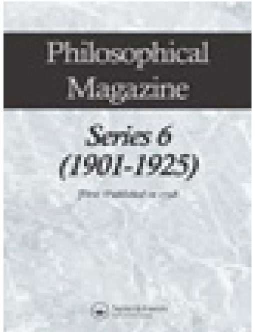

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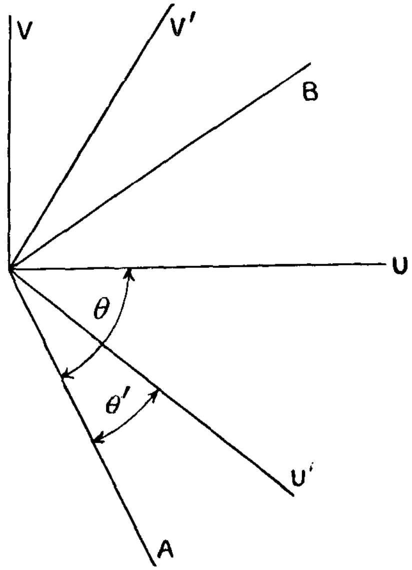
XXI. *On Ellipsoidal Lenses.*

By R. J. Sowter, *B.Sc., A.R.C.Sc.**

THIS note extends the treatment of thin ellipsoidal or astigmatic lenses, by the author's method †, and gives a simple solution for complex problems of the following types :—"To determine the astigmatic pencil, after refraction of an astigmatic pencil by an ellipsoidal lens." And "to find the ellipsoidal lens equivalent to two cylindrical lenses placed a definite distance apart, with their axes inclined at any angle." The method of treatment can be applied to crossed ellipsoidal lenses, in contact, or separated, and is applicable in general to astigmatic pencils.

It has been shown by Prof. S. P. Thompson ‡, and by the author independently, that for obliquely-crossed cylindrical lenses an important double-angled parallelogram of powers can be constructed. This parallelogram is associated with the resolution of a lens-power P into the two powers $P \cos^2 \theta$, $P \sin^2 \theta$, at right angles. It is here "generalized."

Fig. 1.



If an astigmatic pencil having two focal lines at distances $u\ v$ from an ellipsoidal lens of focal powers A, B is refracted by the lens, which is further defined in position with respect

* Communicated by the Physical Society: read April 14, 1905.

† "On Astigmatic Lenses," Proc. Phys. Soc. vol. xvii.

‡ Proc. Phys. Soc. vol. xvi., and Phil. Mag. March 1900.

to the focal lines by θ, then by resolving along two directions at right angles and adopting the curvature notation, we get :—

$$-(U\cos^2\theta+V\sin^2\theta)+U'\cos^2\theta'+V'\sin^2\theta'=A\ . \quad \text{(i.)}$$

$$-(U\sin^2\theta+V\cos^2\theta)+U'\sin^2\theta'+V'\cos^2\theta'=B\ . \quad \text{(ii.)}$$

where

$$U=\frac{1}{u}, \quad V=\frac{1}{v},$$

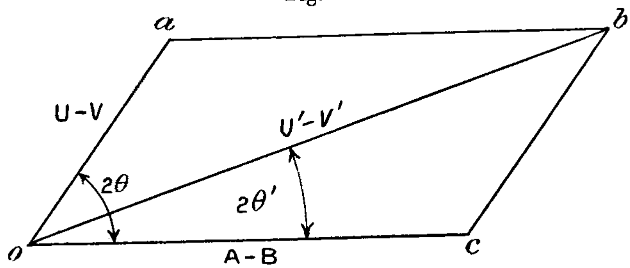
and U', V', θ' define the focal lines in the refracted pencil. Fig. 1 shows the orientation of the powers, or principal curvatures, with respect to each other. Equations (i.) and (ii.) give :—

$$-(U+V)+U'+V'=A+B \quad . \quad . \quad . \quad \text{(I.)}$$

$$-(U-V)\cos 2\theta+(U'-V')\cos 2\theta'=A-B\ . \quad \text{(II.)}$$

This last equation shows that the power parallelogram *o a b c*, shown in fig. 2, has its sides equal to $(U-V)$, $(A-B)$, its

Fig. 2.



angle 2θ, the diagonal $(U'-V')$, and the inclination of the diagonal $2\theta'$. The parallelogram affords the equation .—

$$\frac{U'-V'}{U-V}=\frac{\sin 2\theta}{\sin 2\theta'}. \quad . \quad . \quad . \quad . \quad \text{(III.)}$$

Equations I., II., III., or the parallelogram *o a b c* together with the simple equation (I.), give a complete solution. The signs to be adopted are those that are usual, and such as to make

$$-(U+V)+U'+V'=A+B.$$

The usual *lengthy* solution by the use of the characteristic function resolves itself into that given by Herman* employing Malus' theorem.

If a parallel pencil fall upon the first lens of a crossed and separated cylindrical combination, the equations (i.) (ii.) are

$$-U\cos^2\theta+U'\cos^2\theta'+V'\sin^2\theta'=A,$$

$$-U\sin^2\theta+U'\sin^2\theta'+V'\cos^2\theta'=0,$$

* Geom. Optics, § 174.

where U is the equatorial curvature of the cylindrical wave at the second lens, power A, θ is the angle of crossing of the lenses, and U' V' θ' define the refracted pencil, and consequently the equivalent ellipsoidal lens.

The sides of the parallelogram are A and U.

The following experiments were performed :—

Experiment I. Light from a small aperture having vertical and horizontal cross-wires was brought to a parallel beam by a convex lens, and the beam fell on a convex cylindrical lens of focal length 18·8 cms. supported with its axis vertical. At 8 cms. from this lens another convex cylindrical lens of focal length 25·3 cms. was supported with its axis inclined at 60° to the horizontal. The focal lines were at 8·1 cms., and 140 cms. from the second lens, and the near line was inclined at 21° to the axis of the inclined lens. Equation (I.) gives :—

$$U' + V' = -\cdot 132,$$

and equations II. and III., or the parallelogram, give :—

$$(U' - V') \cos 2\theta' = A + U \cos 2\theta = -\cdot 0858$$

$$(U' - V') \sin 2\theta' = U \sin 2\theta \qquad = -\cdot 0802$$

from which

$$U' - V' = -\cdot 118$$

$$\tan 2\theta' = \ \cdot 935$$

that is,

$$U' = -\cdot 125 \qquad (u' = \ -8 \text{ cms.})$$

$$V' = -\cdot 007 \qquad (v' = -143 \text{ cms.})$$

$$\theta' = 21\tfrac{1}{2}°,$$

values in agreement with those observed.

Experiment II. A beam diverging from a circular aperture having vertical and horizontal cross-wires was made parallel, and fell upon a concave cylindrical lens ($f = 18 \cdot 8$ cms.) supported with its axis vertical. At 10 cms. from this lens, two convex cylindrical lenses were placed in contact and crossed at right angles. Their axes were at 60° and 30° respectively to the horizontal, and their focal lengths were 18·8 cms. and 25·3 cms. The focal lines were at 23 cms. and 70 cms. from the crossed cylinders, and the inclination of one focal line to one of the cylinder axes was 41°.

Equations (i.) and (ii.) give at once :—

$$-U \cos^2 \theta + U' \cos^2 \theta' + V' \sin^2 \theta' = A$$

$$-U \sin^2 \theta + U' \sin^2 \theta' + V' \cos^2 \theta' = B,$$

and the parallelogram, the sides being A − B and U, gives at once :—

$$(U' - V') \sin 2\theta' = U \sin 2\theta,$$

from which we get:—

$$U' + V' = -\cdot 058$$
$$U' - V' = \cdot 0302$$
$$\tan 2\theta' = 8\cdot 11$$

or

$$U' = -\cdot 0139 \qquad (u' = -72 \text{ cms.})$$
$$V' = -\cdot 0441 \qquad (v' = -23 \text{ cms.})$$
$$\theta' = 41\tfrac{1}{2}^\circ.$$

This method of power resolutions, as also the associated "generalized" parallelogram, can only be applied to astigmatic problems in which the wave-fronts after refraction are paraboloidal, or are ellipsoidal and can be assumed paraboloidal in the neighbourhood of some point.

Sturm appears to have been the first to show that all the normals to a paraboloid in the neighbourhood of a point converge to or diverge from two lines at right angles to one another.

XXII. *The Properties of Radium in Minute Quantities.*

To the Editors of the Philosophical Magazine.

GENTLEMEN,—

IN connexion with Mr. Eve's paper on the above subject, the following fact may be of interest.

In December 1903 I prepared a solution by dissolving 1 milligram of a preparation of radium—stated to contain $\frac{1}{1000}$ of its weight of radium barium bromide—in a few c.c. of water. Some of the solution was poured upon a glass plate about 60 sq. cms. in area and the water evaporated. A film was left behind which gradually increased in activity, and at the present time, seventeen months after preparation, is quite as active as it was a week after preparation, and will still excite luminescence on a screen. No sort of care has been taken of the plate. A piece of glass ·25 sq. cm. in area was cut from the plate today and its radioactive power tested and compared with that of 25 grams of fairly good pitchblende. The quantity of active salt upon this area could never have been more than $\frac{1}{200,000}$ milligram, and yet after seventeen months this small amount is at least 20 times as active as the 25 grams of pitchblende.

Yours faithfully,

W. A. DOUGLAS RUDGE.

Woodbridge School, Suffolk,
May 17, 1905.