

ART. XVII.—*The Effects of Changes of Temperature on Permanent Magnets*; by HIRAM B. LOOMIS.

THE following paper is an account of some experiments undertaken to determine more accurately, if possible, the kind of change which takes place when a magnet is heated and cooled after it has been brought to the "permanent state." The subject will be considered under the following heads: I. Historical Sketch. II. Experiments and Calculation of Results. III. Discussion of Results.

I. *Historical Sketch.*

A. *Investigations on the change in magnetic moment due to change in temperature.*—About 1825, Kupfer\* magnetized a steel bar and placed it in a water bath at the temperature of the room. Near it he suspended a magnetic needle and determined the period of 300 swings. The bath was then heated to 100 C. and the period of 300 swings was again determined. The bar was then alternately heated and cooled between these limits of temperature and similar observations taken. Kupfer thus learned that if a permanent magnet is heated above its temperature of magnetization its magnetic moment decreases, that on again cooling the moment increases but not enough to make up for the first loss, and that this is true for the first three or four times it is heated and cooled.

In 1851, Lamont† found that when a permanent magnet was alternately heated and cooled fifteen or sixteen times between fixed limits of temperature, it reached a permanent state in which it had a definite magnetic moment for a given temperature, to which it always returned when brought to that temperature, provided only it had never passed beyond the temperature limits mentioned above. The higher the temperature, the smaller was the magnetic moment.

Riess and Moser‡ also experimented on the change in magnetic moment by swinging magnets in the earth's field and determining the period of vibration. For needles 34 lines long they found the following formula held:

$$I' = I [1 - 0.000324 (t' - t)d],$$

in which  $I$  and  $I'$  are the intensities of magnetization at the temperatures  $t$  and  $t'$  on the Reaumur scale, and  $d$  is the diameter of the magnets. For needles two inches long the numerical factor is 0.000432, showing that the proportional change in intensity of magnetization is greater in shorter magnets.

\* Wiedemann's *Electricität*, iii, p. 753.

† Lamont, *Pogg. Ann.*, lxxxii, p. 440, 1851.

‡ Riess and Moser, *Pogg. Ann.*, xvii, p. 425, 1829.

The temperature limits were  $0^{\circ}$  and  $80^{\circ}$  R. By swinging their magnets at different temperatures they found the change in moment proportional to the difference in temperature, as is shown by their formula.

Prof. G. Wiedemann\* has made some careful investigations on the influence exerted by the temper of the steel and the original intensity of magnetization. He used bars  $22^{\text{cm}}$  long and  $1.35^{\text{cm}}$  in diameter. Before they were magnetized these bars were placed alternately in melting snow and boiling water fifteen times, in order to bring the steel itself as far as possible into such a state that alterations in temperature would produce no structural change. The bars were magnetized in a coil at a temperature of  $0^{\circ}$  C. They were then carefully placed in a box of sheet copper before the needle of a magnetometer and the deflection was observed by telescope and scale. The temperatures of  $0^{\circ}$  and  $100^{\circ}$  C. were obtained by means of melting snow and boiling water. His results for magnets that have reached the permanent state show that in case of hard steel magnets the change in moment is nearly proportional to the moment at  $0^{\circ}$  C., while for tempered and soft steel magnets, the ratio of change to the moment at  $0^{\circ}$  C. increases with the moment. As his results give a good idea of the size of the changes under discussion, I append the following table from his paper.

With reference to the theory of these changes Prof. Wiedemann says: "Besides the permanent effect due to an alteration in temperature there is a temporary change. Each heating diminishes the permanent moment of the molecules. Moreover, for the time being, it loosens the particles of the body and lessens the strain in which they have been placed by the action of external forces, therefore they return a little toward their first position of equilibrium, in which they were held by the forces acting between them before the external forces came into play. Heating thus diminishes the magnetization temporarily; but, on cooling, the molecules return to their former position and the lost magnetization is regained. We can produce entirely analogous phenomena if we change the temperature of bodies which have suffered a change of form (torsion) as a result of mechanical forces, and observe the increase and decrease of this on heating and cooling."

Barus and Stronhal† carefully distinguished the mechanical effect of heating from the purely magnetic effect. They found that a temperature of  $20^{\circ}$  or  $30^{\circ}$  C. above that of the water in which a glass-hard steel rod was dipped in hardening produced quite perceptible annealing effects. This change in the hard-

\* G. Wiedemann, Pogg. Ann., c, p. 235, 1852; ciii, p. 563, 1858; cxxii, p. 355, 1864.

† Bull. U. S. G. S., No. 14, p. 151.

$M_0$	$M_{100}$	$M'_0$	$N_0$	$N_{100}$	$\frac{M_0-M_{100}}{M_0}$	$\frac{M_0-M'_0}{M_0}$	$\frac{M_0-N_0}{M_0}$	$\frac{N_0-N_{100}}{N_0}$
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I. Hard steel bar.

71.5	41.5	44.8	37.	33.2	0.420	0.373	0.483	0.103
134.5	89.2	96.	85.5	77.8	0.321	0.286	0.364	0.090
195.	134.3	146.2	133.3	120.	0.311	0.250	0.316	0.100

II. Tempered steel bar.

44.	27.	30.	29.	27.	0.386	0.318	0.341	0.0690
148.5	107.2	114.5	110.3	101.	0.278	0.229	0.257	0.0814
219.5	165.	179.	172.	156.	0.249	0.184	0.216	0.0930
317.	239.	260.7	251.2	226.	0.246	0.178	0.207	0.1003

Soft steel bar, No. 1.

85.	45.		38.	33.2	0.471		0.553	0.126
141.	73.5		68.5	57.	0.479		0.514	0.168
193.	99.		101.	78.5	0.487		0.478	0.223
209.5	109.5		115.	88.2	0.477		0.451	0.233

Soft steel bar, No. 2.

95.5	49.7	54.2	45.	39.	0.479	0.432	0.529	0.133
136.5	73.	81.5	69.	59.	0.465	0.403	0.495	0.145
174.8	92.5	108.3	93.4	76.	0.471	0.378	0.466	0.186

Very soft steel bar which had been heated and slowly cooled many times.

51.5	34.5	37.			0.330	0.282		
80.5	54.5	58.			0.323	0.279		
113.	76.	82.			0.328	0.274		
159.5	103.3	116.5			0.353	0.270		
181.	113.5	131.			0.373	0.277		

$M_0$  is the intensity of magnetization before any change in temperature has taken place;  $M_{100}$ , when first heated to 100° C. :  $M'_0$ , after being again cooled to 0° C.  $N_0$  and  $N_{100}$  are the intensities of magnetization at the temperatures indicated by the subscripts after the magnet has been heated and cooled fifteen times.

ness of the steel would naturally affect the magnetization. According to their experiments, if a glass hard steel rod is thoroughly annealed by being kept at the temperature of boiling water for a day or two and then magnetized to saturation at the temperature of the room, the loss in magnetization on being heated to the boiling point is relatively small and is nearly independent of the time it is kept there. Nearly the whole change takes place during the first ten minutes. On the other hand, if the bar is not first annealed, the change is much larger and is not complete after twenty-two hours heating.

B. Investigations on the effect of change of temperature on

*the distribution of magnetism.*—Kupfer\* determined at two different temperatures the period of vibration of a short needle placed opposite different parts of a long magnet and found the proportional change in distribution greater at the ends than in the middle of the bar. All his measurements were made before the bar had reached the permanent state.

Poloni† measured the distribution at various temperatures, by slipping a coil from different parts of the magnet to such a distance that the magnet exerted practically no effect and measuring the quantity of electricity thus induced. He worked between the temperatures 0° and 200° C., using an oil bath to obtain his high temperatures. The changes were quite regular between 0° and 180° C., but were very large near 190° C. Between 0 and 180 C., he found that the following formula held:

$$M = A[1 + k^{-1} - k^{-x} - k^{(1+x)}]$$

in which  $M$  is the induction in the magnet at a distance  $x$  from one of the ends;  $l$ , the length of the magnet;  $A$ , a quantity depending only on the temperature, while  $k$  is sensibly constant for a given magnet.

## II. *Experiments and Calculation of Results.*

The existence of a permanent state, in which the moment of a magnet increases or diminishes as its temperature falls or rises, being now well established, the reason for this change becomes an interesting subject for investigation. In the hope of obtaining some clue to its real nature, the following experiments were undertaken. They were planned to determine: first, the change in magnetic moment due to change in temperature in bars of the same cross-section but of different lengths; second, the change in distribution due to change in temperature. The experiments will be considered in the above order.

*A. Experiments on the change of the magnetic moment of magnets of different lengths but of the same cross-section.*—Stubb's steel wire of square cross-section, 0.159<sup>cm</sup> square, was cut into lengths of approximately 5.5<sup>cm</sup>, 8.3<sup>cm</sup>, and 22<sup>cm</sup>. The steel was soft and was used of the same temper as purchased. The bars were annealed in boiling water, magnetized to saturation in a coil, and were in the permanent state when used. The period of vibration in the earth's field was determined at 11° and 99° C. A double box of thin sheet zinc was used to keep the magnets at the required temperatures. In the top of the box was a round opening into the interior, in which a cork holding a thermometer and a glass tube was inserted. Through this tube passed a short wire, which supported the magnet and was suspended from the ceiling by cocoon silk. The suspen-

\* Kupfer, Pogg. Ann., xii, p. 133.

† Poloni, Beibl. v, 802. Atti della R. Acad. dei Lincei, v, p. 262, 1881.

sion was thus quite long and was but little exposed to the action of the heat. The bulb of the thermometer was close to the magnet. In the side of the box was another opening, covered with glass, through which a mirror attached to the magnet was observed, and the time of vibration thus determined. The temperature of the space in which the magnets swung was kept quite constant at 11° C. or 99° C., by passing a current of cold water or steam through the space between the two parts of the double box. For the lower temperature city water was used direct from the faucet.

The magnets and the mirror used in observing their vibrations were weighed. The lengths of the magnets were measured at the ordinary temperature of the room, 18° C. The corrected lengths for 11° and 99° C. were obtained by the following formulæ:

$$\begin{aligned} L_{11} &= L(1 - 7 \times 0.000011), \\ L_{99} &= L(1 + 81 \times 0.000011), \end{aligned}$$

in which  $L_{11}$  and  $L_{99}$  are the lengths at the temperatures indicated by the subscripts,  $L$  is the observed length, and 0.000011 is the coefficient of linear expansion of untempered steel. The moments of inertia for the magnets at each temperature were calculated as follows:

$$\begin{aligned} I_{11} &= m \frac{L_{11}^2 + b^2}{12}, \\ I_{99} &= I_{11} \frac{L_{99}^2}{L_{11}^2}, \end{aligned}$$

in which  $m$  is the mass of the magnet and  $b$  its thickness. The mass of the mirror and of the appliances by which it was fastened to the magnet was 0.4245 grms., and calling its radius of gyration 0.2<sup>cm</sup> the moment of inertia due to it was 0.0170, which was added to that of the magnets. No correction was made for the rest of the suspending apparatus; its mass was always less than 0.10 grms., and it consisted principally of a piece of fine wire about 25<sup>cm</sup> long; and its radius of gyration was exceedingly small, being a fraction of the diameter of the wire. The silk suspension was about 3.5 meters long. No allowance was made for the effect of torsion, as the magnet could be turned 360° without producing enough difference in azimuth to be detected by a telescope and scale at a distance of 3 meters. The formulæ for the magnetic moments are:

$$M_{11} = \frac{4I_{11}'\pi^2}{T^2H}, \quad M_{99} = \frac{4I_{99}'\pi^2}{T^2H},$$

in which  $I'$  denotes the total moment of inertia of the system at the temperature indicated;  $T$ , the period of a complete vibration; and  $H$ , the horizontal intensity of the earth's magnetic force.  $H$  was taken as equal to 0.2. It has been deter-

mined by several previous observers in the room in which the work was done; and as there were no local disturbing iron masses, that part of the building being kept free from iron, it may be taken as within one half of one per cent of correct.

*Specimen Calculation.*—Magnet No. 9.

Length at 18° C. ....	16.40 <sup>cm</sup>
Mass .....	3.609 <sup>grms</sup>
Period of vibration at 11° C. ....	10.435 <sup>sec</sup>
Period of vibration at 99° C. ....	10.812 <sup>sec</sup>
$L_{11} = 16.40 (1 - 7 \times 0.000011)$ .....	16.399 <sup>cm</sup>
$L_{99} = 16.40 (1 + 81 \times 0.000011)$ .....	16.414 <sup>cm</sup>
$I_{11} = m \frac{L_{11} + b^2}{12} = \frac{3.609 (268.92 + 0.02)}{12}$ .....	80.883
$I_{99} = I_{11} \frac{L_{99}^2}{L_{11}^2} = 80.883 \frac{269.43}{268.91}$ .....	81.034
$I_{11}' = 80.883 + 0.017$ .....	80.900
$I_{99}' = 81.034 + 0.017$ .....	81.051
$M_{11} = \frac{4I_{11}'\pi^2}{T_{11}^2H} = \frac{4 \times 80.9 \times 9.8696}{108.889 \times 0.2}$ .....	146.64
$M_{99} = \frac{4I_{99}'\pi^2}{T_{99}^2H} = \frac{4 \times 81.051 \times 9.8696}{116.899 \times 0.2}$ .....	136.80

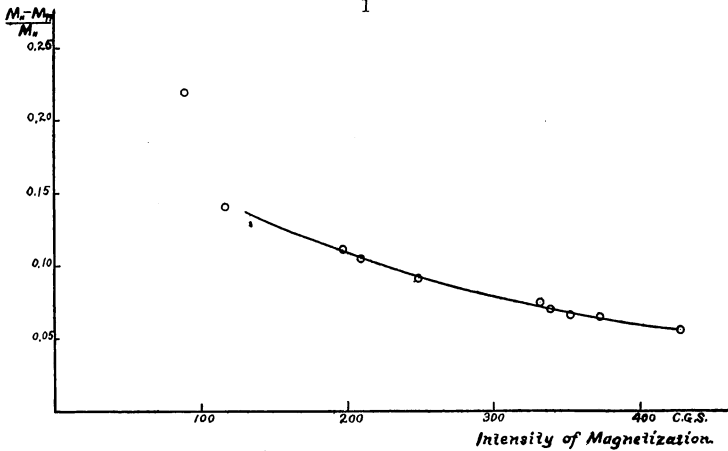
The results of this series of experiments are given in the following table: and figure 1 gives a curve in which  $\frac{M_{11}-M_{99}}{M_{11}}$  is plotted as a function of the intensity of magnetization.

*Magnets of Square Cross-Section.* (0.159<sup>cm</sup> square).

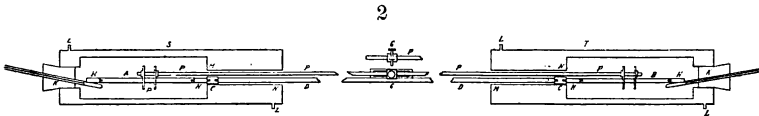
Number of magnet.	Length of magnet.	Moment at 11° C. $M_{11}$	Moment at 99° C. $M_{99}$	$\frac{M_{11}-M_{99}}{M_{11}}$	Intensity of magne- tization.
3	21.53	232.92	219.88	0.056	427
5	22.00	207.08	193.64	0.065	372
9	16.40	146.64	136.80	0.067	352
15	16.31	139.76	129.28	0.075	338
4	11.23	94.76	88.04	0.071	333
6	10.80	72.41	65.72	0.092	249
7	8.15	43.04	38.48	0.105	208
8	8.17	40.48	35.92	0.112	196
13	5.49	16.72	14.44	0.141	117
12	5.40	12.04	9.84	0.220	88

B. *Experiments on the change in distribution due to change in temperature.*—The apparatus employed was suggested by the late Prof. Henry A. Rowland, under whose direction the investigation was conducted, and will be best understood from the diagram in fig. 2. A and B are two cylindrical soft steel magnets (Stubb's steel of the same temper as when purchased) 30.1<sup>cm</sup>

long, and 0.55<sup>cm</sup> in diameter. They were magnetized to saturation in a coil, the magnetic circuit being completed by an iron casting of suitable size and shape. They were then brought to the permanent state by alternate heating and cooling. In both ends of each, holes were bored and threads cut. The depth of these holes in magnet A was 8<sup>mm</sup> at each end. In magnet B



the hole at the south end was 11<sup>mm</sup> deep; that at the north end 7<sup>mm</sup>. In the experiment the magnets were placed perpendicular to the earth's field. Pieces of brass rod, H,H,H,H, of the same diameter as the magnets were screwed into their ends and acted as guides for the two coils (to be described presently), so that after the coils had been slipped off the magnets, they could be slipped back again without trouble. DD is a brass



rod about 1.5 meters long, holding two magnets together in the position shown in the figure. C,C are pieces of non-conducting material to keep the magnets from changing temperature by conducting along the rod DD. E and F are two coils of very fine wire wound on paper tubes which just fit the magnets. They consisted of 150 turns (five layers of 30 turns each), and were about 7<sup>mm</sup> wide. Their frames were joined by a brass rod, PP, of such length that when the coil E was at the center of the magnet B, the coil F was also at the center of the magnet A. By means of this rod they could be moved simultaneously over corresponding portions of the two magnets. At G is a gauge which regulates the distance the coils are moved at a time, so that as they are moved step by step from one end of

the bar to the other, the steps will be of equal length. By loosening a screw the coils may be moved from the middle of the magnets to either end at one step. The cross-sections of two cylindrical double boxes, made of sheet zinc, are indicated at S and T. At K, K are openings in which corks holding thermometers were inserted. At L, L are openings into the spaces between the two parts of the double boxes. Through these a current of steam or cold water was passed to keep the space containing the magnets at the requisite temperature. The temperatures employed were  $14^{\circ}\text{C.}$  and  $99^{\circ}\cdot 5\text{C.}$  City water direct from the faucet was used to produce the lower temperature, and a fairly constant temperature was easily maintained. NM, NM, are other openings by which the magnets were introduced and through which the bars DD and PP passed. They were about  $2\cdot 5\text{cm}$  in diameter and  $20\text{cm}$  long, and were stuffed with cotton, the better to maintain the temperature of the interior. The exploring coils were connected up with an ordinary astatic galvanometer of rather low resistance in such a way that the currents induced by moving them along the magnets opposed each other. An earth inductor and a resistance box were included in the circuit, and in each experiment the galvanometer readings were standardized by the earth inductor. Beginning at the middle of the magnets, the coils were moved step by step to one end, the throw of the needle being observed for each step. The coils were moved so far in the last step that practically no lines of induction passed through them, as was determined by experiment. Similar observations were made for the other half of the magnets. In this way was measured the excess of the number of lines of induction passing from a certain section of one magnet into the air over that passing out of the corresponding section of the other magnet, thus giving the difference of distribution in the two magnets. These measurements are taken first when A and B are both at  $14^{\circ}\text{C.}$ , and again when A is at  $14^{\circ}\text{C.}$  but B at  $99^{\circ}\cdot 5\text{C.}$  The difference between the two sets after they have been reduced to the same scale by the earth inductor readings is evidently the change in distribution in B due to the change in temperature. By this method the quantity observed is of about the same magnitude as the quantity we desire to obtain; the greatest throw of the needle was but little over twice the largest difference obtained on subtracting the two sets of observations.

To get the distribution of magnetism of the bars, the difference in distribution was first measured as above indicated, then the connection of the coils was changed so that the currents induced in the two coils were in the same direction, and the sum of the distributions was obtained in the same way. In



this case extra resistance had to be added from the resistance box to keep the readings on the scale.

*Calculation of Results.*

The formula for the ballistic galvanometer is

$$Q = k \sin \frac{\theta}{2}$$

in which  $Q$  is the quantity of electricity;  $k$ , a constant factor; and  $\theta$ , the angular throw of the needle. The observations were made with telescope and scale. Letting  $d$  represent the observed throw and  $r$  the distance of the mirror from the scale,

$$\tan 2\theta = \frac{d}{r}$$

Expanding  $\sin \frac{\theta}{2}$  we have

$$\sin \frac{\theta}{2} = \frac{d}{2r} \left[ 1 - 0.344 \left( \frac{d}{r} \right)^2 + \right]$$

This formula was used in reducing the large readings.

The earth inductor readings were taken frequently and varied but little throughout the experiment. The throw due to the earth inductor when both magnets were at  $14^{\circ}$  C. was 39.5 scale divisions; when one magnet was at  $99.5^{\circ}$  C. and the other at  $14^{\circ}$  C., it was 37.8. Corresponding throws of the needle due to the slipping of the exploring coils over various portions of the magnets were averaged. The average throws taken when one magnet was hot and the other cold was multiplied by  $\frac{39.5}{37.8} = 1.045$  to reduce to the same scale as the readings taken when both magnets were at  $14^{\circ}$  C. Corresponding measurements were then subtracted to get the difference of distribution caused by change in temperature in terms of the scale divisions. The reduction to absolute measurement was as follows: The effective area of the earth inductor, as determined by previous observers, was 20,716<sup>sq</sup> cm. The total number of lines of induction cut by turning the earth inductor was  $2HA = 8,286.4$ . The throw was 39.5 scale divisions, therefore each scale division of throw caused by the movement of the exploring coils corresponded to  $8,286.4 \div (39.5 \times 150) = 1.398$  C.G.S. lines of induction. The factor 150 is due to the 150 turns of the exploring coils. The change in distribution as given in scale divisions was then multiplied by 1.398, giving the change in distribution in C.G.S. lines of induction for each 2.17<sup>cm</sup> of length, that being the distance the coils were moved at each step. In determining the difference of distribution the angles observed were quite small; the largest was

less than  $2^\circ$ , making the angular throw of the needle less than one degree. The deflections in scale divisions were therefore taken as proportional to  $\sin \frac{\theta}{2}$ . The error in case of the largest reading would not exceed one part in 3,500; and as the results are obtained by subtraction of two throws, the error in the result may be neglected.

*Specimen Calculation.*

Magnet A.—Middle to North End.

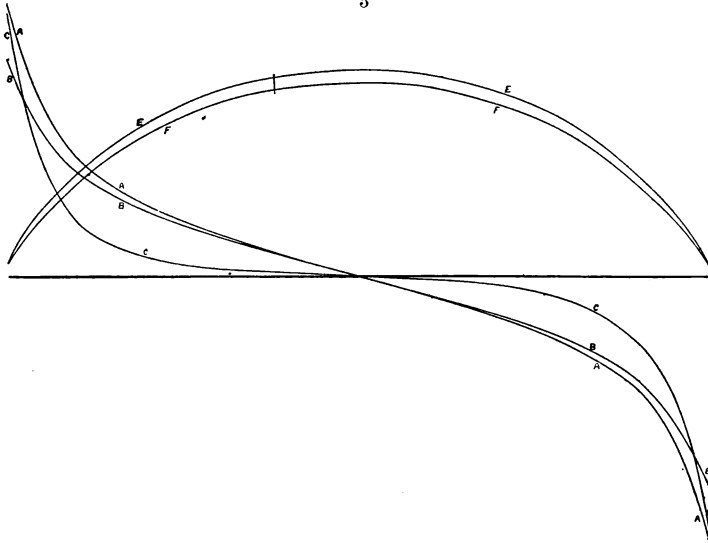
Steps.	Average observed throw at 14° C.	Average observed throw at 99°·5 C.	Corrected throw at 99°·7 C.	Difference in distribu- tion in scale divisions.	Difference in distribu- tion in C.G.S. lines of inductions.
I	18·0	17·5	18·3	— 0·3	— 0·4
II	18·4	18·1	18·9	— 0·5	— 0·7
III	11·8	12·6	13·2	— 1·4	— 2·0
IV	— 7·2	— 4·2	— 4·4	— 2·8	— 3·9
V	— 34·3	— 27·2	— 28·4	— 5·9	— 8·3
VI	— 42·0	— 27·7	— 28·9	— 13·1	— 18·3
VII	— 49·0	— 19·4	— 20·3	— 28·7	— 40·2
End	— 5·4	— 2·2	— 2·3	— 3·1	— 4·3

In getting the sum of the distributions it was found necessary to make use of the formula on p. 187, because the angles were too large to take  $\sin \frac{\theta}{2}$  proportional to  $\tan 2\theta$ . The corrected readings were reduced to absolute measure in the way just described. Thus we have the sum and difference of the linear distribution of the two magnets in absolute measure. One half the sum plus one half the difference gives the distribution of one magnet; and one half the sum minus one half the difference gives that of the other. The distribution at the higher temperature was obtained by subtracting the change in distribution due to the heating from the distribution at the lower temperature. The induction at each point of the magnet was obtained by adding up the number of lines of induction passing out of the magnet beyond that point.

Tables of the results for magnets A and B are given on pages 192 and 193. The first column gives the distance of the exploring coils from the centers of the magnets at the end of each step. The second and fourth columns give the number of C.G.S. lines of induction passing out from the magnet at 14° C. and 99°·5 C. respectively in the step of the coil shown in the first column. The third column gives the change in distribution. The fifth and sixth columns give the magnetizations at the two temperatures, i. e., the number of C.G.S. lines of induction per square centimeter of cross-section passing through the mag-

net at the point indicated. In these columns two values are given for the middle point of the magnet, calculated from the two ends, and serve to indicate the degree of accuracy attained. The second and third columns, from which all the others are calculated, give the means of at least five or six separate determinations which agree well among themselves. The results were further checked by slipping the coils from the middle of the magnets clear off each end at both temperatures. The variation between this measurement and the others was always less than one half of one per cent. This was considered quite

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good, as it was impossible to slip the coils over this whole distance as quickly as they were slipped over the small divisions. In figures 3 curves for magnet A are given as follows: AA is the distribution curve at  $14^{\circ}$  C. BB is the distribution curve at  $99^{\circ}.5$  C. CC is a curve showing the change in distribution due to change in the temperature ( $14^{\circ}-99^{\circ}.5$  C.). The scale of ordinates is ten times that in AA and BB. EE is a curve giving the induction in the magnet at  $14^{\circ}$  C. FF is a curve giving the induction in the magnet at  $99^{\circ}.5$  C.

The tables on pages 192 and 193 together with the curves for distribution give us sufficient data to calculate the magnetic moments of the magnets A and B at both temperatures. In the tables we are given  $n$ , the number of lines of induction issuing from little divisions of the bar throughout its length, as well as the distance  $d$  of these divisions from the center of the magnet. A first approximation to the moment is given by the formula

$$M = \frac{1}{4\pi} \sum nd.$$

To this value the following correction was added: If AB in figure 4 is the length of one of these divisions of the magnet, and CD a part of the distribution curve supposed to be straight, then the area ABDC represents the number of lines of induction issuing from the magnet in the length AB. Let F be the center of gravity of the triangle CED. It is evident that the portion of the distribution represented by the triangle should be multiplied by the abscissa of F, not of H, therefore a correction  $\sum \text{area CED} \times GH$  was added to the summation already given. From this calculation the following results were obtained:

	Magnets.	A.	B.
$M_{14}$	Moment at 14° C.	2298	2060
$M_{99.5}$	Moment at 99° 5 C.	2140	2018
$M_{14} - M_{99.5}$	Loss.	158	142
$\frac{M_{14} - M_{99.5}}{M_{14}}$	Proportional loss.	0.0687	0.0689
	Intensity at 14° C.	322	289
	Intensity at 99° 5 C.	300	283

The magnetic moments were also determined by the method used in the first part of this investigation with the following results:

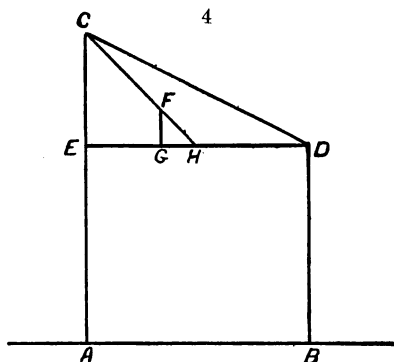
	Magnets.	A.	B.
$M_{14}$	Moment at 14° C.	2359	2091
$M_{99.5}$	Moment at 99° 5 C.	2197	1947
$M_{14} - M_{99.5}$	Loss.	166	146
$\frac{M_{14} - M_{99.5}}{M_{14}}$	Proportional loss.	0.0686	0.0687
	Intensity at 14° C.	339	298
	Intensity at 99° 5 C.	316	277

The difference between these two sets of values is considerable, amounting to two per cent in the case of magnet A. This may be due to the fact that only an approximation could be made to the moments of inertia of the magnets because of the holes in the ends, where a slight error would affect the result materially, as the distance from the center was above 15<sup>cm</sup>. The moments of inertia were calculated by dividing the magnet into two parts, an inner core and an outer shell extending beyond the core at both ends. On the other hand, it is to be noticed that the ratios  $\frac{M_{14} - M_{99.5}}{M_{14}}$  differ by less than one part in 300. In this ratio the moment of inertia of the magnet is eliminated.

In his paper on magnetic distribution, Prof. Rowland gives the following formula for the linear distribution in a magnet

$$\lambda = \frac{\oint (e^{rx} - e^{-rx})}{4\pi\sqrt{RR'}(e^{rb} + e^{-rb})},$$

in which  $R$  is the resistance of unit length of the rod,  $R'$  is the resistance of the medium along unit length of the rod,  $2b$  is the length of the magnet,  $x$  is the distance from the center of the magnet, and  $r = \sqrt{\frac{R}{R'}}$ .



At  $14^{\circ}$  C. the formula for magnet A may be written

$$4\pi\lambda = 167 \frac{e^{0.125x} - e^{-0.125x}}{e^{0.125b} + e^{-0.125b}};$$

at  $99^{\circ}.5$  C.,

$$4\pi\lambda = 161 \frac{e^{0.1242x} - e^{-0.1242x}}{e^{0.1242b} + e^{-0.1242b}}.$$

### III. Discussion of Results.

The first series of experiments shows that the proportional change in magnetic moment due to change in temperature is greater for short than for long magnets; and that the magnet having the greater intensity of magnetization suffers the less proportional change. This last result is not in agreement with the observations of Wiedemann given on page 180. A little consideration will show, however, that Wiedemann's method cannot be relied upon to give accurate results for the change in magnetic moment. He placed a magnetometer needle before the magnet and observed the deflection of the needle for two temperatures of the magnet. If the center of magnetic attraction had remained at the same point of the magnet during the

temperature changes, his method would have given correct results; but, as will appear from the second part of this investigation, the ends lose a greater proportion of their magnetism than does the middle, and therefore the center of magnetic

*Magnet A.*

Distance from center of magnet.	Distribution at 14° C.	Change in distribution.	Distribution at 99°·5.	I <sub>14</sub> Induction at 14° C.	I <sub>99·5</sub> Induction at 99°·5.	$\frac{I_{14}-I_{99·5}}{I_{14}}$
	70·1	— 4·3	65·8			
15·20	428·5	— 40·2	388·3	296·	278·	·060
13·03	285·4	— 18·3	267·1	2106·	1918·	·089
10·86	204·9	— 8·3	196·6	3309·	3044·	·080
8·68	146·8	— 3·9	142·9	4175·	3875·	·071
6·51	91·2	— 2·0	89·2	4795·	4479·	·065
4·34	56·6	— 0·7	55·9	5180·	4586·	·062
2·17	18·6	— 0·4	18·2	5419·	5092·	·060
0	— 17·9	0·	— 17·9	{ 5498· 5453·	{ 5169 } 5126 }	·060
2·17	— 55·5	+ 0·4	— 55·1	5377·	5050·	·060
4·34	— 96·9	+ 2·5	— 94·4	5143·	4818·	·063
6·51	— 142·1	+ 3·2	— 138·9	4734·	4420·	·066
8·68	— 199·9	+ 6·2	— 193·7	4134·	3834·	·072
10·86	— 276·3	+ 14·8	— 261·5	3289·	3015·	·083
13·03	— 432·6	+ 39·2	— 393·4	2122·	1910·	·100
15·20	— 69·7	+ 11·2	— 58·5	294·	247·	·159

attraction travels towards the middle of the magnet as the temperature rises. Since the attraction varies inversely with the square of the distance, and since the quantities to be determined are obtained by subtracting the observed quantities, it is evident that the change in magnetic distribution renders Prof. Wiedemann's method inaccurate. This error does not enter in the present investigation, in which the magnetic moments were

determined by swinging the magnets in the earth's field. The results obtained are summed up in the table on page 184.

The second series of experiments shows that the proportional change in distribution of magnetism due to change of tempera-

Magnet B.

Distance from center of magnet.	Distribution at 14°	Change in distribution.	Distribution at 99°·5.	I <sub>14</sub> Induction at 14°.	I <sub>99·5</sub> Induction at 99°·5.	$\frac{I_{14}-I_{99·5}}{I_{14}}$
	61·0	— 3·9	+ 57·1			
15·20	359·5	—39·6	+319·9	258·	242·	·062
13·03	227·6	—15·1	+212·5	1777·	1594·	·103
10·86	159·7	— 6·0	+153·7	2738·	2491·	·090
8·68	139·4	— 3·4	+136·0	3413·	3141·	·079
6·51	108·1	— 2·1	+106·0	4002·	3716·	·071
4·34	81·4	— 1·5	+ 80·0	4458·	4163·	·066
2·17	43·5	— 0·1	+ 43·4	4802·	4501·	·062
0				{ 4986· 4950·	{ 4685· 4650·	·060
	3·6	+ 0·7	+ 4·3			
2·17	— 36·2	+ 1·1	— 35·1	4965·	4668·	·060
4·34	— 79·8	+ 2·0	— 77·8	4812·	4520·	·060
6·51	—128·3	+ 2·4	—125·9	4475·	4191·	·063
8·68	—188·6	+ 6·6	—182·0	3933·	3659·	·069
10·86	—266·3	+19·0	—247·3	3136·	2890·	·078
13·03	—406·4	+35·4	—371·0	2011·	1845·	·082
15·20	— 69·7	+ 4·1	— 65·6	294·	277·	·057

ture is greatest at the ends and least in the middle of the magnet. A glance at the tables given above, or at the curves in fig. 3, will show this. This is different from the result obtained by Poloni, who found the proportional change sensibly constant throughout the magnet. It could not be expected that the small difference noticed here could be detected by his method, which consisted in measuring two large quantities and

subtracting them in order to obtain a small difference. In the method employed in this research the quantities measured differed but little in size from the quantities desired, and much greater accuracy was easily obtained.

The following is suggested as the explanation; Prof. Ewing has recently made an important addition to Weber's theory of magnetism, in claiming that the forces which hold the little molecular magnets in position are largely the mutual attractions and repulsions of these molecular magnets among themselves. In applying Ewing's theory to the case in hand let us consider a row of magnetic molecules ABC, etc.

A B C                  H I J K L

J is held in position by the action of H, I, etc. on the one side and K, L, etc. on the other side, while A has only B, C, etc. to act upon it. It is evident that the force holding J in position is greater than that acting on A. Suppose the bar of which this line of molecules is a part is heated. If in this process the energy of vibration of A and J receive equal increments, it is evident that the increase in the amplitude of vibration of A will be greater than that of J. The magnetic moment contributed by each molecule is the moment of the molecule resolved along the direction of magnetization of the bar. The moment contributed by A would therefore suffer a larger proportional loss on heating than that contributed by J, and so the loss would be greatest at the ends. This explanation will also account for the fact that the proportional change in magnetic moment is greater in short magnets, because in short magnets the end regions of these lines of molecular magnets will naturally form a larger part of the whole line than in long magnets. We should also expect these lines of molecular magnets to be longer when the intensity of magnetization was greater. This would account for the fact that the change in magnetic moment due to temperature changes is less the greater the intensity of magnetization. This appears from the curve given in fig. 1. There are other facts which point in the same direction: When a magnet is heated before it has reached the permanent state, Kupfer found, as already stated, that the proportional permanent loss was greatest at the ends. In some rough tests I have made on this point, heating the bar almost to redness, I have found the permanent proportional loss at the ends nearly twice as great as at points near the center of the magnet. This would naturally follow from Ewing's theory, for the force holding the end magnetic molecules in position being less, they would be more easily set into such violent vibration as to swing out of one position of equilibrium into another.

Western Springs, Ill.