



LXIV. Additions to the theory of eclipses, and the methods of calculating their results

Professor Bessel

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rectness of the views taken of this stratum in the foregoing notice. That this patch of sandstone, which is now upwards of six miles from the nearest point of the same rock, once formed part of a continuous stratum, we cannot doubt, nor that the intervening portion has been removed by the operation of water, that mighty agent which has been employed universally in modifying the surface of the globe. It is difficult to obtain an idea of the extent of force necessary, but it is, nevertheless, as probable, that such a removal of this bed may have taken place, as that the strata on the high side of the dyke have been removed, which, when the slip took place, must have presented at this point, a face of rock, upwards of one thousand feet high.

LXIV. *Additions to the Theory of Eclipses, and the Methods of calculating their Results.* By Professor BESSEL.

[Concluded from page 347.]

LET us now suppose that ϕ' , μ denote the latitude and longitude of the zenith; D , A the longitude and latitude of the star; let (μ) and (ϕ') express the right ascension and declination of the zenith, and ε the obliquity of the ecliptic. We then obtain these equations:

$$\begin{aligned}\sin \phi' &= \sin (\phi') \cos \varepsilon - \cos (\phi') \sin (\mu) \sin \varepsilon \\ \cos \phi' \sin \mu &= \sin (\phi') \sin \varepsilon + \cos (\phi') \sin (\mu) \cos \varepsilon \\ \cos \phi' \cos \mu &= \cos (\phi') \cos (\mu), \text{ from which we derive the} \\ \text{following expressions for } u \text{ and } v \text{ by } (\phi') \text{ and } (\mu) \\ u &= r \sin (\phi') \sin \varepsilon \cos A + r \cos \phi' [\cos A \sin (\mu) \cos \varepsilon - \\ &\quad \sin A \cos (\mu)] \\ v &= r \sin (\phi') \cdot [\cos D \cdot \cos \varepsilon - \sin D \sin \varepsilon \sin A] \\ &\quad - r \cos (\phi') [\sin (\mu) (\cos D \sin \varepsilon + \sin D \cdot \cos \varepsilon \sin A) + \\ &\quad \cos (\mu) \sin D \cos A]\end{aligned}$$

The differential quotients of $r \cos (\phi')$ and $r \sin (\phi')$ give therefore, if β retains the above signification,

$$\begin{aligned}\frac{du}{d \cdot e^2} &= \frac{1}{2} \beta^2 \cdot u - \beta \cdot \sin \varepsilon \cos A \\ \frac{dv}{d \cdot e^2} &= \frac{1}{2} \beta^2 \cdot v - \beta (\cos D \cos \varepsilon - \sin D \cdot \sin \varepsilon \cdot \sin A)\end{aligned}$$

We have therefore, calculating with longitudes and latitudes, and referring $N + \psi$ to the same, for the term dependent on Δe^2 , this expression:

$$\begin{aligned}-\omega \sin \pi \cdot \Delta e^2 \left\{ \frac{1}{2} \beta^2 [x \cos \psi + \frac{n}{s} (t - d - \tau) \sin \psi - k] + \right. \\ \left. N.S. \text{ Vol. 8. No. 48. Dec. 1830. } \quad 3 H \quad \beta \sin\end{aligned}$$

$$\beta \sin \varepsilon \cos A \cos (N + \psi) - \beta (\cos D \cos \varepsilon - \sin D \sin \varepsilon \sin A) \sin (N + \psi) \}.$$

All the parts taken together, give the following complete development of formula (6).

$$(11) \dots d = t - T + \frac{ms}{n} \frac{\cos (M - N - \psi)}{\cos \psi} - h \cdot \frac{\cos (N + \psi)}{\sin \psi} \cos \delta \Delta \alpha \\ + h \frac{\sin (N + \psi)}{\sin \psi} \Delta \delta + h \cdot \frac{1}{\sin \psi} \cdot \omega \sin \pi \cdot \Delta k \\ - h \left\{ \frac{x}{\tan \psi} + \frac{n}{s} (t - d - \tau) \right\} \cos \pi \cdot \Delta \pi \\ - h \left\{ \frac{1}{2} \beta^2 \left(\frac{x}{\tan \psi} + \frac{n}{s} (t - d - \tau) - \frac{k}{\sin \psi} \right) - \frac{\beta V}{\sin \psi} \right\} \omega \sin \pi \cdot \Delta e^2$$

in which V , if we calculate with right ascension and declination, $= \cos D \sin (N + \psi)$, and if we calculate with longitudes and latitudes

$$= (\cos D \cdot \cos \varepsilon - \sin D \sin \varepsilon \sin A) \sin (N + \psi) - \sin \varepsilon \cos A \cdot \cos (N + \psi).$$

It appears from this formula that there are combinations between the quantities $\Delta \alpha$, $\Delta \delta$, &c. in which they affect the result, or that several of them will appear united by the equality of their coefficients. This is expressed by this formula:

$$(12) \dots d = t - T + \frac{ms}{n} \frac{\cos (M - N - \psi)}{\cos \psi} + h \varepsilon + \frac{h \xi}{\tan \psi} + \frac{h \eta}{\sin \psi} - h E \S - h F \cdot i$$

in which $\varepsilon = \sin N \cdot \cos \delta \cdot \Delta \alpha + \cos N \cdot \Delta \delta$

$$\xi = -\cos N \cdot \cos \delta \cdot \Delta \alpha + \sin N \Delta \delta - x \cdot \cos \pi \cdot \Delta \pi$$

$$\eta = \omega \sin \pi \cdot \Delta k$$

$$\S = \cos \pi \cdot \Delta \pi$$

$$i = \omega \sin \pi \cdot \Delta \cdot e^2$$

and E and F represent the above-given coefficients of $h \S$ and $h i$. This form is the most simple in which the influence of the corrections of the elements of calculation can be represented.

The unknown quantity ε cannot be determined by observations of an occultation of a star, except when d is known for at least one place of observation, for it entirely unites with the difference of meridians. The second term proportional to the reciprocal of the tangent ψ is the same which is usually added to the time of conjunction deduced from an observation as correction on account of the latitude (or declination); but it appears from the expression for its coefficient ξ , that this term involves $\Delta \alpha$, $\Delta \delta$, and $\Delta \pi$. The dependence of this term on $\Delta \alpha$ might appear contradictory to the first method above explained

[1],

[1], as the time of conjunction was there determined without being affected by an error in longitude (or right ascension), whereas such an error here affects the difference of meridians, and consequently likewise the time of conjunction. But it is to be observed that this contradiction is only apparent, and may be considered as arising from the omission of that term of the correction which is to be added to the time of the conjunction calculated by that method, and which depends on the error (x) of the assumed difference of meridians as explained in [2]; if this correction is added and x eliminated by the comparison of the time of conjunction found with the one resulting from the tables and $\Delta \alpha$, the dependence on this quantity will likewise be perceived. Both methods only differ by being made to depend on different unknown quantities. The determination of all the 5 quantities $\epsilon, \zeta \dots$ by observations of an occultation of a star, is mathematically possible; but it is easily perceived that if these quantities are thus kept separate, small errors of observation will greatly affect their determination, unless the observations were made at the most proper spots of the globe, and not confined to the small part of its surface contained between the observatories of Europe. The advantageous separation of the two last quantities from the others, requires, for example, that at two of the places of observation the times of the phænomenon should be very different, which will be the case if at the one it takes place a little after the rising of the moon, and at the other a little before her setting; the last is separated from the rest only by the difference of the value of β for the different places of observation.

The difficulty of producing a concurrence of favourable circumstances induces the belief that the determination of the excentricity of the meridians of the earth for which observations of occultations of stars have been proposed (without, however, sufficiently developing the greatest possible advantages to be obtained by them), may always be founded on more successful methods. Besides this difficulty, the mountains projecting on the limb of the moon, and other probable deviations from the globular form, may and will spoil observations good in themselves; the immersion and emersion can rarely both be observed with accuracy; and lastly, the advantageous selection of the places of observation is much restricted by the presence of the sun above the horizon. I believe, therefore, that the calculation of the influence of all the five unknown quantities will only have an interest for the purpose of judging how far they may affect the results of the calculation, but not for their determination.

[11.] In most cases only ϵ and ζ will be determined by the observation; in particular cases, likewise η ; but the others will be considered as evanescent. My experience proves that this object, a very limited one when compared with the complete determination of all unknown quantities, is generally so difficult to be attained that, in most cases, a good meridian observation of the moon is very acceptable in order to diminish the uncertainty which the occultation alone leaves behind. The comparison of it with the observations of the occultation is most easy when the right ascension and declination have been employed in the calculation. The quantities ϵ and ζ having been found by observations of an occultation, we have

$$(13) \dots \begin{cases} \cos \delta \Delta \alpha = \epsilon \sin N - \zeta \cos N \\ \Delta \delta = \epsilon \cos N + \zeta \sin N \end{cases}$$

If these quantities denote the errors of right ascension and declination, and if it be required to find those of longitude and latitude, or *vice versa*, the well-known formulæ by which these calculations may be effected are to be applied. The complete formulæ [11] and [12] will show in every case how far the errors of the tables determined on the supposition of η , ϑ , z being evanescent might be altered by these quantities. This connection might be determined generally from one of the two formulæ in particular cases, *ex. gr.* if $\Delta \alpha$ and $\Delta \delta$ have been determined by the combination of the observations of immersion and emersion made at one place; but it appears to be more convenient to calculate the coefficients for both phenomena, and to derive the result required from their numerical values.

[12.] I shall now generally consider the problem of eclipses, and suppose that both bodies have a parallax and a diameter. The determination of the most convenient form of the general equation [2] is then less apparent than in the particular case of an occultation of a fixed star; but even then formulæ may be found combining convenience of calculation with perfect correctness. Although the method of approximation, explained by Lagrange, is sufficient for practice, yet the importance of a theory which has been so often treated, will be an apology for resuming it again.

The expression $(a b' - a' b) (c' \sin \pi - c \sin \pi') - (a c' - a' c) (b' \sin \pi - b \sin \pi') + (b c' - b' c) (a' \sin \pi - a \sin \pi')$ is identically $= 0$: if we put, therefore,

$$\begin{aligned} c' \sin \pi - c \sin \pi' &= G \sin d \\ b' \sin \pi - b \sin \pi' &= G \cos d \cos a \\ a' \sin \pi - a \sin \pi' &= G \cos d \sin a \end{aligned}$$

and

and substitute d and a for the arbitrary quantities u and v used in the transformation of the sum of three squares, we shall have $(a'b' - a'b)^2 + (a'c' - a'c)^2 + (b'c' - b'c)^2 = [(a'b' - a'b) \cos d + (a'c' - a'c) \sin d \cos a - (b'c' - b'c) \sin d \sin a]^2 + [(a'c' - a'c) \sin a + (b'c' - b'c) \cos a]^2$, and the expression which forms the first part of the equation (2) is thus reduced to the sum of two squares.

The angles d and a by which this is effected may be considered, the first as the declination (or latitude), the second as the right ascension (or longitude) of a point of the sphere of the heavens, which may be easily demonstrated to be the point in which the great circles passing through the true and apparent places, respectively, of the bodies, intersect each other. For in the expressions by which d and a have been determined, the last parts of the expressions [1] of a, b, c, a', b', c' vanish; so that we have

$$(14) \dots \begin{cases} G \sin d &= \sin \pi \sin D - \sin \pi' \sin \delta \\ G \cos d \cdot \cos a &= \sin \pi \cos D \cos A - \sin \pi' \cos \delta \cos \alpha \\ G \cos d \cdot \sin a &= \sin \pi \cos D \sin A - \sin \pi' \cos \delta \sin \alpha \end{cases}$$

In these equations is contained the condition that the three points concerned in it, viz. the two true places of the bodies and the point determined by d and a , are situated in a great circle; this condition may be reduced to the form in which it is usually represented, by eliminating $G, \sin \pi, \sin \pi'$, which is done by multiplying the three equations respectively by

$$-\frac{\sin(\alpha - A)}{\cos \delta}, + \frac{\tan d}{\cos \delta} \sin A - \frac{\tan D}{\cos \delta} \sin a, - \frac{\tan d}{\cos \delta} \cos A + \frac{\tan D}{\cos \delta} \cos a, \text{ and we shall have}$$

(15) $0 = \tan \delta \sin(A - \alpha) - \tan D \sin(\alpha - a) + \tan d \sin(a - A)$ the usual form of the condition above mentioned. But as we have likewise

$$\begin{aligned} G \sin d &= \sin \pi \cdot \Delta' \cdot \sin D' - \sin \pi' \cdot \Delta \cdot \sin \delta' \\ G \cos d \cdot \cos a &= \sin \pi \cdot \Delta' \cdot \cos D' \cos A' - \sin \pi' \cdot \Delta \cos \delta' \cdot \cos \alpha' \\ G \cos d \cdot \sin a &= \sin \pi \cdot \Delta' \cdot \cos D' \cos A' - \sin \pi' \cdot \Delta \cos \delta' \cdot \sin \alpha \end{aligned}$$

And as these equations have the same form as the preceding ones, the point determined by d and a is likewise situated in the great circle passing through the apparent places. Substituting for a, b, c, a', b', c' their expressions in [1] we obtain

$$a'b' - a'b = \cos \delta \cdot \cos D \sin(\alpha - A) - G \cdot r \cos \phi' \cos d \cdot \sin(\mu - \alpha)$$

$$a'c' - a'c = \cos d \cdot \sin D \sin \alpha - \cos D \cdot \sin \delta \sin A - G[r \cos \phi' \sin d \sin \mu - r \sin \phi' \cos d \cdot \sin \alpha] \quad b'c' -$$

$$b c' - b' c = \cos \delta \sin D \cos \alpha - \cos D \sin \delta \cos A -$$

$G [r \cos \phi' \sin d \cdot \cos \mu - r \sin \phi' \cos d \cdot \cos a]$
and consequently,

$$(a b' - a' b) \cos d + (a c' - a' c) \sin d \cdot \cos a - (b' c' - b c) \sin d \sin a \\ = -\sin \delta \cos D \sin d \sin (A - \alpha) + \cos \delta \sin D \sin d \times \\ \sin (\alpha - a) + \cos \delta \cos D \cos d \sin (\alpha - A) - G \cdot r \cos \phi' \sin (\mu - a)$$

and adding the product of equation [15] by $\cos \delta \cos D \sin d$

$$= \frac{\cos D}{\cos d} \cos \delta \sin (\alpha - A) - G \cdot r \cos \phi' \sin (\mu - a)$$

We likewise obtain

$$(a c' - a' c) \sin a + (b c' - b' c) \cos a = \cos \delta \sin D \cos (\alpha - a) \\ - \sin \delta \cos D \cos (A - \alpha) + G [r \sin \phi' \cos d - r \cos \phi' \sin d \times \\ \cos (\mu - a)].$$

The second part of equation [2], viz. $(a' \sin \varrho \pm a \sin R)^2$
 $+ (b' \sin \varrho \pm b \sin R)^2 + (c' \sin \varrho \pm c \sin R)^2$ is more conveni-
 ent for calculation, if represented in its irrational form. It is
 the square of $\Delta \cdot \Delta' \sin \Sigma = \sin \varrho \sqrt{(\Delta'^2 - \sin R^2)} \pm$
 $\sin R \sqrt{(\Delta^2 - \sin \varrho^2)}$ where

$$\Delta^2 = a^2 + b^2 + c^2 = 1 - 2 r \sin \pi \cdot \cos \gamma + r^2 \sin^2 \pi^2$$

$$\Delta'^2 = a'^2 + b'^2 + c'^2 = 1 - 2 r \sin \pi' \cdot \cos \gamma' + r^2 \sin^2 \pi'^2$$

$\cos \gamma$ and $\cos \gamma'$ being written for

$$\sin \phi' \sin \delta + \cos \phi' \cos \delta \cos (\mu - \alpha) \text{ and}$$

$$\sin \phi' \sin D + \cos \phi' \cos D \cos (\mu - A). \text{ If we denote,}$$

therefore,

$$\sqrt{[\cos \varrho^2 - 2 r \sin \pi \cos \gamma + r^2 \sin^2 \pi^2]} \text{ by } \lambda$$

$$\sqrt{[\cos R^2 - 2 r \sin \pi' \cos \gamma' + r^2 \sin^2 \pi'^2]} \text{ by } \lambda'$$

the required part is $(\lambda' \sin \varrho \pm \lambda \sin R)^2$.

[13.] The equation [2] becomes by substituting these trans-
 formations of its several parts :

$$(16) \dots \left(\frac{\lambda' \sin \varrho \pm \lambda \sin R}{G} \right)^2 \\ = \left\{ \frac{\cos D}{\cos d} \cdot \frac{\cos \delta \sin (d - A)}{G} - r \cos \phi' \sin (\mu - a) \right\}^2 \\ + \left\{ \frac{\sin \delta \cos D \cos (A - \alpha) - \cos \delta \sin D \cos (\alpha - a)}{G} - r (\sin \phi' \cos d - \right. \\ \left. \cos \phi' \sin d \cos (\mu - a)) \right\}^2$$

It has consequently induced the form $k^2 = (P - u)^2 + (Q - v)^2$,
the same which takes place for the case of occultations of
 fixed stars. The difference between the general equation and
 the particular case consists in this, that in the former there is,
 instead of the constant k , a variable one dependent on the place
 of

of observation and the angles γ and γ' (the zenith distances of the bodies); that P and Q likewise involve d and a , and that u and v contain these angles instead of D and A. It is therefore not necessary to give particular methods for the calculation of eclipses, be they either eclipses of the sun or transits of the inferior planets over the disc of the sun, as all these phænomena may be treated after the method which I have developed for the occultations of stars.

From the formulæ [14] results

$$G^2 = \sin^2 \pi - 2 \sin \pi \cdot \sin \pi' \cdot \cos \sigma + \sin^2 \pi'^2$$

$$\text{tang} (A - a) = \frac{\sin \pi' \cos \delta \sin (\alpha - A)}{\sin \pi \cdot \cos D - \sin \pi' \cos \delta \cos (\alpha - A)}$$

where σ stands for the geocentric distance of the two bodies. For a solar eclipse we may put

$$G = \sin \pi - \sin \pi'$$

$$a = A - \frac{\sin \pi'}{\sin \pi} (\alpha - A)$$

$$d = D - \frac{\sin \pi'}{\sin \pi} (\delta - D) \text{ without causing in the cal-}$$

ulation any perceptible deviation from the truth. The quantities whose introduction has so much contracted the formulæ will then be found almost without calculation, and the calculation of solar eclipses will in point of ease present only insignificant differences from those of occultations of stars. We have here another confirmation of the remark which one has so often occasion to make,—that the rigorous mathematical solution of astronomical problems ceases to require more difficult calculations than the approximately correct ones, as soon as one has succeeded in representing the former in its true shape.

F. W. BESSEL.

LXV. *Continuation of the Table of Atomic Weights, and Notice of a new Scale of Equivalents.* By Mr. JOHN PRIDEAUX, Member of the Plymouth Institution.

To the Editors of the Philosophical Magazine and Annals.

Gentlemen,

Plymouth, Aug. 8th, 1830.

I BEG leave now to send you the Table of acids and bases, and a description of the scale, which has already intruded on so many of your pages.

Table