On a Method of Comparing the Electrical Capacities of two Condensers

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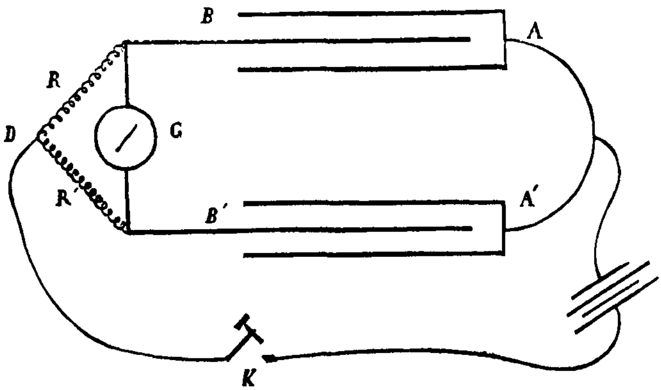
XXII. *On a Method of Comparing the Electrical Capacities of two Condensers.* By R. T. Glazebrook, *M.A., Fellow of Trinity College, and Demonstrator of Experimental Physics at the Cavendish Laboratory, Cambridge* *.

The following is a well-known method of comparing the capacities of two condensers:—

Let A, A′ be the outer coatings, B, B′ the inner of two condensers. Connect A A′ together, and to one pole of a battery. Connect B to a resistance R and to one pole of a



galvanometer, B′ to another resistance R′ and to the other pole of the galvanometer. Connect the other ends of R R′ together and to a key K, and let the second screw of the key be in connexion with the other pole of the battery. Let C C′ be the capacities of the condensers. On depressing the key the condensers are charged; and it is easy to show that, if $CR=C'R'$, no current passes through the galvanometer.

If, then, we adjust R until no current is observed on making contact, R′ remaining unaltered, we can find the ratio of C to C′.

I propose to discuss the more general problem of finding the current through the galvanometer when the equation $CR=C'R'$ is not fulfilled, and hence to obtain the conditions of sensibility.

Let V_1 be the potential of A, V_2 of the other pole of the battery, V of B, V′ of B′, at time t. Let G be the galvanometer resistance.

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Let ι = current in R;
ι' = " R';
ι_1 = current into condenser A;
ι_1' = " " B;
x = current through galvanometer;
Q, Q' = the quantities in the condensers.

Let us further suppose that there is a small leakage through the condensers, $\rho\,\rho'$ being their resistances.

Then we have

$$\left.\begin{aligned} \iota &= \frac{V_2-V}{R}, \quad \iota' = \frac{V_2-V'}{R'}, \quad x = \frac{V-V'}{G}, \\ \iota_1 &= \iota - x, \quad \iota_1' = \iota' + x, \\ Q &= C(V-V_1), \quad Q' = C'(V'-V_1), \\ \iota_1 &= \frac{dQ}{dt} + \frac{V-V_1}{\rho}, \\ \iota_1' &= \frac{dQ'}{dt} + \frac{V'-V_1}{\rho'}, \end{aligned}\right\} \quad . \quad . \quad (1)$$

From these we obtain:—

$$Gx + R\iota - R'\iota' = 0; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$CR\frac{d\iota}{dt} + \frac{R+\rho}{\rho}\left\{\iota - \frac{V_2-V_1}{R+\rho}\right\} - x = 0; \quad . \quad . \quad . \quad (3)$$

$$C'R'\frac{d\iota'}{dt} + \frac{R'+\rho'}{\rho'}\left(\iota' - \frac{V_2-V_1}{R'+\rho'}\right) + x = 0. \quad . \quad . \quad . \quad (4)$$

Assume

$$\iota - \frac{V_2-V_1}{R+\rho} = Ae^{-nt},$$

$$\iota' - \frac{V_2-V_1}{R'+\rho'} = A'e^{-nt},$$

$$x = Be^{-nt}.$$

On substituting we have

$$A\left\{\frac{R+\rho}{\rho} - CRn\right\} - B = 0, \quad . \quad . \quad . \quad . \quad (5)$$

$$A'\left\{\frac{R'+\rho'}{\rho'} - C'R'n\right\} + B = 0, \quad . \quad . \quad . \quad (6)$$

$$GB + RA - R'A' = 0, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

whence

$$\left.\begin{aligned} &A\left\{G\left(\frac{R+\rho}{\rho} - RCn\right)+R\right\} - R'A'=0, \\ &A'\left\{G\left(\frac{R'+\rho'}{\rho'}-R'C'n\right)+R'\right\} -RA=0. \end{aligned}\right\} \quad . \quad (8)$$

Eliminating A A′ we arrive at the quadratic,

$$n^2-n\left\{\frac{1}{G}\left(\frac{1}{C}+\frac{1}{C'}\right)+\frac{1}{R'C'}\left(\frac{R+\rho}{\rho}\right)+\frac{1}{RC}\left(\frac{R'+\rho'}{\rho'}\right)\right\}$$
$$+\frac{1}{GRR'CC'}\left\{\frac{G(R+\rho)(R'+\rho')}{\rho\rho'}+\frac{R(R'+\rho')}{\rho'}\right.$$
$$\left.+\frac{R'(R+\rho)}{\rho}\right\}=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Let n_1 n_2 be the roots, and let

$$\left.\begin{aligned} &G\left(\frac{R+\rho}{\rho} -RCn\right)+R=\lambda, \\ &G\left(\frac{R'+\rho'}{\rho'}-R'Cn\right)+R'=\lambda'. \end{aligned}\right\}. \qquad (10)$$

Then from (8) we have

$$\left.\begin{aligned} \lambda_1 A_1 + \lambda_2 A_2 &= R'(A_1'+A_2'), \\ \lambda_1' A_1'+\lambda_2' A_2' &= R(A_1+A_2), \end{aligned}\right\} \quad . \quad . \quad (11)$$

where A_1, λ_1 &c. denote the values of A, λ corresponding to n_1, n_2.

Also initially

$$\iota=\frac{V_2-V_1}{R}, \qquad \iota'=\frac{V_2-V_1}{R'}; \quad . \quad . \quad . \quad . \quad (12)$$

therefore, putting $t=0$ and $V_2-V_1=E$ in the equations

$$\iota=\frac{V_2-V_1}{R+\rho}+A_1e^{-n_1t}+A_2e^{-n_2t}$$

$$\iota'=\frac{V_2-V_1}{R'+\rho'}+A_1'e^{-n_1t}+A_2'e^{-n_2t},$$

we find

$$\left.\begin{aligned} A_1+A_2 &= \frac{E\rho}{R(R+\rho)}, \\ A_1'+A_2' &= \frac{E\rho'}{R'(R'+\rho')}. \end{aligned}\right\} \quad . \quad . \quad . \quad (13)$$

Solving for A_1 A_2, we get

$$A_1=\frac{E}{R(n_1-n_2)}\left\{\frac{1}{GC}\left(\frac{\rho}{R+\rho}+\frac{\rho'}{R'+\rho'}\right)\right.$$

$$\left.+\frac{1}{R}\left(\frac{1}{RC}-\frac{n_2\rho}{R+\rho}\right)\right\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

$$A_2=-\frac{E}{R(n_1-n_2)}\left\{\frac{1}{GC}\left(\frac{\rho}{R+\rho}-\frac{\rho'}{R'+\rho'}\right)\right.$$

$$\left.+\frac{1}{R}\left(\frac{1}{RC}-\frac{n_1\rho}{R+\rho}\right)\right\}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

and similar values for A_1' A_2'.

Hence we find

$$Gx=E\left\{\frac{R'}{R'+\rho'}-\frac{R}{R+\rho}\right\}$$

$$+\frac{E}{n_1-n_2}\left[\left(\frac{1}{R'C'}-\frac{1}{RC}\right)(e^{-n_1t}-e^{-n_2t})\right.$$

$$+\left\{\left(n_2-\frac{1}{GC}-\frac{1}{GC'}\right)e^{-n_1t}-\left(n_1-\frac{1}{GC}-\frac{1}{GC'}\right)e^{-n_2t}\right\}$$

$$\left.\times\left\{\frac{\rho}{R+\rho}-\frac{\rho'}{R'+\rho'}\right\}\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

Let us take the case in which there is no leakage first, $\rho=\rho'=\infty$, and

$$x=\frac{E}{G(n_1-n_2)}\left(\frac{1}{R'C'}-\frac{1}{RC}\right)(e^{-n_1t}-e^{-n_2t}). \quad . \quad . \quad (17)$$

Thus, if $RC=R'C'$, we see that x is zero for all values of t.

The total effect on the galvanometer, since the time of charging is short, is proportional to the quantity of electricity which passes. To find this we integrate the value of x with respect to t from 0 to τ, and suppose τ so large that

$$e^{-n_1\tau}=e^{-n_2}=0.$$

Then, if P be the quantity,

$$P=\frac{E(R'C'-RC)}{GRR'CC'n_1n_2};$$

also

$$n_1n_2=\frac{G+R+R'}{GRR'CC'};$$

$$\therefore \quad P=\frac{E(R'C'-RC)}{G+R+R'}; \quad . \quad . \quad . \quad . \quad . \quad (18)$$

and if H be the strength of the field in which the needle hangs, T the time of a complete oscillation, k the galvanometer-constant, and α the angle through which the needle is turned,

$$P = \frac{HT}{\pi k} \sin\frac{\alpha}{2};$$

$$\therefore \quad \sin\frac{\alpha}{2} = \frac{\pi k}{HT} \cdot \frac{E(R'C'-RC)}{(G+R+R')}, \quad . \quad . \quad . \quad (19)$$

which leads to the condition that, when there is no throw of the galvanometer, $R'C'=RC$.

We proceed to inquire what resistances will give the most accurate value for the capacity C in terms of C′, the known capacity of a standard condenser when using a given galvanometer. Let us suppose the adjustment made by varying R, and determine the error δa produced in α by an error δR in R. Then, remembering that, when the adjustment is perfect, $RC=R'C'$ and $a=0$, if δR is the error from perfect adjustment, we have

$$\delta a = -\frac{2}{HT}\frac{E\pi kC\delta R}{(G+R+R')}; \quad . \quad . \quad . \quad . \quad (20)$$

and if δC is the error in the capacity, since $C=\frac{R'C'}{R}$,

$$\delta C = -\frac{R'C'\delta R}{R^2} = \frac{HT(G+R+R')R'C'}{2E\pi kCR^2}\delta a;$$

or, since $CR=C'R'$,

$$\delta C = \frac{HT(G+R+R')}{2E\pi kR}\delta a. \quad . \quad . \quad . \quad . \quad (21)$$

Now k varies as the number of turns in the galvanometer, and so also does G;

$$\therefore \quad K=\mu G,$$

$$\therefore \quad \delta C = \frac{HT\delta a}{2E\pi\mu}\left\{\frac{1}{G}+\frac{1}{R}+\frac{R'}{GR}\right\}; \quad . \quad . \quad (22)$$

and if we suppose that we are liable to an error δa in a, the error in C is least when the resistances R and R′ are both high.

Thus it is best to use, with a given galvanometer, high resistances R and R′.

We arrive at the same result if we make the adjustments by varying R′ instead of R.

Again, let us suppose that we have a galvanometer with a given channel, and we wish to fill it with wire so as to be most sensitive. Let V be the volume of the channel, y the radius of the wire, l its length, ρ its specific resistance, and suppose we neglect the thickness of the covering; then

$$4\rho y^2 = \mathrm{V},$$

$$\mathrm{G} = \frac{\rho l}{\pi y^2} = \frac{4\rho l^2}{\mathrm{V}},$$

$$k = g\rho,$$

where g depends only on the form and dimensions of the channel. We must therefore make

$$\frac{\frac{4\rho l^2}{\mathrm{V}} + \mathrm{R} + \mathrm{R}'}{l}$$

a minimum. We find

$$\frac{4\rho}{\mathrm{V}} - \frac{\mathrm{R}+\mathrm{R}'}{l^2} = 0; \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

and we get finally

$$\mathrm{G} = \mathrm{R} + \mathrm{R}'.$$

Now for a given value of G, R and R′ must be as high as possible; therefore we must make the resistance of our galvanometer as high as possible.

Returning to the general case in which there is a leakage in the condensers, and putting $t=\tau$, τ being so large that the terms involving $e^{-n\tau}$ may be neglected, we get

$$x = \frac{\mathrm{E}}{\mathrm{G}}\left\{\frac{\mathrm{R}'}{\mathrm{R}'+\rho'} - \frac{\mathrm{R}}{\mathrm{R}+\rho}\right\}. \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Thus there is a steady current through the galvanometer, and the needle is permanently deflected.

Again, if $t=0$, $x=0$. But let us suppose t very small, so that, on expanding e^{-nt}, powers higher than the first may be neglected, and find the initial current. We find

$$x = \frac{\mathrm{E}}{\mathrm{G}}\left\{\frac{\mathrm{R}'\mathrm{C}'-\mathrm{RC}}{\mathrm{RC}\,\mathrm{R}'\mathrm{C}'} + \frac{1}{\mathrm{G}}\left(\frac{1}{\mathrm{C}} + \frac{1}{\mathrm{C}'}\right)\left(\frac{\rho}{\mathrm{R}+\rho} - \frac{\rho'}{\mathrm{R}'+\rho'}\right)\right\}t, \quad (25)$$

which reduces, if we neglect powers of $\frac{1}{\rho}$ above the first, to

$$x = \frac{\mathrm{E}}{\mathrm{G}}\left\{\frac{\mathrm{R}'\mathrm{C}'-\mathrm{RC}}{\mathrm{RC}\,\mathrm{R}'\mathrm{C}'} + \frac{1}{\mathrm{G}}\left(\frac{1}{\mathrm{C}} + \frac{1}{\mathrm{C}'}\right)\left(\frac{\mathrm{R}'}{\rho'} - \frac{\mathrm{R}}{\rho}\right)\right\}t. \quad . \quad (26)$$

By altering R the sign of this may be made to change, and thus the initial current may be in the same or in the opposite direction to the final. In practice this is indicated by a short kick of the needle in one direction, followed by a deflection in the other.

On the same assumptions as to ρ, ρ', let us find the quantity of electricity which passes through the galvanometer in time τ, τ being very short, but yet so long that $e^{-n\tau}$ may be neglected. Integrating (16) and calling the quantity P, we have

$$P=\frac{E}{G+R+R'}\left[\{R'C'-RC\}\left\{1-\frac{G\left(\frac{R}{\rho}+\frac{R'}{\rho'}\right)+RR'\left(\frac{1}{\rho}+\frac{1}{\rho'}\right)}{G+R+R'}\right\}\right.$$
$$\left.-\left\{\frac{R'}{\rho'}-\frac{R}{\rho}\right\}\{RC+R'C'\}\right]+\frac{E}{G}\left(\frac{R'}{\rho'}-\frac{R}{\rho}\right)\tau. \quad . \quad (27)$$

If τ be very short, we may neglect the last term compared with the others, and, to the same degree of approximation with respect to ρ ρ, we get as the condition of no kick,

$$R'C'-RC=\left(\frac{R'}{\rho'}-\frac{R}{\rho}\right)(R'C'+RC),$$

or

$$\frac{RC}{R'C'}=1-2\left(\frac{R'}{\rho'}-\frac{R}{\rho}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (28)$$

Again, let k, as before, be the galvanometer-constant, and δ the permanent deflexion. Then, from (25),

$$\frac{E}{G}\left\{\frac{R'}{\rho'}-\frac{R}{\rho}\right\}=k\tan\delta, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

$$\frac{RC}{R'C'}=1-\frac{2Gk}{E}\tan\delta\,; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

and this equation enables us to determine the capacity.

Let us suppose the adjustment made by varying R. Then, starting from a position in which the first kick is in an opposite direction to the final deflexion, adjust R until that kick is just reduced to zero, and the spot of light moves off gradually in the one direction, and after some oscillations comes to rest. Then, if δ is the deflexion of the galvanometer-needle the capacity is

$$C=\frac{C'R'}{R}\left\{1-\frac{2Gk}{E}\tan\delta\right\}.$$

Unless the leakage is considerable, the correction will be very small.

In measuring the capacity of many condensers, the difficulty is increased by the phenomenon of electric absorption. In fact the condenser has no true capacity; for the charge produced by a given electromotive force depends on the time during which that force has acted. We may, however, take the capacity as the ratio of the instantaneous charge to the electromotive force producing it; and in this case (contact with the battery being maintained only for a very short time) we may perhaps look on electric absorption as a kind of conduction through the substance of the condenser. We must suppose that the resistance to the conduction is a function of the time, which becomes indefinitely great after a not very long interval, but which we may perhaps treat as sensibly constant during the time for which contact is maintained; and if ρ_0, ρ'_0 be the values of this resistance during that interval, and, as before, we may neglect $\frac{1}{\rho_0^2}$ &c. and higher powers, we have, as the value of the capacity,

$$C=\frac{R'C'}{R}\left\{1-2\left(\frac{R'}{\rho'_0}-\frac{R}{\rho_0}\right)\right\}.$$

Thus a small correction should be applied to the value $\frac{R'C'}{R}$, depending on the rate of absorption during the interval for which contact is maintained with the battery. An approximation to this quantity may be obtained by charging the condenser for some time with a battery of known electromotive force, and then allowing it to discharge itself at small intervals of time through the galvanometer. On the whole, however, the results of measurements made, neglecting this correction, are fairly satisfactory.

The capacity of a paraffin condenser was determined by several observers during the past term at the Cavendish Laboratory. Their results differed by from $\frac{2}{3}$ to 1 per cent. The standard used was not in all cases the same; and the measures obtained by one observer, comparing this same condenser with two different standards, differed by about $\frac{1}{3}$ per cent. It seemed possible to determine within 10 ohms, when each of the resistances R, R′ was about 5000 ohms, the value of R for which the initial kick was zero.