. It may conduce to a better understanding of some of these important and difficult questions if without dogmatism I

* Communicated by the Author.

† Phil. Mag. vol. x. p. 574 (1905).

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