

cavities in the interior of the basaltic rocks on this coast, though they are frequent on the surface exposed to the air.

THE last variety of whinstone enumerated by Dr RICHARDSON is the Ochrous, which makes, as he says, a conspicuous figure in the stupendous precipices along the coast of Antrim. It is disposed in extensive strata of every thickness, from an inch to twenty-four feet, and varies in colour, from a bright minium to a dull ferruginous brown.

THREE remarks are made by Dr RICHARDSON, that are undoubtedly of importance, and show that this stone is merely basalt in a certain state of decomposition.

1. THE ochrous strata are extensive; they remain always parallel to the basalt strata which they separate; they unite to the basalt without interrupting its solidity; the change from the one to the other is sudden, and the lines of demarkation are distinct. The ochrous stone is never found but contiguous to other basalt.

2. THE substances imbedded in the ochrous rock, and in basalts, are exactly the same; calcareous spar, zeolite, chalcedony, &c.

3. AMONG the varieties which this rock presents, there may be found every intermediate stage between found basalt and perfect ochre. The change is often partial, beginning with veins and slender ramifications.

A L G E B R A.

RULE for reducing to a Continued Fraction the Square Root of any given Integer Number, not a Square. By JAMES IVORY, Esq. Communicated 10th January 1801.

1. LET N be the given integer number, and take n the root of the square next less than N ; and, for the sake of uniformity, put $P^{\circ} = 1$, $R^{\circ} = 0$, $\mu = n$.

2. TAKE

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Rule for reducing a square root to a continued fraction.

2. TAKE $P' = N - n^2$ for a divisor, and $2n - R^{\circ} = 2n$ for a dividend. Let the quotient be μ' , and remainder R' .

3. TAKE $P'' = P' - (R^{\circ} - R')$ μ' , for a divisor, and $2n - R'$ for a dividend. Let the quotient be μ'' and the remainder R'' .

4. TAKE $P''' = P' - (R' - R'') \times \mu''$ for a divisor, and $2n - R''$ for a dividend. Let the quotient be μ''' , and the remainder R''' .

5. THESE operations may be continued without end; the divisor P^{ρ} being found from the formula $P^{\rho} = P^{\rho-2} - (R^{\rho-2} - R^{\rho-1}) \times \mu^{\rho-1}$: the corresponding dividend being $2n - R^{\rho-1}$; and the quotient of the division being denoted by μ^{ρ} , and the remainder by R^{ρ} . But it will only be necessary to continue these operations till we arrive at a value $P^{\rho} = P^{\circ} = 1$, which will always necessarily be the case. After this, the series of numbers, $\mu^{\rho+1}, \mu^{\rho+2}, \mu^{\rho+3}$, will necessarily be the same as the numbers $\mu', \mu'', \mu''', \&c.$; μ^{ρ} , continually repeated in their order.

THE rule may be shortly expressed in algebraic language, thus:

$$\begin{array}{ll}
 P^{\circ} = 1; & \mu = n \times 1 + R^{\circ} = n; \\
 P' = N - n^2; & 2n - R^{\circ} = 2n = P' \times \mu' + R'; \\
 P'' = 1 - \mu'(R^{\circ} - R') = 1 + \mu'R'; & 2n - R' = P'' \times \mu'' + R''; \\
 P''' = P' - \mu''(R' - R''); & 2n - R'' = P''' \times \mu''' + R'''; \\
 P^{iv} = P'' - \mu'''(R'' - R'''); & 2n - R''' = P^{iv} \times \mu^{iv} + R^{iv}; \\
 \text{and so on.} &
 \end{array}$$

HAVING thus found the numbers, $\mu, \mu', \mu'', \mu''', \mu^{iv}$, we shall have the continued fraction sought,

$$\sqrt{N} = \mu + \frac{1}{\mu' + \frac{1}{\mu'' + \frac{1}{\mu''' + \frac{1}{\mu^{iv}} + \&c.}}}$$

And the fraction may be continued indefinitely, by repeating the denominators $\mu', \mu'', \dots \mu^{\rho}$, continually in their order.

EXAMPLE

EXAMPLE I. To reduce the square root of 13 to a continued fraction.

THE operation will be as under :

$$N = 13, n = 3, 2n = 6.$$

$$\begin{aligned} P^0 &= 1, \mu = 3, R^0 = 0. \\ P' &= 13 - 9 = 4; \quad \frac{6-R^0}{4} = \frac{6}{4} = 1 + \frac{2}{4}; \quad P' = 4, \mu' = 1, R' = 2. \\ P'' &= 1 + \mu'R' = 1 + 2 = 3; \quad \frac{6-R'}{3} = \frac{4}{3} = 1 + \frac{1}{3}; \quad P'' = 3, \mu'' = 1, R'' = 1. \\ P''' &= 4 - 1 \times (2 - 1) = 3; \quad \frac{6-R''}{3} = \frac{5}{3} = 1 + \frac{2}{3}; \quad P''' = 3, \mu''' = 1, R''' = 2. \\ P^{IV} &= 3 - 1 \times (1 - 2) = 4; \quad \frac{6-R'''}{4} = \frac{4}{4} = 1 + \frac{0}{4}; \quad P^{IV} = 4, \mu^{IV} = 1, R^{IV} = 0. \\ P^V &= 3 - 1 \times (2 - 0) = 1; \quad \frac{6-R^{IV}}{1} = \frac{6}{1} = 6 + \frac{0}{1}; \quad P^V = 1, \mu^V = 6, R^V = 0. \end{aligned}$$

Here I stop, because, $P^V = P^0 = 1$; and I conclude that the fraction sought is formed by the numbers, $\mu', \mu'', \mu''', \mu^{IV}, \mu^V$; that is, by the numbers 1, 1, 1, 1, 6, continually repeated in their order. Thus,

$$\sqrt{13} = 3 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{6 + \frac{1}{1}}}}}} \&c.$$

EXAMPLE II. To reduce $\sqrt{61}$ to a continued fraction :

$$N = 61, n = 7, 2n = 14.$$

$$\begin{aligned} P^0 &= 1, \mu = 7, R^0 = 0. \\ P' &= 61 - 49 = 12; \quad \frac{14-R^0}{12} = \frac{14}{12} = 1 + \frac{2}{12}; \quad P' = 12, \mu' = 1, R' = 2. \\ P'' &= 1 + 1 \times 2 = 3; \quad \frac{14-2}{3} = \frac{12}{3} = 4 + \frac{0}{3}; \quad P'' = 3, \mu'' = 4, R'' = 0. \\ P''' &= 12 - 4 \times (2 - 0) = 4; \quad \frac{14-0}{4} = \frac{14}{4} = 3 + \frac{2}{4}; \quad P''' = 4, \mu''' = 3, R''' = 2. \\ P^{IV} &= 3 - 3 \times (0 - 2) = 9; \quad \frac{14-2}{9} = \frac{12}{9} = 1 + \frac{3}{9}; \quad P^{IV} = 9, \mu^{IV} = 1, R^{IV} = 3. \\ &P^V \end{aligned}$$

$$\begin{aligned}
 P^v &= 4 - 1 \times (2 - 3) = 5; \frac{14-3}{5} = \frac{11}{5} = 2 + \frac{1}{5}; P^v = 5, \mu^v = 2, R^v = 1. \\
 P^{vi} &= 9 - 2 \times (3 - 1) = 5; \frac{14-1}{5} = \frac{13}{5} = 2 + \frac{3}{5}; P^{vi} = 5, \mu^{vi} = 2, R^{vi} = 3. \\
 P^{vii} &= 5 - 2 \times (1 - 3) = 9; \frac{14-3}{9} = \frac{11}{9} = 1 + \frac{2}{9}; P^{vii} = 9, \mu^{vii} = 1, R^{vii} = 2. \\
 P^{viii} &= 5 - 1 \times (3 - 2) = 4; \frac{14-2}{4} = \frac{12}{4} = 3 + \frac{0}{4}; P^{viii} = 4, \mu^{viii} = 3, R^{viii} = 0. \\
 P^{ix} &= 9 - 3 \times (2 - 0) = 3; \frac{14-0}{3} = \frac{14}{3} = 4 + \frac{2}{3}; P^{ix} = 3, \mu^{ix} = 4, R^{ix} = 2. \\
 P^x &= 4 - 4 \times (0 - 2) = 12; \frac{14-2}{12} = \frac{12}{12} = 1 + \frac{0}{12}; P^x = 12, \mu^x = 1, R^x = 0. \\
 P^{xi} &= 3 - 1 \times (2 - 0) = 1; \frac{14-0}{1} = \frac{14}{1} = 14 + \frac{0}{1}; P^{xi} = 1, \mu^{xi} = 14, R^{xi} = 0.
 \end{aligned}$$

HERE I stop, because $P^{xi} = P^0 = 1$. And the series of numbers fought is, 1, 4, 3, 1, 2, 4, 1, 3, 4, 1, 14.

ON turning to page 378 of the English edition of EULER'S *Algebra*, it will be found that the table there given consists of the two series of numbers, $P^0, P', P'', \&c.$ and $\mu, \mu', \mu'', \mu''', \&c.$

THIS rule is the more worthy of notice, that it proceeds by certain definite arithmetical operations: whereas the method of M. DE LA GRANGE determines the numbers, $\mu, \mu', \mu'', \&c.$ by appreciating the value of certain expressions to the nearest unit, or by a process that is in some measure tentative, and therefore not strictly analytical.

SURGERY.

Mr RUSSEL read an account of a singular variety of Hernia which occurred to him while he was delivering clinical lectures in conjunction with Dr BROWN and Mr THOMSON. Mr THOMSON dissected the parts with great care and accuracy, and discovered certain peculiarities, which makes the knowledge of this

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A singular
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