Forced Extensional Totalization in Linear Continuum Dynamics

Lance R. Williams*
Department of Computer Science
University of New Mexico

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Abstract

Continuum physical theories model states as real- or complex-valued fields and dynamics as linear operators on infinite-dimensional spaces. Under *explanatory realism*, an ontic interpretation incurs two semantic commitments: (i) real-valued physical magnitudes must *denote* relative to the theory's admissible state interface, and (ii) the theory must be *semantically closed* under its own evolution and readout rules. Denotation is interface-relative: it requires the existence of a total *continuous* witness on names. Equivalently, it requires *bounded input dependence* at each fixed finite precision. Failure of bounded input dependence is captured by *forced extensional totalization (FET)*, in which even coarse output precision cannot be stabilized by any finite prefix of a state name. We give a physics-facing construction of FET for the three-dimensional wave equation with Kirchhoff point readout. We construct a countable family of smooth, compactly supported, finite-energy initial states localized on pairwise disjoint shrinking latitude belts on the Huygens sphere, and restrict the admissible domain to states containing *at most one* localized contingency. Each nonzero belt perturbation is normalized to contribute the same order-one amount to the readout, while the zero state contributes none. Under any non-oracular state interface satisfying a minimal locality-of-access condition, no finite prefix can rule out activation of some sufficiently deep belt. The induced point readout therefore behaves like the existential predicate "some belt is active", exhibits FET at the zero state, and fails to denote. Semantic closure fails: the evolution–readout pipeline yields a localized point magnitude that is non-denoting relative to the fixed state interface. The result isolates a semantic obstruction to ontic readings of continuum dynamics. It is independent of computability, predictability, or measurement feasibility, and it holds despite smooth compact support, finite energy, and classical well-posedness.

*ORCID 0009-0006-5733-5495

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1 Introduction

Any theory that aspires to describe the world as it is, instead of merely organizing observations, incurs semantic obligations. In particular, an *ontic* interpretation of a theory commits it to the existence of the entities and magnitudes it posits, and to semantic coherence of the laws governing them. These commitments are usually left implicit. The aim of this paper is to make them explicit, and to show that, when taken seriously, they place nontrivial constraints on continuum dynamics.

The guiding perspective is *explanatory realism.* Under explanatory realism, a physical theory is required not only to reproduce observations, but to supply a coherent semantics for its intended ontic domain. Once this stance is adopted, two minimal semantic commitments follow.

- **Denotation of physical magnitudes.** If a theory treats a real-valued quantity as an ontic physical magnitude, then relative to the theory's fixed admissible presentation of states the magnitude must admit a *total continuous* semantic witness supporting finite-precision approximation (continuity-on-names in represented-space semantics) [Weihrauch, 2000, Schröder, 2002, Kreitz and Weihrauch, 1985].

- **Closure under evolution.** If a theory treats certain states as physically admissible, then its fundamental dynamics must preserve admissibility. That is, physically admissible states must evolve to physically admissible states under the theory's evolution rule (a total self-map on the admissible domain).

These commitments constrain different aspects of a theory, but they arise from a single semantic demand: *coherence under finite access.* A physical theory does not present its states as Platonic objects. It presents them through a state interface, some representation of a state as a name, such that finite prefixes correspond to finite-precision access [Weihrauch, 2000, Schröder, 2002]. Once such an interface is fixed, explanatory realism requires that neither magnitudes nor laws demand, as a condition of intelligibility, the resolution of *information not settled at any finite stage of access*, i.e. information not determined by any finite-precision description of the state.

First, *denotation* is a stability requirement on *readout.* At the intuitive level, it is nothing more than the requirement that *finite information about the state eventually pins down finite precision of the magnitude.* A physical state in a continuum theory cannot be accessed as an extensional Platonic object; it is accessed only through progressively refined finite descriptions. If a quantity $M(s) \in \mathbb{R}$ is treated as an ontic magnitude, then for each requested accuracy 2^{-N} there must exist some finite access depth at which all states still compatible with the available information already force agreement on $M(s)$ to within 2^{-N}. If no such depth exists—if some fixed coarse accuracy can never be settled by any finite prefix because arbitrarily fine unresolved contingencies retain $O(1)$ leverage on the readout—then the magnitude is not semantically determinate relative to the state interface.

Second, *closure under evolution* is a coherence requirement on *law.* If D is the class of states the theory declares physically admissible, then the fundamental evolution rule must be a total self-map $F : D \to D$. That is: admissible states must not evolve into states that the theory itself cannot regard as admissible. Here admissibility is understood semantically and interface-relatively: D is not merely a mathematical function space, but the theory's intended ontic domain as presented through its fixed finite-access interface [Weihrauch, 2000, Schröder, 2002].

In short, both commitments reduce to a single organizing idea: *the theory must not demand semantic miracles.* Neither readout nor evolution is allowed to force the extraction (or implicit resolution) of *information not settled at any finite stage of access* in order to make sense of what the theory itself treats as physically real.

The relevance of these commitments becomes apparent in the analysis of continuum dynamics. The seminal work of Pour-El and Richards [1989] shows that even well-posed linear partial differential equations can conflict with standard notions of effective describability. Their Theorem 1 establishes that (for the wave equation in $\mathbb{R}^3$) there exist computable initial data whose solutions take noncomputable values at individual spacetime points, despite the evolution being mathematically well posed [Pour-El and Richards, 1989]. This exposes a tension between continuum dynamics and effective computation. By itself, however, it does not yet constitute a denotation obstruction in the finite-access sense developed here: noncomputability is an effectivity failure, whereas denotation concerns continuity-on-names and finite-precision stability relative to an interface [Weihrauch, 2000, Schröder, 2002].

Closely related phenomena were identified by Moore and others in the study of undecidability and long-term unpredictability in continuous dynamical systems [Moore, 1990, 1991]. These analyses likewise demonstrate that idealized continuum dynamics can embed undecidable behavior and defeat effective prediction, even when the governing equations are simple and deterministic. Again, the issue there is typically effectivity (undecidability/noncomputability) rather than semantic failure in the witness/continuity sense.

A stronger obstruction appears in Theorem 2 of Pour-El and Richards [1989]. There, computable initial data evolve to solutions whose restrictions to compact regions fail to admit any admissible name in the represented function space once those regional restrictions are treated as single physical magnitudes [Pour-El and Richards, 1989]. From the standpoint of explanatory realism, and under the standard requirement that such regional restrictions denote as single semantic objects, this is naturally read as a failure of denotation and hence as a violation of semantic closure [Weihrauch, 2000, Schröder, 2002].

The present paper isolates the same underlying failure mechanism in a more elementary and more localized form. We give a self-contained construction showing that linear continuum evolution, even when applied to smooth, compactly supported, finite-energy initial data, can force *forced extensional totalization* (FET) at a *single point readout*. Throughout, we treat pointwise field values as legitimate ontic magnitudes, in keeping with standard continuum field ontology: a single field value at a spacetime event is a physical magnitude whose semantic status must be coherent relative to the fixed state interface.

The construction uses the standard three-dimensional wave equation together with point readout. The key technical fact is geometric: in three spatial dimensions, Huygens' principle and Kirchhoff's formula imply that a point value at (x_0, t_0) is a fixed spherical aggregation of the initial data on the light-sphere $\{\, |x - x_0| = t_0 \,\}$ [Evans, 2010, John, 1982]. This intrinsic spherical aggregation acts as a *semantic amplifier*.

The central twist is that no elaborate encoding is needed. We distribute a countable family of independent localized "belt" perturbations on the light-sphere, with disjoint supports accumulating at the equator. But we restrict attention to a maximally tame admissible domain: the initial condition contains *at most one nonzero contingency*. Either none of the belts is active, or exactly one belt is active. If a belt is active, the point readout takes the value 1; if no belt is active, the point readout is 0. Thus the readout encodes a simple predicate:

"is there any active localized contingency at all?"

Under any admissible non-oracular state presentation satisfying a minimal locality-of-access constraint (made explicit in Appendix B), this predicate is *non-denoting* at the all-zero state.

Every finite prefix of the state-name remains compatible both with "no belt is active" and with "some belt is active at deeper index n," while the point readout differs by $O(1)$ between these cases. Equivalently, approximation of the point magnitude to even a fixed coarse precision requires extensional resolution of an unbounded family of independent possibilities. This is FET in its starkest possible form.

This result instantiates, in a continuum-dynamical setting, a semantic divide familiar from witness-based semantics. Classical determination of an object does not guarantee semantic availability through an admissible interface. As emphasized by Bauer in his analysis of realizability and the Kleene tree, an object may be classically determined yet lack any witness in a witness-based semantics [Bauer, 2006]. In the same spirit, the magnitude constructed here is classically specified yet fails to denote under admissible semantics. The obstruction arises not from noncomputability per se, but from *forced extensional totalization* and the resulting loss of bounded input dependence.

A compact finite-access semantics is developed in Section 2. That section fixes the represented-space setting, defines denotation as a finite-precision stability condition on compatible-state neighborhoods, isolates forced extensional totalization (FET) as an explicit witness pattern for non-denotation, and states the corresponding closure requirement for an evolution–readout pipeline as denotation preservation across the admissible domain [Weihrauch, 2000, Schröder, 2002, Kreitz and Weihrauch, 1985]. In particular, the two explanatory-realist commitments announced above are re-expressed there in the precise language used by the reductio.

The remainder of the paper implements a single construction inside that framework. Section 3 reviews the wave equation in $\mathbb{R}^3$ and Kirchhoff's formula, emphasizing the geometric fact that point readout at (x_0, t_0) is an intrinsic spherical aggregation of the initial data on the light-sphere $\{|x - x_0| = t_0\}$ [Evans, 2010, John, 1982]. Section 4 constructs the belt-localized family of smooth perturbations on that light-sphere and defines the reduced admissible domain in which the initial condition contains at most one active belt. Section 5 states and proves the main theorem: for any admissible non-oracular state interface satisfying a minimal locality-of-access condition, the induced evolution–readout magnitude exhibits FET at a name of the all-zero state, hence fails to denote there. By the closure definition of Section 2, this yields failure of semantic closure for the corresponding evolution–readout pipeline on the chosen admissible domain. Section 6 discusses scope, interpretation, and standard objections.

2 Finite-access semantics for continuum magnitudes

A continuum field theory can be written in a perfectly ordinary mathematical form: a state s evolves under deterministic dynamics, and a real number $x \in \mathbb{R}$ is obtained by applying a readout functional to that state. Nothing here requires semantic philosophy.

The semantic problem appears only when we insist on an explanatory realist reading: the theory is not merely an instrument for organizing measurements, but a description of what actually *is*. Under such an ontic reading, real-valued magnitudes are supposed to refer to definite numbers, and the theory's admissible states are supposed to form a coherent domain of physical possibility. Yet continuum states contain infinitely many degrees of freedom, so access to a state can only proceed through partial information. A semantic account must therefore answer two related questions:

(i) Denotation. When a theory asserts a real-valued magnitude $M(s) \in \mathbb{R}$, what must be true for that value to be *determinate relative to finite access to s*?

(ii) Closure. When a theory declares a class D of states physically admissible, what must be true for that admissibility and semantic determinacy to be *preserved under the theory's own evolution and readout pipelines*?

This section develops a compact finite-access semantics in which the relevant questions can be stated precisely. It begins by explaining why explanatory realism requires a minimal finite-precision stability condition on magnitudes: partial information about the state must constrain the values of quantities in an informationally meaningful way. The remainder of the section introduces the representational and topological machinery needed to formulate this condition precisely and to state, in Subsections 2.4 and 2.7, the two semantic commitments that will drive the reductio. The framework is standard in represented-space semantics and Type–2 analysis [Weihrauch, 2000, Schröder, 2002, Kreitz and Weihrauch, 1985, Myhill, 1957], but is presented here in the minimal form required for the construction.

2.1 Why a finite-precision stability condition is unavoidable

Under explanatory realism, a physical magnitude is intended to be a quantity whose value is fixed by the physical state of the system, and that determination must be expressible relative to the theory's admissible interface to states. Given the uncontroversial nature of this premise, the remaining task is to identify the weakest condition under which the magnitude can be said to be determined by the state.

Admissible interfaces to states convey information through finite descriptions. A finite prefix of a state description is understood as expressing partial information about the state, and this interpretation must be uniform in the sense that the same prefix represents the same partial state information wherever it occurs. This view of representations as carrying information via finite prefixes is standard in represented–space semantics and computable analysis Weihrauch [2000], Schröder [2002]. Under this interpretation, the only way a magnitude can be determined by the state in an informationally meaningful sense is that partial state information constrains its value.

This leads to a minimal semantic requirement, made precise in the next subsection: for every desired precision, there must exist a finite amount of state information that restricts the magnitude to a correspondingly small range. This property will be called *finite-precision stability* in what follows. It does not assume computability, invertibility, or experimental accessibility; it requires only that the magnitude depend on the state in a way that is finitely stabilizable. Conditions of this type correspond to continuity of functionals in analysis and to the continuity of realizers in Type–2 computability, where output precision can depend only on finite input information Rudin [1991], Evans [2010], Weihrauch [2000].

Conditions of this form are not peculiar to the present framework. In analysis, physical magnitudes are modeled as continuous functionals of states; continuity expresses precisely that small changes in the state produce bounded changes in the quantity Rudin [1991], Evans [2010]. In the theory of differential equations and mathematical physics, meaningful quantities are required to depend continuously on initial data, a principle central to classical well-posedness in the sense of Hadamard and standard PDE treatments Courant and Hilbert [1962], Kreyszig [1978]. In computable analysis and represented-space semantics, realizers of well-defined quantities are necessarily continuous with respect to the underlying representation topology, because only finite information about the input can be used to produce any given output precision Weihrauch [2000], Schröder [2002]. The condition introduced here can thus be viewed as a common semantic core underlying analysis, computable semantics, and physical modeling practice.

What is ruled out are magnitudes whose values depend on state distinctions that no finite information about the state can constrain. Such quantities are not determined by the state in any informationally meaningful sense and are indistinguishable from oracle variables attached to the theory. Allowing such assignments would collapse the distinction between physical quantities and arbitrary hidden labels, rendering the notion of denotation vacuous. The formal definitions given in the next section should therefore be understood not as a strengthening of ordinary semantics, but as an explicit formulation of this minimal coherence requirement.

2.2 The semantic map: from a state name to a real name

Finite access is modeled by giving states and real numbers through *names*. A name is an infinite string that reveals information progressively through finite prefixes [Weihrauch, 2000, Pour-El and Richards, 1989].

State interface. Let D be the set of physically admissible initial states. A *state interface* is a representation

$$\delta_D :\subseteq \Sigma^\omega \to D,$$

where Σ is a finite alphabet (e.g. $\{0,1\}$), and $p \in \Sigma^\omega$ is a *name* of the state $s = \delta_D(p)$. The intended reading is that finite prefixes $p{\restriction}m$ encode finite interface access [Weihrauch, 2000, Schröder, 2002].

In what follows, "admissible" is used in the standard represented-spaces sense: δ_D (and likewise $\delta_\mathbb{R}$ below) is intended to be topologically adequate, so that a function between the underlying spaces is continuous exactly when it admits a continuous realizer on names [Weihrauch, 2000, Schröder, 2002].

Real interface. To model finite access to a real number, fix an admissible representation

$$\delta_\mathbb{R} :\subseteq \Sigma^\omega \to \mathbb{R},$$

for example the usual Cauchy representation [Weihrauch, 2000].

Readout. A (real-valued) magnitude is a map from states to real numbers

$$M : D \to \mathbb{R}.$$

In this paper the relevant magnitude is point readout at a fixed spacetime event after evolution; we will discuss this special case soon. For now, M can be any physical magnitude.

Together these ingredients determine the semantic relationship between an input name p and an output name q:

$$p \xrightarrow{\delta_D} s = \delta_D(p) \xrightarrow{M} x = M(s) \xrightarrow{\delta_\mathbb{R}^{-1}} q,$$

where q is intended to be a $\delta_\mathbb{R}$-name of the real number x. This picture can be drawn as a commutative diagram. The purpose is not to introduce new structure, but to make explicit the semantic task: produce a real-name from a state-name in a way compatible with finite access.

$$\begin{array}{ccc} \mathrm{dom}(\delta_D) & \xrightarrow{R} & \mathrm{dom}(\delta_\mathbb{R}) \\ \delta_D \downarrow & & \downarrow \delta_\mathbb{R} \\ D & \xrightarrow{M} & \mathbb{R} \end{array}$$

where $p \in \mathrm{dom}(\delta_D)$, $q \in \mathrm{dom}(\delta_\mathbb{R})$ and $q = R(p)$. A map R turns a state-name p into a real-name q while respecting the intended meaning of both representations:

$$\delta_\mathbb{R}(R(p)) \;=\; M(\delta_D(p)). \tag{1}$$

When (1) is satisfied, we say that R is a *witness on names*. This is the standard witness/realizer viewpoint in represented spaces and Type–2 semantics [Weihrauch, 2000, Schröder, 2002, Kreitz and Weihrauch, 1985]. Crucially, (1) must be read correctly. It does *not* say that real numbers are secretly strings. It says that if we claim $M(\delta_D(p))$ is an ontic magnitude, then there must exist a stable interface-respecting map R that produces names of that magnitude from finite access to the state-name.

2.3 Prefix neighborhoods and compatible states

Fix a name $p \in \mathrm{dom}(\delta_D)$ and a prefix length m. The *cylinder* determined by that prefix is the set of all names consistent with the same finite-access information:

$$[p\!\upharpoonright_m] := \{\, r \in \Sigma^\omega : r\!\upharpoonright_m = p\!\upharpoonright_m \,\}.$$

Among admissible names, this cylinder corresponds to a family of compatible states:

$$U_{p,m} := \left\{\delta_D(r) \;:\; r \in \mathrm{dom}(\delta_D) \;\wedge\; r\!\upharpoonright_m = p\!\upharpoonright_m\right\} \;\subseteq\; D. \tag{2}$$

Thus $U_{p,m}$ is what the interface has *not ruled out* at depth m.

When the reader knows only the prefix $p\!\upharpoonright_m$, the theory must treat *every* state in $U_{p,m}$ as still semantically possible relative to that finite access. This cylinder-based viewpoint is standard in the represented-spaces semantics of continuity-on-names [Weihrauch, 2000, Schröder, 2002].

In the reductio construction below, the key semantic leverage comes from a single interface requirement: finite access to a state must not behave like an *extensional oracle* that decides arbitrarily deep localized contingencies. This is the minimal locality-of-access idea needed for denotation to have nontrivial content (compare Appendix B).

Definition 2.1 (Non-oracular locality of finite access at s_0)**.** Let $s_0 \in D$ be a distinguished reference state and let $p_0 \in \mathrm{dom}(\delta_D)$ be a name with $\delta_D(p_0) = s_0$. We say that δ_D satisfies *non-oracular locality of finite access at* s_0 if no finite prefix of p_0 decides all tail contingencies uniformly, in the following cylinder sense:

$$\forall m \in \mathbb{N} : \quad U_{p_0,m} \not\subseteq \{s_0\}. \tag{3}$$

Equivalently, every finite-information neighborhood of s_0 remains compatible with at least one admissible alternative state $s \neq s_0$.

Definition 2.1 is deliberately weak. It does not impose computability, effectivity, or any particular coding scheme. It states only that finite prefixes represent finite access rather than prepackaged answers to extensional questions. Without such a restriction, denotation becomes representation-engineering (Proposition B.1).

Lemma 2.2 (Prefix neighborhoods cannot decide tail contingencies at s_0)**.** *Assume δ_D satisfies non-oracular locality of finite access at s_0 in the sense of Definition 2.1. Let p_0 be any name of s_0. Then for every prefix depth $m \in \mathbb{N}$,*

$$\exists s \in U_{p_0,m} : \quad s \neq s_0.$$

Proof. Immediate from (3). □

Lemma 2.2 is purely semantic. In later sections we instantiate it using an explicit countable family of pairwise disjoint localized perturbation states whose point readout has an $O(1)$ jump between the reference state and any perturbation.

2.4 Denotation as finite-precision stability

The semantic issue is not whether $M : D \to \mathbb{R}$ exists as a set-theoretic function. It is whether the value of M is stable under finite access to names.

Semantic commitment 1 (Denotation). If M is treated as an ontic real-valued magnitude, explanatory realism requires that M be semantically accessible through the theory's fixed state interface. In this setting, that requirement is expressed as continuity-on-names of a witness map R satisfying (1), equivalently as the finite-precision stability condition below [Weihrauch, 2000, Schröder, 2002].

Definition 2.3 (Denotation at a name)**.** Let $p \in \mathrm{dom}(\delta_D)$ name the state $s = \delta_D(p)$. We say the magnitude M *denotes at* p if for every $N \in \mathbb{N}$ there exists a prefix depth m such that all states still compatible with that prefix yield outputs agreeing to within 2^{-N}:

$$\forall N \in \mathbb{N}\ \exists m \in \mathbb{N} : \quad \mathrm{diam}\big(M(U_{p,m})\big) \ < \ 2^{-N}. \tag{4}$$

In words: to determine N bits of the output, there must exist some finite interface depth after which the available prefix information already forces those N bits.

This diameter criterion is not ad hoc: it is exactly the name-level continuity condition expressed in cylinder form [Weihrauch, 2000, Schröder, 2002].

Proposition 2.4 (Continuity-on-names $\iff$ diameter shrinkage)**.** *Fix representations δ_D and $\delta_{\mathbb{R}}$. Let $M : D \to \mathbb{R}$ and let $p \in \mathrm{dom}(\delta_D)$. The following are equivalent:*

1. *There exists a* continuous *witness map $R : \mathrm{dom}(\delta_D) \to \mathrm{dom}(\delta_{\mathbb{R}})$ satisfying* (1) *(at p).*
2. *The diameter shrinkage condition* (4) *holds (i.e. M denotes at p).*

Proof sketch. In the represented-spaces framework, continuity of R means that for each output precision level, the first N symbols of the output name $R(p)$ depend only on some finite prefix length m of the input name p [Weihrauch, 2000]. But agreeing on the first m symbols is exactly membership in the same cylinder $[p\!\restriction_m]$, which corresponds to the compatible state set $U_{p,m}$. Thus continuity is equivalent to: at some finite m, all compatible states yield outputs lying in an interval of diameter $< 2^{-N}$. □

Proposition 2.4 is the bridge between (a) the witness equation (1), and (b) the finite-access geometry of compatible-state sets $U_{p,m}$.

2.5 Forced extensional totalization (FET)

The failure mode exploited by this paper is the strongest possible violation of finite-precision stability.

Definition 2.5 (Forced extensional totalization (FET))**.** We say that the pipeline exhibits *forced extensional totalization (FET)* at the name p if there exists some fixed precision threshold 2^{-N} such that *no* finite prefix depth settles even that coarse precision:

$$\exists N \in \mathbb{N}\ \forall m \in \mathbb{N}: \quad \operatorname{diam}\big(M(U_{p,m})\big) \ \geq \ 2^{-N}. \tag{5}$$

Equivalently, there exists $\varepsilon > 0$ such that for every m there are two states consistent with the same depth-m prefix whose readout values differ by at least ε. That is, within *every* finite-information neighborhood of the same purported state, the magnitude can still flip by an $O(1)$ amount.

Unlike computability pathologies in the sense of Turing-style effectiveness [Turing, 1936], FET does not require subtle encodings, infinite series, or slow convergence. It is simply the statement that an unbounded family of mutually independent tail contingencies retains undominated leverage on the readout.

Corollary 2.6 (FET $\Rightarrow$ non-denotation)**.** *If FET holds at a name p for the pipeline $(\delta_D, M, \delta_{\mathbb{R}})$, then no total continuous witness map R exists at p. Equivalently, the magnitude M is* non-denoting *at p.*

Proof. If FET holds then (4) fails, i.e. the diameter does not shrink below a fixed threshold at any finite prefix depth. By Proposition 2.4, the failure of diameter shrinkage is exactly the failure of continuity-on-names for any witness map R satisfying (1). □

This corollary is the formal semantic lever used in the reductio: once we engineer a physical magnitude M for which the compatible-output diameter remains $O(1)$ in every prefix neighborhood, non-denotation follows immediately.

2.6 Dynamics and post-evolution readout

In applications to continuum dynamics, the magnitude of interest is not usually a primitive readout on initial states. It is a *post-evolution* readout.

Let F_{t_0} denote an evolution map (e.g. for the wave equation) from initial data to the field at time t_0. Let E_{x_0} denote a real-valued readout (e.g. evaluation at a spatial point x_0). Then the magnitude studied in this paper is the composite

$$M_{(t_0,x_0)} \ := \ E_{x_0} \circ F_{t_0}. \tag{6}$$

Semantically, nothing changes: denotation and FET are defined only in terms of the induced map from initial states to real numbers. Thus in all definitions above one may take

$$M := M_{(t_0,x_0)}.$$

Accordingly, the witness equation becomes

$$\delta_{\mathbb{R}}(R(p)) \ = \ M_{(t_0,x_0)}(\delta_D(p)),$$

and FET/non-denotation is decided by the prefix-diameter behavior of

$$\operatorname{diam}\big(M_{(t_0,x_0)}(U_{p,m})\big) \qquad \text{as} \qquad m \to \infty.$$

2.7 Semantic closure as denotation preservation

We can now state the second semantic commitment imposed by explanatory realism in the notation developed above. Denotation is a local, name-relative stability requirement for a single magnitude. Closure is the corresponding global coherence requirement: the theory's own evolution and readout pipelines must preserve denotation on the admissible domain.

Semantic commitment 2 (Closure). If D is the theory's class of admissible states and (D, δ_D) is the fixed finite-access presentation of those states, then explanatory realism requires that the theory's evolution and readout rules do not drive admissible states to non-denoting magnitudes. In the present setting, this requirement is naturally expressed as denotation preservation for the induced evolution–readout pipeline.

Let $F : D \to X$ be a total evolution map (e.g. F_{t_0} for the wave equation), let $E : X \to \mathbb{R}$ be a real-valued readout functional (e.g. point evaluation E_{x_0}), and define the induced magnitude $M := E \circ F : D \to \mathbb{R}$.

Definition 2.7 (Semantic closure for an evolution–readout pipeline)**.** Fix an admissible state presentation (D, δ_D) and an admissible real presentation $\delta_{\mathbb{R}}$ [Weihrauch, 2000, Schröder, 2002]. We say the theory is *semantically closed* for the evolution–readout pipeline (F, E) on (D, δ_D) if the induced magnitude $M := E \circ F$ *denotes at every admissible state name*, i.e.

$$\forall p \in \mathrm{dom}(\delta_D)\ \forall N \in \mathbb{N}\ \exists m \in \mathbb{N} : \quad \mathrm{diam}\big(M(U_{p,m})\big) < 2^{-N},$$

where $U_{p,m}$ is the compatible-state set defined in (2).

Remark 2.8 (Failure of semantic closure)*.* Semantic closure fails as soon as there exists an admissible state name $p \in \mathrm{dom}(\delta_D)$ at which M does not denote. Equivalently, by Corollary 2.6, semantic closure fails if the evolution–readout pipeline exhibits FET at some admissible name.

The reductio construction therefore has a simple semantic structure: engineer an admissible domain D, a non-oracular state interface δ_D, and an evolution–readout magnitude $M = E \circ F$ such that FET holds at a name of some admissible state. Corollary 2.6 then yields non-denotation, and Definition 2.7 yields failure of semantic closure.

3 Continuum dynamics and a one-bit fan-in readout

This section isolates the physical mechanism used in the construction. The key mechanism is an *existential* fan-in readout:

> *The initial condition contains at most one localized contingency. If any contingency is present, the point readout returns* 1*; otherwise it returns* 0*.*

The sting is that this is not a tail-sensitivity effect. It is not about convergence. It is about a single hidden localized degree of freedom that may lie beyond any finite interface prefix, yet whose effect at readout is $O(1)$.

We work with the standard wave equation in $\mathbb{R}^3$ and exploit only one fact: point readout $u(x_0, t_0)$ is a fixed linear functional of the initial data localized on the sphere $|x - x_0| = t_0$ (Huygens' principle and Kirchhoff formula) [Courant and Hilbert, 1962, Hadamard, 1923, Evans, 2010, Taylor, 2011].

3.1 The wave equation in $\mathbb{R}^3$

Let $u : \mathbb{R}^3 \times [0, \infty) \to \mathbb{R}$ satisfy the (unit-speed) wave equation

$$\partial_t^2 u \;=\; \Delta u, \tag{7}$$

with Cauchy data at $t = 0$:

$$u(x, 0) = f(x), \qquad \partial_t u(x, 0) = g(x), \tag{8}$$

where

$$f, g \in C_c^\infty(\mathbb{R}^3).$$

For such data the solution exists globally and remains smooth [Courant and Hilbert, 1962, Evans, 2010, Taylor, 2011]. All states used in the construction will be of this benign kind: smooth, compactly supported, finite energy.

3.2 Huygens' principle and point readout as a linear functional

Fix a spacetime event (x_0, t_0) with $t_0 > 0$. In three spatial dimensions, solutions satisfy Huygens' principle: the value $u(x_0, t_0)$ depends only on the initial data on the sphere

$$S(x_0, t_0) := \{y \in \mathbb{R}^3 : |y - x_0| = t_0\}.$$

More precisely, Kirchhoff's formula gives an explicit representation. For smooth compactly supported Cauchy data (f, g) one has

$$u(x_0, t_0) = \frac{\partial}{\partial t}\bigg|_{t=t_0} \left(\frac{t}{4\pi} \int_{S^2} f(x_0 + t\omega)\, d\omega \right) + \frac{t_0}{4\pi} \int_{S^2} g(x_0 + t_0\omega)\, d\omega, \tag{9}$$

where $d\omega$ denotes surface measure on the unit sphere S^2 [Courant and Hilbert, 1962, Hadamard, 1923, Evans, 2010, Taylor, 2011]. Thus even though we read out a *point value*, the PDE enforces a fixed geometric aggregation over a sphere.

For the belt constructions below it is convenient to isolate the dependence on *angular* data explicitly. Write spherical coordinates about x_0 as $y = x_0 + r\omega$ with $r \geq 0$ and $\omega \in S^2$, and consider shell-supported displacement data of the form

$$f(r, \omega) = \eta(r - t_0)\, h(\omega), \qquad g \equiv 0,$$

where $\eta \in C_c^\infty(\mathbb{R})$ is a fixed radial bump supported in a thin neighborhood of 0, and $h \in C^\infty(S^2)$ is arbitrary. Evaluating (9) at $t = t_0$ depends only on the restriction of f and its radial derivatives on the readout sphere $r = t_0$. For the separable ansatz above one has

$$f(t_0, \omega) = \eta(0)\, h(\omega), \qquad \partial_r f(t_0, \omega) = \eta'(0)\, h(\omega),$$

and similarly for any higher radial derivatives that appear after expanding the time derivative in (9). Thus the entire radial-derivative dependence collapses to fixed coefficients determined by η (through $\eta(0), \eta'(0), \ldots$), leaving a bounded linear functional of h alone:

$$u(x_0, t_0) = \int_{S^2} K(\omega)\, h(\omega)\, d\omega, \tag{10}$$

for some smooth kernel $K \in C^\infty(S^2)$ determined by t_0 and by the fixed choice of the radial bump η. A clean explicit realization of such shell-localized data is given in Appendix A.

For our purposes it suffices to record the following structural fact.

Proposition 3.1 (Point readout as a continuous linear functional)**.** *Fix* (x_0, t_0) *with* $t_0 > 0$. *There exists a continuous linear functional*

$$\Lambda : C_c^\infty(\mathbb{R}^3) \times C_c^\infty(\mathbb{R}^3) \to \mathbb{R}$$

such that for every smooth compactly supported initial state $s = (f, g)$*, the corresponding solution satisfies*

$$u_s(x_0, t_0) = \Lambda(f, g).$$

Moreover, $\Lambda(f, g)$ *depends only on the restriction of* (f, g) *to the sphere* $S(x_0, t_0)$ *and its radial derivatives there.*

This is standard PDE theory: it is the content of Kirchhoff's formula and the associated Huygens' principle in odd spatial dimension 3 [Courant and Hilbert, 1962, Hadamard, 1923, Evans, 2010, Taylor, 2011]. The important point is conceptual: even though we read out a *point value*, the PDE enforces a fixed geometric aggregation over a sphere.

3.3 A countable family of independent "belt" perturbations

We now describe the localized components used in the construction. Fix an (arbitrary) countable family of pairwise disjoint open sets

$$B_1, B_2, \dots \subset S(x_0, t_0),$$

which we call *belts*, accumulating only at a set of measure zero. Concretely, one may take the latitude belts shown in Figure 1, shrinking toward the equator. For each n we will choose a smooth compactly supported initial state

$$s_n = (f_n, g_n) \in C_c^\infty(\mathbb{R}^3) \times C_c^\infty(\mathbb{R}^3)$$

such that:

1. **Localization**. The restriction of (f_n, g_n) to $S(x_0, t_0)$ is supported inside the belt B_n.
2. **Independence**. The belts are disjoint, so no point on the sphere lies in more than one belt.
3. **Uniform readout effect**. The point readout at (x_0, t_0) is *the same for every belt*:

$$\Lambda(f_n, g_n) = u_{s_n}(x_0, t_0) = 1. \tag{11}$$

Condition (11) is the critical feature. Each belt, if activated, contributes an $O(1)$ jump at readout. Because Λ is linear, (11) can be enforced by scaling each belt-localized perturbation appropriately. For any belt-supported (f_n', g_n') with $\Lambda(f_n', g_n') \neq 0$, define

$$(f_n, g_n) := \frac{1}{\Lambda(f_n', g_n')}(f_n', g_n'),$$

which yields $\Lambda(f_n, g_n) = 1$. The existence of nontrivial smooth belt-supported data with nonzero point readout is immediate from the explicit Kirchhoff representation of Λ and the density of C_c^∞ data localized near the sphere [Courant and Hilbert, 1962, Hadamard, 1923, Evans, 2010, Taylor, 2011].

3.4 The restricted domain: at most one active belt

We now impose the restriction that makes the argument maximally sharp: the admissible initial states in the construction contain *at most one* active belt perturbation. Define the countable family of states

$$s_0 := (0,0), \qquad s_n := (f_n, g_n) \;\; (n \geq 1),$$

and define the reduced domain

$$D_{\leq 1} := \{s_0\} \;\cup\; \{s_n : n \geq 1\}. \tag{12}$$

This is the continuum-dynamical analogue of the toy bitstream domain

$$\{b \in \{0,1\}^{\mathbb{N}} : \sum_n b_n \leq 1\},$$

but now realized by smooth compactly supported wave-equation data.

3.5 The fan-in magnitude

We can now package the construction into a single post-evolution point magnitude. Define

$$M_* : D_{\leq 1} \to \mathbb{R}, \qquad M_*(s) := u_s(x_0, t_0) = \Lambda(f, g).$$

That is, M_* takes an admissible initial state $s = (f, g)$ and returns the evolved field value at the fixed spacetime event (x_0, t_0).

By construction of the reduced domain $D_{\leq 1} = \{s_0\} \cup \{s_n : n \geq 1\}$ and the uniform normalization of the belt states, we have

$$M_*(s_0) = 0, \qquad M_*(s_n) = 1 \text{ for all } n \geq 1. \tag{13}$$

Thus the wave equation together with Kirchhoff point readout implements, on the restricted domain $D_{\leq 1}$, the simplest possible *existential fan-in*:

> *if there exists an active belt, the readout is* 1*; if no belt is active, the readout is* 0*.*

At this stage the *physics* content of the construction is complete. The remaining ingredient is semantic and concerns only the state interface δ_D. To obtain forced extensional totalization at s_0, one does *not* need any metric smallness argument or any claim that the belt states converge to s_0 in a particular function-space topology. One needs only the finite-access locality requirement that the interface does not decide the extensional existential predicate "some belt is active" at any finite prefix depth. In the cylinder language of Section 2, this is exactly the *local underdetermination at* s_0 hypothesis (Theorem 5.2), discussed further in Appendix B.
Once that single non-oracularity condition is imposed, the semantic conclusion is immediate: every finite-prefix neighborhood of the zero state s_0 remains compatible with some belt state s_n (with n arbitrarily large), while the readout differs by $O(1)$ between these cases. Because (13) is a uniform jump, this yields forced extensional totalization at s_0 in its strongest possible form.

4 Angular localization: a countable family of one-bit perturbations

This section constructs the geometric ingredients for the existential fan-in construction. We build a countable family of smooth, compactly supported initial states

$$s_0 = (0, 0), \qquad s_n = (f_n, g_n) \ \ (n \geq 1),$$

with the following properties:

1. each s_n is localized on its own disjoint "latitude belt" on the Huygens sphere $S(x_0, t_0)$, so these perturbations remain independently specifiable at the interface level;
2. each s_n has the *same* $O(1)$ *effect* at readout:

$$u_{s_n}(x_0, t_0) = 1 \qquad \text{for every } n \geq 1;$$

3. in the reduced domain $D_{\leq 1} = \{s_0\} \cup \{s_n : n \geq 1\}$, there is at most one active contingency.

Intuitively, each belt perturbation is smooth, compactly supported, and well-posed in the PDE sense. The semantic obstruction arises only when point readout collapses a countable family of independently local contingencies into a single bit:

"is *some* belt active?"

Under any non-oracular interface, every finite-prefix neighborhood of the zero state remains compatible with activating a sufficiently deep belt, while the readout jump remains 1.

4.1 Shell localization and reduction to angular data

Fix the readout event (x_0, t_0) with $t_0 > 0$. Let (r, ω) denote spherical coordinates about x_0 with

$$r = |x - x_0|, \qquad \omega \in S^2.$$

Choose once and for all a radial bump function

$$\eta \in C_c^\infty(\mathbb{R}), \qquad \eta \geq 0, \qquad \operatorname{supp}(\eta) \subset (-\delta, \delta)$$

for some small fixed $\delta > 0$, with normalization

$$\int_{\mathbb{R}} \eta(\rho)\, d\rho = 1.$$

Define the spherical-shell localization factor

$$\eta_{t_0}(r) := \eta(r - t_0),$$

so η_{t_0} is supported in a thin fixed shell around the sphere $r = t_0$. We will consider initial displacement data of the form

$$f(r, \omega) = \eta_{t_0}(r)\, h(\omega), \qquad g \equiv 0, \tag{14}$$

where $h : S^2 \to \mathbb{R}$ is smooth and carries all degrees of freedom. This keeps the PDE discussion transparent: everything is localized onto the Huygens sphere while retaining smoothness and compact support in $\mathbb{R}^3$ [Courant and Hilbert, 1962, Evans, 2010, Taylor, 2011, Lee, 2013].

Figure 1: **Disjoint latitude belts accumulating at the equator.** The sphere is shown with latitudes at heights $z = \pm C/n$. Each belt B_n supports a single *one-bit perturbation state* s_n. The belts shrink and accumulate at the equator while remaining pairwise disjoint. This arrangement makes the perturbations independently local and smooth, while allowing arbitrarily "deep" belts beyond any fixed finite resolution of the state interface.

4.2 Latitude belts with disjoint supports

We now define a countable family of disjoint latitude belts on S^2. Write $\omega = (\omega_1, \omega_2, \omega_3) \in S^2$ and let

$$z(\omega) := \omega_3 \in (-1, 1)$$

denote latitude height. Define the shrinking intervals

$$J_n^+ := \left(\frac{1}{n+1}, \frac{1}{n} \right), \qquad J_n^- := -J_n^+,$$

and enumerate the collection $\{J_n^+, J_n^-\}_{n \geq 1}$ as a single sequence $\{J_n\}_{n \geq 1}$ in any fixed order. Define the corresponding latitude belts

$$B_n := \{\omega \in S^2 : z(\omega) \in J_n\}.$$

These belts are pairwise disjoint, each has nonempty interior, and they accumulate only at the equator $z = 0$ (see Figure 1).

4.3 Smooth belt bumps

For each $n \geq 1$, choose a smooth cutoff function

$$\psi_n \in C^\infty(S^2), \qquad 0 \leq \psi_n \leq 1, \qquad \mathrm{supp}(\psi_n) \subset B_n, \qquad \psi_n \not\equiv 0.$$

This is standard smooth-manifold theory: every nonempty open set on a smooth manifold admits a nontrivial C^∞ bump function supported within it [Lee, 2013]. Define the angular profile

$$h_n(\omega) := \psi_n(\omega),$$

and the associated initial state

$$s_n = (f_n, g_n), \qquad f_n(r, \omega) := \eta_{t_0}(r)\, h_n(\omega) = \eta_{t_0}(r)\psi_n(\omega), \qquad g_n \equiv 0. \tag{15}$$

Each f_n is $C_c^\infty(\mathbb{R}^3)$ and supported in a thin spherical shell, and its restriction to the Huygens sphere $r = t_0$ lies entirely within the belt B_n. This yields the key locality property: perturbations indexed by different n are disjointly localized on the sphere.

4.4 Uniform transfer to the point readout

Section 3 established that point readout at (x_0, t_0) defines a continuous linear functional on initial data:

$$u_s(x_0, t_0) = \Lambda(f, g)$$

(Kirchhoff/Huygens) [Courant and Hilbert, 1962, Hadamard, 1923, Evans, 2010, Taylor, 2011]. We now enforce the decisive uniformity condition: *every belt perturbation produces the same $O(1)$ effect at readout.*

Lemma 4.1 (Uniform readout normalization)**.** *For each $n \geq 1$ there exists a scalar $a_n \neq 0$ such that if we replace the state $s_n = (f_n, g_n)$ of (15) by the rescaled state*

$$s'_n = (f'_n, g'_n), \qquad (f'_n, g'_n) := a_n(f_n, g_n),$$

then the evolved point value at (x_0, t_0) satisfies

$$u_{s'_n}(x_0, t_0) = 1.$$

Proof. Fix n. Because the point readout is a nontrivial continuous linear functional of the initial data on the sphere $r = t_0$ (Kirchhoff/Huygens), and because ψ_n may be chosen arbitrarily subject only to $\psi_n \not\equiv 0$ and $\mathrm{supp}(\psi_n) \subset B_n$, we may choose ψ_n so that $\Lambda(f_n, g_n) \neq 0$ [Courant and Hilbert, 1962, Hadamard, 1923, Evans, 2010, Taylor, 2011]. Then set

$$a_n := \frac{1}{\Lambda(f_n, g_n)}.$$

Linearity of Λ yields

$$u_{s'_n}(x_0, t_0) = \Lambda(f'_n, g'_n) = a_n \Lambda(f_n, g_n) = 1.$$

□

We henceforth rename s'_n as s_n for simplicity, and assume:

$$u_{s_n}(x_0, t_0) = 1 \tag{16}$$

for all $n \geq 1$. In particular, each belt perturbation has full-scale effect at readout.

4.5 The reduced one-bit domain

Finally, define the reduced domain of admissible states used in the theorem:

$$D_{\leq 1} := \{s_0\} \cup \{s_n : n \geq 1\}, \qquad s_0 := (0,0).$$

This is the "at most one active belt" restriction. It eliminates any possibility that the result is an artifact of combining many contingencies in a single state.

Under this restriction, the readout magnitude becomes

$$M_*(s) := u_s(x_0, t_0),$$

and by construction

$$M_*(s_0) = 0, \qquad M_*(s_n) = 1 \ (n \geq 1).$$

The semantic obstruction proved in the next section arises because, under any physically meaningful non-oracular interface, every finite-prefix neighborhood of the zero state s_0 remains compatible with activating some sufficiently deep belt s_n. Since the readout jump is 1, this yields forced extensional totalization at finite precision in the strongest possible form.

5 Main theorem: FET induced by linear continuum dynamics

This section states and proves the central result. Section 4 constructed a countable family of smooth, compactly supported, finite-energy initial states

$$s_0 = (0,0), \qquad s_n = (f_n, g_n) \ \ (n \geq 1),$$

each localized on a distinct shrinking latitude belt B_n on the Huygens sphere. Each belt perturbation produces the same $O(1)$ contribution to point readout:

$$u_{s_n}(x_0, t_0) = 1 \quad (n \geq 1), \qquad u_{s_0}(x_0, t_0) = 0.$$

The semantic obstruction is interface-relative: under any physically meaningful non-oracular presentation of states, no finite prefix of a name compatible with s_0 can exclude that a sufficiently deep belt perturbation is active. As a result, the point-readout magnitude exhibits *forced extensional totalization* (FET) at s_0.

Throughout, we use the denotation and FET criteria developed in Section 2, which are standard in represented spaces and Type–2 semantics [Weihrauch, 2000, Schröder, 2002, Kreitz and Weihrauch, 1985]. Once FET is established for the readout pipeline, non-denotation follows immediately by Corollary 2.6.

5.1 A concrete locality-of-access example

Before stating the theorem, we give a concrete illustration of the only semantic hypothesis used below: a minimal locality-of-access (non-oracle) condition at the zero state.

Example 5.1 (Why the locality-of-access hypothesis is physically natural)**.** To see why the "local underdetermination at s_0" condition is minimal and physically natural, consider a standard kind of finite access to smooth, compactly supported initial data $(f, g) \in C_c^\infty(\mathbb{R}^3)$.

A finite interface prefix $p\upharpoonright_m$ may be thought of as encoding only coarse localized information about (f, g): for example, finitely many sampled values and derivatives of f and g on a finite spatial grid, with bounded spatial extent and bounded resolution (not necessarily effectively bounded, only finite). Equivalently, $p\restriction_m$ constrains (f,g) only through finitely many local linear functionals (point evaluations, local averages, jet coefficients on a finite set of sites, etc.).

Under any such finite-access scheme, agreement on a prefix cannot decide *existential tail facts* about arbitrarily fine, spatially localized degrees of freedom. In particular, no finite prefix compatible with the zero state can rule out the possibility that a smooth bump is present on a sufficiently thin latitude belt B_n on the Huygens sphere $S(x_0,t_0)$, with n so large that the belt lies below the interface's resolved angular scale. Thus for every prefix length m, one can choose n large enough that there exist names p_0 of s_0 and p_n of s_n whose prefixes agree:

$$p_0\restriction_m = p_n\restriction_m, \qquad \delta_D(p_0)=s_0, \quad \delta_D(p_n)=s_n.$$

This is exactly the locality-of-access hypothesis used in Theorem 5.2.

We now state the main theorem in a form that explicitly lists the pipeline ingredients.

5.2 Statement of the main theorem

Theorem 5.2 (FET at a point readout in continuum dynamics)**.** *Fix (x_0,t_0) with $t_0>0$ and consider the wave equation in $\mathbb{R}^3$ with smooth, compactly supported initial data and evolution operator*

$$F_{t_0} : (f,g) \mapsto u(\cdot,t_0).$$

(Existence, uniqueness, and regularity on smooth compactly supported data are classical; see, e.g., Courant and Hilbert [1962], Hadamard [1923], Evans [2010], Taylor [2011].)
Let $D_{\le 1}$ be the reduced admissible domain defined in Section 4.5:

$$D_{\le 1} := \{s_0\} \cup \{s_n : n \ge 1\}, \qquad s_0=(0,0), \qquad s_n=(f_n,g_n),$$

where each s_n is smooth, compactly supported, finite-energy, supported on a single latitude belt B_n, and normalized so that

$$u_{s_n}(x_0,t_0)=1 \quad (n\ge 1), \qquad u_{s_0}(x_0,t_0)=0. \tag{17}$$

Let $(D_{\le 1},\delta_D)$ be any admissible state presentation intended to reflect ordinary finite-information access to such localized smooth data in the represented-spaces sense [Weihrauch, 2000, Schröder, 2002], and assume δ_D satisfies the following minimal locality-of-access condition:

(Local underdetermination at s_0) *For every prefix length $m\in\mathbb{N}$ there exists an index $n\ge 1$ and names $p_0,p_n \in \mathrm{dom}(\delta_D)$ such that*

$$\delta_D(p_0)=s_0, \qquad \delta_D(p_n)=s_n, \qquad p_0\restriction_m = p_n\restriction_m .$$

Define the point-readout magnitude

$$M_* : D_{\le 1} \to \mathbb{R}, \qquad M_*(s) := u_s(x_0,t_0).$$

Then the readout pipeline $(\delta_D, M_*, \delta_{\mathbb{R}})$ exhibits forced extensional totalization *(FET) at s_0 in the sense of Definition 2.5. Consequently:*

1. *the point magnitude* M_* *is* non-denoting at s_0 *relative to the fixed interface* $(D_{\leq 1}, \delta_D)$*, by Corollary 2.6;*

2. *the evolution–readout pipeline* (F_{t_0}, E_{x_0}) *fails* semantic closure *on* $(D_{\leq 1}, \delta_D)$ *in the sense of Definition 2.7.*

Remark 5.3 (What the hypothesis means). The "local underdetermination at s_0" hypothesis says only this: no matter how much finite interface information a state-name prefix provides, it does not rule out that a sufficiently deep belt perturbation s_n is active. This is the minimal non-oracular locality-of-access constraint required for denotation to have nontrivial content, in the same spirit as the cylinder-based continuity conditions standard in represented-space semantics [Weihrauch, 2000, Schröder, 2002].

5.3 Proof overview

The proof is extremely simple once the construction is in hand.

1. Section 4 constructed belt-localized smooth states s_n whose point readout at (x_0, t_0) is uniformly normalized to 1 using linearity of the Kirchhoff/Huygens readout functional [Courant and Hilbert, 1962, Hadamard, 1923, Evans, 2010, Taylor, 2011].

2. The reduced domain $D_{\leq 1}$ ensures that each state contains *at most one* localized contingency.

3. The locality-of-access hypothesis implies that every finite-prefix neighborhood of the zero state s_0 contains some belt state s_n.

4. Since $M_*(s_0) = 0$ and $M_*(s_n) = 1$, the readout diameter is 1 inside every finite-prefix neighborhood of s_0. This is exactly the witness pattern for FET (Definition 2.5).

5. By Corollary 2.6 (via the continuity-on-names ⇔ diameter-shrinkage equivalence in Proposition 2.4), we conclude that M_* fails to denote at s_0 and semantic closure fails.

5.4 Proof of Theorem 5.2

Proof. Let $M_* : D_{\leq 1} \to \mathbb{R}$ be the point readout magnitude

$$M_*(s) := u_s(x_0, t_0).$$

By construction (17) we have

$$M_*(s_0) = 0, \qquad M_*(s_n) = 1 \quad \text{for all } n \geq 1.$$

Fix an arbitrary prefix length $m \in \mathbb{N}$. By the local underdetermination hypothesis at s_0, there exist an index $n \geq 1$ and names $p_0, p_n \in \mathrm{dom}(\delta_D)$ such that

$$\delta_D(p_0) = s_0, \qquad \delta_D(p_n) = s_n, \qquad p_0 \restriction_m = p_n \restriction_m .$$

Let $U_{p_0,m}$ be the compatible-state set defined in (2). Then $s_0, s_n \in U_{p_0,m}$, and hence

$$\mathrm{diam}\big(M_*(U_{p_0,m})\big) \;\geq\; |M_*(s_n) - M_*(s_0)| \;=\; |1 - 0| \;=\; 1.$$

Now take $N = 1$, so that $2^{-N} = 2^{-1}$. The inequality above implies that for *every* prefix length m,

$$\mathrm{diam}\big(M_*(U_{p_0,m})\big) \;\geq\; 1 \;>\; 2^{-1}.$$

Therefore the pipeline $(\delta_D, M_*, \delta_{\mathbb{R}})$ exhibits *forced extensional totalization* at p_0 in the sense of Definition 2.5. By Corollary 2.6, M_* is non-denoting at the name p_0 naming s_0 relative to the fixed state interface $(D_{\leq 1}, \delta_D)$.
Finally, by Definition 2.7, semantic closure for the evolution–readout pipeline (F_{t_0}, E_{x_0}) requires denotation of the induced post-evolution magnitude at *every* admissible name. Since M_* fails to denote at p_0, semantic closure fails on $D_{\leq 1}$. □

5.5 Corollary: failure of semantic closure for the admissible domain

Corollary 5.4 (Semantic non-closure under wave evolution with point readout). *Under the hypotheses of Theorem 5.2, the wave evolution together with point readout fails semantic closure on the admissible domain $D_{\leq 1}$: there exists an admissible initial state $s_0 \in D_{\leq 1}$ such that the evolved point magnitude $u_{s_0}(x_0, t_0)$ does not denote relative to the fixed interface $(D_{\leq 1}, \delta_D)$.*

Proof. Immediate from Theorem 5.2. □

Remark 5.5 (Where the "sting" lives). The corollary can be read in the bluntest possible way: even when the initial condition contains *at most one* localized contingency, and even though the PDE is linear and well posed on smooth data, a localized point readout can demand a scalar magnitude whose value is semantically underdetermined at all finite interface depths. The physical theory permits the magnitude to be either 0 or 1 inside every finite-information neighborhood of the same purported state. Under explanatory realism, that is a failure of semantic coherence.

6 Interpretation, scope, and objections

This section explains what the theorem means and (equally important) what it does *not* mean. The construction of Sections 3–5 is deliberately elementary. It avoids the usual distractions (computability, chaotic sensitivity, regional statehood, function-space semantics) and instead targets a single question:

> *If continuum field values are ontic magnitudes, and if admissible states are presented through a physically meaningful (non-oracular) interface, can a well-posed linear PDE preserve semantic closure under point readout?*

Theorem 5.2 answers *no*. Even a single hidden local contingency is enough.

6.1 What the result does not claim

Theorem 5.2 is a semantic obstruction, not a computational or operational one. To prevent confusion, we list explicitly what is *not* being claimed.

- **Not an effective-computation claim.** The result does not assert that the wave equation computes anything in the algorithmic sense [Turing, 1936, Pour-El and Richards, 1989]. No Type–1 or Type–2 machine is implemented by the PDE, and no effective procedure is extracted from the dynamics.
- **Not a measurement-feasibility claim.** The result does not depend on whether the relevant initial data can be prepared, whether the readout can be measured, or whether any experimental apparatus could resolve the contingency in practice. Nothing here is epistemic; the issue is semantic determinacy relative to a fixed finite-access interface [Weihrauch, 2000, Schröder, 2002].

- **Not a claim about noncomputable initial data.** Every state used in the construction is smooth, compactly supported, and of finite energy. No pathological distributions and no function-space singularities are employed (cf. the standard well-posedness theory for the wave equation on smooth data [Courant and Hilbert, 1962, Hadamard, 1923, Evans, 2010, Taylor, 2011]).

- **Not a "halting problem" stunt.** The present argument does not depend on the halting set, Chaitin's Ω, or any diagonalization or incompleteness phenomenon [Turing, 1936, Chaitin, 1975, 1987, Downey and Hirschfeldt, 2010]. In particular, the obstruction is *not* that the readout computes a noncomputable real, nor that the evolution hides an oracle in fine-scale initial data. Rather, the obstruction is purely semantic: a point readout induces an *existential fan-in* over a countable family of disjoint localized degrees of freedom,

  "is *any* localized contingency active?"

  and this fan-in has diameter 1 inside every finite-prefix neighborhood of the zero state under any non-oracular finite-access interface. There is no tail suppression, no infinite weighted sum, and no appeal to noncomputability.

In short, the theorem targets semantic coherence of *ontic* continuum magnitudes under a fixed state interface. It is entirely orthogonal to questions of computability and prediction.

6.2 Why point evaluation is the right battleground

The construction deliberately uses the simplest possible observable: a single pointwise field value $u(x_0, t_0)$. This choice is not rhetorical convenience but ontological necessity. Continuum field theories (classical or quantum) typically treat the field as assigning a definite value to each spacetime event. At minimum, the theory treats the map

$$(x, t) \;\mapsto\; u(x, t)$$

as meaningful, and pointwise field values are routinely interpreted as physical magnitudes (even if idealized); see, e.g., classical discussions of field quantities and operational meaning [Bridgman, 1927, Chang, 2004].

Once one takes that point ontology seriously, denotation becomes non-negotiable: if $u(x_0, t_0)$ is treated as a real-valued magnitude, then under explanatory realism it must denote relative to the theory's state interface. That commitment is made precise in the finite-access semantics of Section 2, built on standard represented-space notions of admissibility and continuity-on-names [Weihrauch, 2000, Schröder, 2002, Myhill, 1957, Kreitz and Weihrauch, 1985].

In this sense, point evaluation is the cleanest battleground: it is the least committal readout that still tracks what a continuum field ontology literally asserts. Theorem 5.2 shows that even this minimal readout can force semantic failure.

6.3 What changes if you weaken ontology or weaken readout

The most common response to semantic non-closure results is to retreat.

(i) Deny pointwise ontology. A critic may say: "point values are not physical; only smeared or averaged quantities matter." If one takes this line, the present theorem is simply *not addressed*, for it targets precisely the pointwise ontology that the critic has abandoned.

That retreat is coherent—but it is not innocent. It amounts to asserting that the continuum theory does not in fact treat $u(x, t)$ as an ontic magnitude, despite its formal language. One must then specify what *does* denote and why, and one must explain how subsystems and localized records are to be recovered without pointwise statehood. This is no longer standard continuum ontology.

(ii) Restrict readout to smoothing functionals. A second retreat is to permit only observables of the form

$$\int u(x, t_0)\, \phi(x)\, dx$$

for test functions ϕ with compact support. Such readouts suppress localized encoding by design, and they often admit strong continuity properties. Mathematically, these are the standard "smeared field" or distributional pairing observables familiar in PDE and distribution theory [Hörmander, 1983].

Again, that retreat may be coherent, but it is a substantive modification of the ontology. The critic is no longer interpreting the continuum field as assigning ontic magnitudes pointwise, but only through fixed coarse-graining.

(iii) Allow oracle presentations of states. A third response is semantic rather than physical: build extensional answers into finite prefixes of state names. Under such interfaces, any magnitude can be made denoting by construction.

This is a coherent move in pure logic, but it trivializes denotation as a finite-access constraint. If finite prefixes are permitted to encode globally pre-decided extensional facts about arbitrarily deep degrees of freedom, then the continuity-on-names requirement of Section 2 loses all discriminatory force: the interface itself can simply *stipulate* the value of every putative magnitude. In that case explanatory realism no longer constrains continuum ontology; it has been satisfied vacuously by a semantic oracle (compare the general role of admissibility as an "interface adequacy" condition [Weihrauch, 2000, Schröder, 2002]).

Accordingly, the present paper treats oracle interfaces as inadmissible as physical semantics.

In short: the theorem is not fragile. It can be evaded only by retreating from at least one of three standard assumptions:

1. pointwise continuum ontology;
2. localized readout as a legitimate magnitude;
3. non-oracular finite-information state interfaces.

Any of these retreats may be defendable, but each is a substantive philosophical and ontological move.

6.4 Relation to Pour–El and Richards

The work of Pour–El and Richards [Pour-El and Richards, 1989] is the canonical source for the observation that well-posed linear PDEs can produce semantic and computational obstructions when interpreted ontically.

Their results are often summarized as "computable initial data can evolve into noncomputable values." That summary is correct but incomplete: it concerns effective computation rather than semantic denotation.

The more serious semantic phenomenon appears in their Theorem 2, where a solution may fail to have an admissible name as an element of a represented function space once regional restrictions are treated as single magnitudes. Under explanatory realism, that is naturally read as a failure of denotation and hence of semantic closure; see also the represented-space perspective on admissible function-space names [Weihrauch, 2000, Schröder, 2002].

The present paper is not a reinterpretation of Pour–El and Richards and does not depend on their machinery. Instead it isolates a simpler obstruction in a more local form. No function-space semantics, regional statehood assumption, or global field restriction is needed: semantic failure already arises at the level of a single point magnitude.

In this sense, the relationship is one of mechanism rather than dependency: both results expose semantic non-closure, but the present construction shows that the obstruction does not require regional magnitudes. Point readout alone suffices.

6.5 Relation to realizability and witness semantics

The semantic structure of the theorem mirrors a well-known phenomenon in logic: classical determination does not guarantee witness-supported existence.

In realizability semantics, an object may exist uniquely in the classical sense yet fail to exist in the witness-based sense. Bauer emphasizes this contrast using the Kleene-tree phenomenon: every finite stage is benign, and classical logic determines a unique infinite path, but no computational witness can select it [Bauer, 2006, 2025].

The present construction exhibits an analogous pattern in a physical setting. Each localized belt contingency is benign in isolation. The domain $D_{\leq 1}$ is extremely tame: every admissible state contains at most one localized perturbation, and the PDE dynamics are linear and well posed [Courant and Hilbert, 1962, Evans, 2010, Taylor, 2011]. Yet the readout $u(x_0, t_0)$ induces an *existential fan-in*:

"is there any active belt?"

and that fan-in defeats witness-supported denotation through the physical state interface. At the zero state, every finite-information neighborhood contains both possibilities (readout 0 and readout 1), yielding FET. Thus the continuum-theoretic obstruction should be read as a physical analogue of a general semantic divide:

> *classical existence/determinacy does not imply witness-supported semantic availability through an admissible interface.*

This is exactly the philosophical point that explanatory realism forces into the foreground [Psillos, 1999, Chakravartty, 2007]. If continuum magnitudes are to be ontic, then their semantic access must be coherent through the same interface that presents admissible states. Theorem 5.2 shows that linear continuum dynamics can violate that coherence in the most elementary localized setting.

6.6 Relation to ultraviolet limits in physical theory

The present result is a semantic one: it identifies an incompatibility between continuum state ontologies and the minimal denotational requirements imposed by explanatory realism. It does not establish the existence of a fundamental length scale, nor does it presuppose any specific model of quantum gravity. Nevertheless, the conclusion is consonant with a widely held expectation in modern physics that continuum field descriptions cannot be ontologically exact.

In many areas of theoretical physics, independent considerations motivate the idea that continuum degrees of freedom must be limited at sufficiently small scales. These include the breakdown of classical field theory at high energies, the appearance of natural length scales associated with quantum gravity, and entropy bounds suggesting finite information capacity per region [Weinberg, 1995a, Peskin and Schroeder, 1995, Susskind, 1995, Bekenstein, 1981]. Such arguments are typically dynamical or thermodynamic in character. The present argument is of a different kind: it is semantic rather than dynamical, and it operates already at the level of linear continuum dynamics with smooth, finite-energy states.

The obstruction identified here arises because continuum state spaces permit law-relevant localized contingencies whose influence on certain magnitudes cannot be finitely stabilized under any admissible finite-access interface. This failure of semantic closure occurs even in perfectly well-posed, reversible partial differential equations. The issue is therefore not ultraviolet divergence, nonlinearity, or quantum effects, but the unbounded informational capacity of the continuum state ontology itself. In this respect the result is closer in spirit to arguments based on finite information density or entropy bounds than to perturbative renormalization alone [Bekenstein, 1981, Susskind, 1995].

Taken together, these considerations provide an independent line of pressure toward the view that continuum field descriptions should be understood as effective or mathematical scaffolding rather than as literal ontologies. The argument does not single out a particular alternative, such as discrete structures, band-limited fields, or other finite-information frameworks. Instead, it shows that some restriction of continuum state spaces is required if magnitudes are to denote in a semantically coherent way under explanatory realism.

Thus the semantic constraint identified here aligns with, and reinforces, the broader physical expectation that fundamental theories must incorporate ultraviolet limits, even though the argument itself is entirely classical and representation-theoretic in character.

7 Conclusion

We have exhibited a minimal semantic obstruction to ontic readings of continuum dynamics under explanatory realism. The obstruction is not computational, epistemic, or probabilistic. It is semantic: *forced extensional totalization* (FET) induced by an otherwise standard continuum evolution–readout pipeline.

The construction is deliberately spare. We use the classical three-dimensional wave equation with smooth, compactly supported, finite-energy initial data. Countably many independent localized "belt" perturbations are placed on the light-sphere associated with a point readout (x_0, t_0), with disjoint supports accumulating at the equator. Crucially, the admissible domain is maximally tame: the initial state contains *at most one* nonzero localized contingency. Either no belt is active, or exactly one belt is active.

Nevertheless, Kirchhoff's formula forces a sharp semantic consequence. Point readout is not a primitive coordinate projection; in $\mathbb{R}^3$ it is intrinsically a fixed spherical aggregation of initial data over the light-sphere (Huygens' principle). This aggregation acts as a semantic amplifier. The induced magnitude becomes the simplest possible fan-in predicate: *is any localized contingency active or not?* If none is active, the readout is 0; if one is active, the readout is 1.

Under any admissible non-oracular state presentation satisfying a minimal locality-of-access constraint (Appendix B), this magnitude is *non-denoting* at the all-zero state. Every finite prefix of a state-name compatible with s_0 remains compatible with some belt state s_n, while the induced readout differs by order one. Thus, even at a fixed coarse precision, approximation exhibits *unbounded input dependence* and no total continuous semantic witness exists. By the finite-access semantics of Section 2 (in particular Definition 2.5 and Corollary 2.6), this establishes semantic failure of the induced point magnitude and hence a failure of semantic closure for the corresponding evolution–readout pipeline (Definition 2.7).

The conceptual stakes are straightforward. If pointwise field values are treated as ontic physical magnitudes, then denotation is not optional: ontic magnitudes must be semantically accessible through the theory's fixed finite-information state interface. And if certain states are taken to be physically admissible, then admissibility must be preserved by the theory's own evolution. The present result shows that these two minimal requirements can conflict even in a linear, well-posed, classical setting with smooth compactly supported data. The continuum is not merely hard to compute. Under explanatory realism it can be semantically non-closed.

Accordingly, this result is not a criticism of continuum PDEs as mathematical tools. It is a constraint on *ontic interpretation*. To retain explanatory realism while avoiding semantic non-closure, one must modify at least one element of the standard continuum package: the ontology (for example, deny point magnitudes), the admissible state interface (for example, permit oracle-like names, thereby trivializing denotation), or the kinematics (for example, adopt a finite-information state space). But one cannot keep all three, namely pointwise continuum ontology, non-oracular finite-access semantics, and semantic closure under evolution, without encountering forced extensional totalization.

A Analytic lemma: smooth belt bumps with uniform point readout

This appendix supplies the only analytic ingredient needed for the main construction: a countable family of smooth angular perturbations localized on pairwise disjoint shrinking latitude belts whose propagated contribution to a fixed point readout is *uniformly nontrivial* (indeed, can be normalized to equal 1). The purpose is purely geometric/analytic. No semantic assumptions are used here.

Setup: the angular readout kernel

Fix (x_0, t_0) with $t_0 > 0$ and a nonnegative radial bump $\eta \in C_c^\infty(\mathbb{R})$ supported near 0. For shell-supported initial displacement data of the form

$$f(r, \omega) = \eta(r - t_0)\, h(\omega), \qquad g \equiv 0,$$

Kirchhoff's formula for the 3-dimensional wave equation implies that the point value $u(x_0, t_0)$ depends linearly and continuously on the angular profile h (and, more explicitly, can be written as an integral against a smooth kernel on S^2) [Courant and Hilbert, 1962, Evans, 2010, Taylor, 2011]. In particular, there exists $K \in C^\infty(S^2)$, not identically zero, such that

$$u(x_0, t_0) = \int_{S^2} K(\omega)\, h(\omega)\, d\omega. \tag{18}$$

Here K is determined by (x_0, t_0, η). Since K is continuous and not identically zero, there exists a nonempty open set $U \subset S^2$ and a constant $c_0 > 0$ such that

$$|K(\omega)| \geq c_0 \qquad \text{for all } \omega \in U, \tag{19}$$

and (shrinking U if necessary) we may assume that K has fixed sign on U.

Belt geometry

Let $z(\omega) = \omega_3$ denote the latitude coordinate. Fix a family of pairwise disjoint open intervals $\{J_n\}_{n\geq 1}$ in $(-1, 1)$, accumulating only at 0, and define the corresponding latitude belts

$$B_n := \{\omega \in S^2 : \; z(\omega) \in J_n\}.$$

We assume the belts are chosen so that $\overline{B_n} \subset U$ for all n. (This can always be achieved by placing the belts in a region where K has fixed nonzero sign.)

Uniform belt bumps with prescribed readout coefficient

Lemma A.1 (Uniform belt bumps with prescribed readout). *Let $K \in C^\infty(S^2)$ be nonzero and let $\{B_n\}_{n\geq 1}$ be pairwise disjoint open latitude belts with $\overline{B_n} \subset U$, where U satisfies* (19) *and K has fixed sign on U. Then there exists a sequence $\psi_n \in C^\infty(S^2)$ such that:*

1. $\operatorname{supp}(\psi_n) \subset B_n$ *and* $0 \leq \psi_n \leq 1$*;*
2. $\displaystyle\int_{S^2} K(\omega)\, \psi_n(\omega)\, d\omega \neq 0$ *for every n;*
3. *defining the normalized angular profiles*

$$\varphi_n(\omega) := \frac{\psi_n(\omega)}{\int_{S^2} K(\nu)\, \psi_n(\nu)\, d\nu},$$

we have the exact normalization

$$\int_{S^2} K(\omega)\, \varphi_n(\omega)\, d\omega = 1 \qquad \textit{for every } n. \tag{20}$$

Proof. Fix a nonnegative reference bump $\chi \in C_c^\infty((-1,1))$ such that $\chi \not\equiv 0$ and $0 \leq \chi \leq 1$. For each belt interval $J_n = (a_n, b_n)$ choose a midpoint $z_n \in J_n$ and a half-width $w_n > 0$ such that $(z_n - w_n, z_n + w_n) \subset J_n$. Define

$$\psi_n(\omega) := \chi\left(\frac{z(\omega) - z_n}{w_n}\right).$$

Then $\psi_n \in C^\infty(S^2)$, $0 \leq \psi_n \leq 1$, and $\operatorname{supp}(\psi_n) \subset B_n$. Since $\overline{B_n} \subset U$ and K has fixed sign on U, we have $K(\omega)\psi_n(\omega) \geq 0$ (or ≤ 0) pointwise on S^2 and $\psi_n \not\equiv 0$, hence

$$\ell_n := \int_{S^2} K(\omega)\, \psi_n(\omega)\, d\omega \neq 0.$$

Define $\varphi_n := \psi_n / \ell_n$. Then (20) holds by construction. □

A physically clean family of initial data

Lemma A.1 is exactly what is needed in the main text. Using (18), if we take

$$h(\omega) = \varphi_n(\omega), \qquad f_n(r,\omega) = \eta(r - t_0)\, \varphi_n(\omega), \qquad g_n \equiv 0,$$

then the corresponding solution satisfies the *uniform point-readout effect*

$$u_{(f_n,g_n)}(x_0, t_0) = 1 \qquad \text{for all } n. \tag{21}$$

Thus the point readout cannot distinguish *which* belt was activated: every single-belt perturbation yields the same macroscopic point magnitude at (x_0, t_0).

Remark A.2 (Uniform finiteness and regularity). Each (f_n, g_n) is smooth and compactly supported, with support contained in a fixed thin spherical shell $\operatorname{supp} \eta(\cdot - t_0)$. All regularity properties required by the wave equation (finite energy, etc.) hold automatically [Courant and Hilbert, 1962, Evans, 2010, Taylor, 2011].

B Non-oracular state interfaces and the locality-of-access hypothesis

The main theorem assumes only one semantic hypothesis about the presentation δ_D of physically admissible states: *local underdetermination at the zero state* (Theorem 5.2). This appendix explains (i) why some such hypothesis is unavoidable if denotation is to have nontrivial content, (ii) how unconstrained interfaces trivialize the continuity-on-names requirement, and (iii) why the locality hypothesis used in the theorem is the weakest finite-access assumption compatible with the construction.

Throughout, recall that names live in Baire space Σ^ω with its standard product topology. Basic open sets are *cylinders* determined by prefixes: for a finite string $\sigma \in \Sigma^m$,

$$[\sigma] := \{ p \in \Sigma^\omega : p\restriction_m = \sigma \}.$$

Continuity of a witness map on names is exactly continuity with respect to this cylinder topology [Weihrauch, 2000, Schröder, 2002, Kreitz and Weihrauch, 1985].

B.1 Trivialization: unconstrained interfaces make denotation vacuous

A continuum theory supplies an extensional map $M : D \to \mathbb{R}$ (a magnitude). Under explanatory realism, however, *denotation* is not the existence of M as a set-theoretic function. It is the existence of a witness on names

$$R : \mathrm{dom}(\delta_D) \to \mathrm{dom}(\delta_{\mathbb{R}}) \qquad \text{such that} \qquad \delta_{\mathbb{R}}(R(p)) = M(\delta_D(p)),$$

with R total and continuous on $\mathrm{dom}(\delta_D)$ [Weihrauch, 2000, Schröder, 2002]. Continuity here means bounded input dependence: for each requested output precision, there exists a prefix length after which all compatible state-names force that precision.

If the state presentation δ_D is unconstrained, this semantic content can be destroyed completely.

Proposition B.1 (Trivialization by extensional prepackaging)**.** *Let D be any nonempty set, let $M : D \to \mathbb{R}$ be any function, and fix any admissible real representation $\delta_{\mathbb{R}}$ (e.g. Cauchy names) [Weihrauch, 2000]. Then there exists a representation*

$$\widehat{\delta}_D :\subseteq \Sigma^\omega \to D$$

and a map

$$\widehat{R} : \mathrm{dom}(\widehat{\delta}_D) \to \mathrm{dom}(\delta_{\mathbb{R}})$$

such that:

1. *$\widehat{R}$ is total and continuous;*
2. *for all $p \in \mathrm{dom}(\widehat{\delta}_D)$,*
$$\delta_{\mathbb{R}}(\widehat{R}(p)) \;=\; M(\widehat{\delta}_D(p)).$$

In particular, denotation (continuity-on-names) becomes automatic once the interface is allowed to encode extensional answers into short prefixes.

Proof sketch. Fix any surjection $e : \Sigma^\omega \twoheadrightarrow D$ and define $\widehat{\delta}_D := e$. For each p, let $x := M(\widehat{\delta}_D(p)) \in \mathbb{R}$. Choose some $\delta_\mathbb{R}$-name q_x of x (existence is guaranteed by the surjectivity of $\delta_\mathbb{R}$) [Weihrauch, 2000]. Now define $\widehat{R}(p) := q_x$.

To force continuity, choose the representation $\widehat{\delta}_D$ so that the prefix $p\!\restriction_m$ includes (for each m) the first m symbols of a $\delta_\mathbb{R}$-name for $M(\widehat{\delta}_D(p))$. Equivalently, make the naming scheme itself carry the readout. Then $\widehat{R}$ can output those symbols directly, and the dependence of the first N output symbols on p is finite (indeed, bounded by a linear function of N), which is exactly continuity in the Baire-space (cylinder) topology [Weihrauch, 2000, Schröder, 2002]. □

Remark B.2 (Interpretation). Proposition B.1 is not a pathology; it is a warning. If a state interface is allowed to be engineered to answer readout queries, then continuity-on-names is no constraint at all. Denotation becomes a coding convention rather than a semantic commitment.

B.2 Interface principle: finite access must not decide existential tail facts

In this paper the domain of admissible states is maximally tame:

$$D_{\leq 1} = \{s_0\} \cup \{s_n : n \geq 1\},$$

where $s_0 = (0, 0)$ is the zero state and each s_n is a smooth compactly supported single-belt perturbation supported in a disjoint shrinking latitude belt on the Huygens sphere.

The intended meaning of a state-name prefix is what finite access to the state has revealed so far. In a continuum field theory, physically meaningful finite access must behave like progressive approximation of localized smooth data, not like an oracle that can settle arbitrary extensional questions about the infinite tail. This is exactly the role of admissibility in represented-space semantics: the representation must respect the topology of the domain rather than smuggle nonlocal extensional information into short names [Weihrauch, 2000, Schröder, 2002, Myhill, 1957].

> **Interface principle (non-oracular finite access).** A physically meaningful state representation δ_D must be such that for localized independent perturbations on disjoint shrinking regions, finite prefixes do not decide existential tail facts (for example, "some sufficiently deep belt perturbation is active") uniformly.

This is not an effectivity restriction. The required bounds may be noncomputable. The principle asserts only that the representation respects the topology-of-approximation one expects in continuum physics: coarse information cannot determine arbitrarily fine localized contingencies.

Remark B.3 (Motivation from standard practice in quantum field theory). The locality-of-access principle imposed here is not an ad hoc technical trick. It is a minimal semantic formalization of a norm that is already standard in the practice of fundamental physics: physically meaningful descriptions are required to respect locality and finite-resolution structure, and are not permitted to treat arbitrarily fine-grained contingent structure as uniformly decidable from coarse information.

Wilsonian renormalization and effective field theory provide the canonical framework in which this locality-respecting attitude is made explicit at the level of theory construction. In particular, they institutionalize the methodological demand that a physical description should not require global extensional information about infinitely many independent microscopic contingencies in order to make macroscopic sense. For classic discussions see, e.g., Wilson and Kogut [1974], Polchinski [1984], Weinberg [1995b].

B.3 The locality-of-access hypothesis as a cylinder-topology statement

Theorem 5.2 uses a single formal hypothesis on δ_D. We repeat it here, now expressed in topological terms.

> **Local underdetermination at s_0.** For every prefix length m there exists $n \geq 1$ and names $p_0, p_n \in \mathrm{dom}(\delta_D)$ such that
>
> $$\delta_D(p_0) = s_0, \qquad \delta_D(p_n) = s_n, \qquad p_0 \restriction_m = p_n \restriction_m .$$

Equivalently:

> **Cylinder formulation.** For every cylinder neighborhood $[p_0 \restriction_m]$ of any name p_0 of s_0, the set $\delta_D([p_0 \restriction_m] \cap \mathrm{dom}(\delta_D))$ contains some nonzero belt state s_n.

This is precisely the claim that within the cylinder topology on name space, every finite-information neighborhood of the all-zero state remains compatible with the activation of some sufficiently deep localized perturbation. That is exactly what is meant by "finite access cannot rule out arbitrarily fine local contingencies."

Remark B.4 (Why this is the weakest non-oracle condition needed here)*.* The locality hypothesis does not assume any particular metric, norm, computability, or regularity of δ_D beyond its intended finite-access role. It merely rules out interfaces that encode the extensional predicate

$$\exists n \geq 1 \quad (s = s_n)$$

into bounded prefix length. Such oracle interfaces would invalidate the semantic point of the paper by making the readout denoting by design (Proposition B.1). In other words, the hypothesis is just enough to prevent the representation from collapsing denotation into a naming convention [Weihrauch, 2000, Schröder, 2002].

Remark B.5 (Minimality of the locality hypothesis)*.* One may ask whether the local underdetermination hypothesis used in Theorem 5.2 is stronger than necessary. In a precise sense, it is minimal for the present construction. The theorem proves non-denotation by exhibiting FET at s_0, and FET is a universal quantification over prefix depths:

$$\exists N \ \forall m : \ \mathrm{diam}\big(M(U_{p_0,m})\big) \geq 2^{-N}.$$

Accordingly, any hypothesis sufficient to force FET must ensure that every cylinder neighborhood of a name of s_0 remains compatible with at least one nonzero belt state. If one weakens the locality requirement to a merely infinitely-often or measure-theoretic condition (for example, only for a subsequence of prefix lengths), then one no longer obtains FET. At best one obtains intermittent instability, which is compatible with denotation. Conversely, strengthening the locality requirement is unnecessary. The theorem does not assume any metric regularity, norm control, effectivity, or computable modulus of underdetermination. It requires only that finite prefixes behave as finite access rather than as an oracle deciding the extensional existential predicate "some sufficiently deep belt is active." This is exactly the weakest cylinder-topology statement that keeps denotation non-vacuous and prevents the trivialization mechanism of Proposition B.1.

With this hypothesis in place, the FET mechanism in the main theorem becomes inevitable. Since the construction ensures

$$M_*(s_0) = 0, \qquad M_*(s_n) = 1 \quad (n \geq 1),$$

every cylinder neighborhood of s_0 contains admissible states whose readouts differ by 1. Hence the induced readout pipeline has unbounded input dependence at fixed finite precision and admits no continuous witness on names at s_0 [Weihrauch, 2000, Schröder, 2002].

In short, the locality-of-access hypothesis is not a semantic embellishment. It is the minimal formal expression of the idea that finite prefixes represent finite access rather than extensional oracles. Without it, denotation can always be forced by representation engineering (Proposition B.1), and the semantic commitments of explanatory realism lose their constraining force.

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