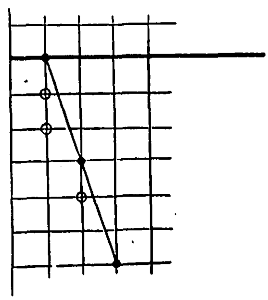
Hence, if the equations

$$F = 0, \quad F_w = 0, \quad F_{ww} = 0, \quad F_z = 0$$

can be solved for y, z, w, and if the functions of x so obtained satisfy

$$dy = z\,dx, \quad dz = w\,dx,$$

then the corresponding locus is a singular solution of the second kind, and is an osculating envelope of singular solutions of the first kind.



That these conditions are necessary in order that both kinds of singular solution may exist appears from the diagram; for the side of the polygon nearest the origin must indicate *two* expansions with leading index 3.

APPENDIX (a).

(SESSION 1900–1901.)

In the list of exchanges, No. 56 (p. 3) should be deleted, as the copy in question is given to the College by a member of the Society.

At the December meeting the Treasurer declined to have a resolution passed for the adoption of his report. The Auditor was thanked for his services by letter from the Secretary.

Mr. J. H. Michell sends the following remarks for insertion in the Appendix :—

There is an oversight in § 3 of my paper "On Transmission of Stress, &c." (*Proc.*, Vol. XXXI., p. 183), which perhaps ought to be pointed out.

The method proposed implies the continuity of the functions $(\lambda+3\mu)\theta-2\mu w'_z$, ϖ, &c., across the boundary between the parts of $z=0$, over which the different conditions hold. This continuity will not in general exist. Cases (*c*), (*d*) are covered by Boussinesq's solution, as I have pointed out. The difficulty is with (*a*), (*b*). Whether the proposed method is feasible when the true continuity conditions (u, v, w continuous) are introduced requires further examination.

The following is an abstract of the communication made by Mr. Tucker (p. 311) entitled "The Brocardal Properties of some Associated Triangles":—

A_1AB_1, B_1BC_1, C_1CA_1 are drawn perpendicular to $A\Omega$, $B\Omega$, $C\Omega$, cutting the circumcircle in B', C', A' respectively; and, in like manner, A_2AC_2, B_2BA_2, C_2CA_2 perpendicular to $A\Omega'$, $B\Omega'$, $C\Omega'$, cutting the circle in C'', A'', B'' respectively. The equation to A_1B_1 is

$$\beta\cos\omega+\gamma\cos(A-\omega)=0, \qquad \text{(i.)}$$

and to A_2B_2 is
$$\alpha\cos\omega+\gamma\cos(B-\omega)=0. \qquad \text{(ii.)}$$

The points A_1, A_2 are given by

$$\cos(C-\omega)\cos(A-\omega),\quad -\cos\omega\cos(A-\omega),\quad \cos^2\omega, \qquad \text{(iii.)}$$

and by
$$\cos(A-\omega)\cos(B-\omega),\quad \cos^2\omega,\quad -\cos\omega\cos(A-\omega). \qquad \text{(iv.)}$$

We readily get
$$A_1B_1 = c\operatorname{cosec}\omega = A_2B_2; \qquad \text{(v.)}$$

hence the triangles $A_1B_1C_1$ (or Δ_1) and $A_2B_2C_2$ (or Δ_2) are similar to ABC and are congruent to one another.

The points A', A'' are given by

$$-\cos(C-\omega)\cos(B+\omega),\quad \cos\omega\cos(B+\omega),\quad \cos\omega\cos(C-\omega), \qquad \text{(vi.)}$$

$$-\cos(B-\omega)\cos(C+\omega),\quad \cos\omega\cos(B-\omega),\quad \cos\omega\cos(B+\omega), \qquad \text{(vii.)}$$

and the triangles $A'B'C'$ (or Δ'), $A''B''C''$ (or Δ''), and ABC are congruent.

The circles (C_1), (C_2) round Δ_1, Δ_2 respectively, are given by

$$P.\Sigma\alpha\beta\gamma+\cot\omega.\Sigma a\alpha\left[\Sigma a\sin C\cos(B-\omega)\right]=0$$

and
$$P.\Sigma\alpha\beta\gamma+\cot\omega.\Sigma a\alpha\left[\Sigma a\sin B\cos(C-\omega)\right]=0,$$

where
$$P\equiv\operatorname{cosec}\omega.\Pi\sin A.$$

The circumcircle ABC can be readily shown to be the auxiliary circle of the Brocard ellipses of Δ_1, Δ_2.

The cyclic property (circumcentre O) of the six Brocard points of the five triangles and other properties are more elegantly derived from the theory of similar figures.

A referee writes: "We have as corresponding points of two similar figures

$$A,\ B,\ C;\quad A',\ B',\ C';\quad A_1,\ B_1,\ C_1;\ \ldots;$$

$$A'',B'',C'';\quad A,\ B,\ C;\quad A_2,\ B_2,\ C_2;\ \ldots;$$

the centre of similitude being O and the figures being congruent. Again, we may consider the three similar figures

$$A,\ B,\ C,\ \ldots,\ K,\ \Omega,\ \Omega',\ O,\ \ldots;$$

$$A_1, B_1, C_1, \ldots, K_1, \Omega, \Omega_1, O_1, \ldots;$$

$$A_2, B_2, C_2, \ldots, K_2, \Omega_2, \Omega', O_2, \ldots,$$

whose centres of similitude are O, Ω', Ω, and invariable points K, K_1, K_2 (the symmedian points); hence it follows that the symmedian points of $A_1B_1C_1$ and $A_2B_2C_2$ lie on the Brocard circle of ABC."

The accompanying notice of the late M. Hermite has been drawn up, at the request of the Council, by Mr. G. B. Mathews:—

Charles Hermite, whose death occurred on January 14th, 1901, was born at Dieuze, in Lorraine, December 24th, 1822. After school-days spent at Nancy and Paris, he entered the École Polytechnique in 1842, and soon gave evidence of his remarkable genius; for it was in 1843 that he wrote to Jacobi his well-known letter on the theory of Abelian functions, and this was followed in 1844 by another on the transformation of the elliptic functions. Adopting the profession of a teacher, his career was one of uninterrupted success; after holding several minor appointments he was elected in 1862 to a newly founded chair in the École Normale, and in 1869 to the professorship of higher analysis in the Sorbonne, which he held until 1897. Readers of his lithographed course will understand the enthusiasm with which his successor and former pupil, M. Picard, speaks of his charm as a lecturer; in judgment of selection and lucidity of exposition these lectures are unsurpassable, besides showing on every page the impress of an original mind. The festal celebration of Hermite's seventieth birthday showed in an impressive way the regard in which he was held by his pupils and fellow-workers alike; and we may hope that this was a bright day in a life which, like that of many other men of science, was, on the whole, retired and uneventful. It would be tedious to recount all the distinctions bestowed upon the great mathematician by learned societies; but it is proper to recall the fact that in 1871 (December 14th) he was elected foreign member of the London Mathematical Society, to the *Proceedings* of which he contributed three or four short papers.

Hermite was exclusively an analyst, and, above all perhaps, an arithmetician. He was an avowed disciple of Gauss, Jacobi, and Dirichlet; with the last two he frequently corresponded, and received from them an encouragement which must have done much to develop his powers. The brilliant discoveries of Jacobi naturally led Hermite to the study of elliptic and Abelian transcendents: his first two letters to Jacobi have already been referred to, and in subsequent years we have his researches on the elliptic modular functions, and, above all, the memoir *Sur quelques applications des fonctions elliptiques*, which contains, not only a discussion of Lamé's equation, which has become classical, but the extremely important theory of the decomposition of periodic functions into the sum of "simple elements," each with one (simple or multiple) pole. It may be observed that Hermite always adhered to the Jacobian elliptic and theta functions; partly, no doubt, because he had grown accustomed to them, but also, perhaps, because of their more obvious association with arithmetical theories. It is noticeable that the same thing may be said of Kronecker.

The invention of the calculus of invariants by Boole, Cayley, and Sylvester naturally attracted Hermite's attention, and he soon made substantial additions to the theory. To him is due the discovery of the first skew invariant, and the law of reciprocity which has been called after his name. Moreover, he showed the value of the new calculus by applying it to the transformation of elliptic functions, to the transcendental solution of the quintic, to the Tschirnhausen transformation, and to the separation of the roots of equations after the manner of Sturm.

It is in his arithmetical researches that Hermite's genius shows to greatest advantage. Here his familiarity with algebra on the one hand, and transcendent functions on the other, by converging to a focus, enable him to penetrate into depths otherwise inaccessible. His work on the reduction and classification of arithmetical forms is of the very highest importance, and involves a new and ingenious application of continuous parameters. His theory of forms with conjugate complex variables has shown itself capable of important developments, and he has made many beautiful applications of elliptic functions to arithmetic. One of his most famous achievements is the proof that e, the base of natural logarithms, is an essentially transcendental number.

Hermite had a large correspondence, and many of his most remarkable discoveries were communicated by letter to his friends. Göpel was induced to publish his classical memoir on Abelian functions by reading Hermite's first letter to Jacobi; had he not seen this, it is possible that he might have died without giving any of his work to the world.

A portrait of Hermite will be found in the *Annales de l'École Normale Supérieure*, t. XVIII. (1901), together with an excellent account of his scientific work by M. Émile Picard. It is to be hoped that a collected edition of Hermite's mathematical papers will be issued without unnecessary delay.

[It may be interesting to note that M. Hermite communicated the following Questions to the *Educational Times*: references are to the Volumes of the *Reprint*:—

Vol. VII., p. 37, 2254 (and Sylvester); p. 53, 2273. Vol. XXIX., p. 76, 5492. Vol. XLVI., p. 21, 8560; p. 50, 8510 (and solution); p. 63, 8588 (and solution); p. 94, 8633; p. 111, 8717. Vol. XLVII., p. 53, 8863 (and solution). Vol. XLVIII., p. 21, 9072. Vol. LI., p. 33, 9832; p. 49, 9930. Vol. LII., p. 63, 10125; p. 118, 10083. Vol. LIII., p. 127, 10166. Vol. LXVII., p. 91, 10267. Vol. LXV., p. 99, 12337.

R. T.]