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65. Chords of Quickest and Slowest Descent from One Circle to Another, Both Circles Being in a Vertical Plane, and Not Intersecting

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axes of coordinates, the equations to the hyperbola and parabola are

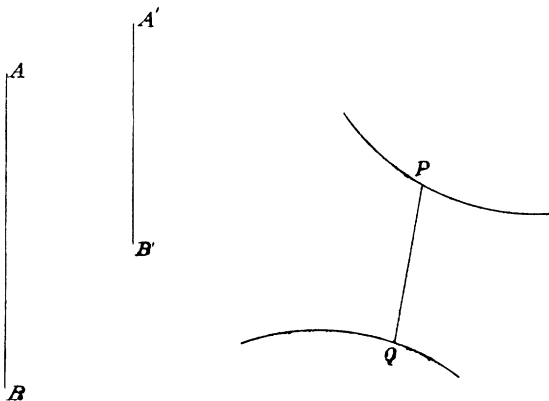
$$a/x + b/y = 1 \quad \text{and} \quad \sqrt{x/a} + \sqrt{y/b} = 1;$$

and if any point  $x', y'$  on the hyperbola be taken for the vertex of a circum-inscribed triangle, the equation to the opposite side of the triangle is

$$xx'/aa' + yy'/bb' = 1.$$

We have thus a fairly simple method for investigating the properties of such triangles. F. S. MACAULAY.

65. *Chords of quickest and slowest descent from one circle to another, both circles being in a vertical plane, and not intersecting.*



As is well known, if  $PQ$  be a chord of quickest or slowest descent from one curve to another in the same vertical plane, the verticals through  $P, Q$  and the normals at  $P, Q$  form a rhombus.

Further, it is easy to see that, if the centres of curvature at  $P$  and  $Q$  both lie outside the verticals, the time in  $PQ$  is a minimum, and if they both lie inside, the time in  $PQ$  is a maximum; while the time is neither a maximum nor a minimum if neither of these conditions is fulfilled.

Let  $A, A'$  be the highest,  $B, B'$  the lowest points of the two circles. Join  $AA', AB', BA', BB'$ . Let  $P, Q$  be the points in which one of these chords cuts the circles again. Then the normals and verticals at  $P, Q$  form a rhombus, as may be seen at once by noting that  $PQ$  always passes through a centre of similitude.

By applying the criterion of the positions of the centres of curvature, we discriminate as follows:

- (1) If the circles are external to each other, the chords through the centre of *inverse* similitude alone give solutions, and
  - (a) If  $B'$  is above  $A$ , the chord  $A'B$  gives a minimum, the chord  $B'A$  a maximum time of descent from the circle  $A'B'$  to the circle  $AB$ .
  - (b) If  $B'$  is below  $A$ , but  $A'$  above  $B$ , the same chords give two minimum solutions, one from the circle  $AB$  to the circle  $A'B'$ , one from the circle  $A'B'$  to the circle  $AB$ .
- (2) If one circle is inside the other, the chords through the centre of *direct* similitude alone give solutions, both minimum, one from the outer circle to the inner, one from the inner circle to the outer.

H. A. ROBERTS.