

## FORM AND EFFICIENCY OF INCANDESCENT FILAMENTS.

BY CHARLES J. REED.

If we pass an electric current through any conductor, as a cylindrical wire, its temperature tends to increase by the transformation of electrical energy into heat. If no heat is allowed to escape, the temperature will increase indefinitely, or until the conductor is melted or otherwise destroyed. If the heat is allowed to escape by radiation alone from the surface of the conductor, its temperature will increase only until the rate of loss by radiation is exactly equal to the rate of transformation. By heat we here include all radiant energy, whether of high or low degree.

As the temperature of an incandescent body increases, not only does the actual quantity of radiant energy increase, but its wave lengths diminish. Hence, as experience has shown, after incandescence is reached, increasing the temperature in a given ratio, increases the light emitted in a much greater ratio.

The exact relation between temperature and luminosity is not known, and it is probably not very simple, if such a relation exists at all. The phenomenon of luminosity is really a physiological one and depends partly upon individual optical capacity; some persons being able to see above, and some below the visible spectrum of the average human eye. The radiant energy we call light is one thing. The sensation of luminosity by which we always estimate light is an entirely different thing. A constant source of light may vary greatly in luminosity, according to the condition of the receptive mechanism and its individuality.

But even ignoring the physiological aspect of the question,

the nearest approach we have to formulæ for radiation are the approximate empirical formulæ of Dulong and Petit and of Stefan ; which are for total radiations of a low temperature and limited range. They cannot apply to light alone, nor even to total radiation of high degree.

It is unfortunate that there exists no instrument more reliable than the retina of the living eye for measuring the intensity of radiant luminiferous energy, and no method of reading the measurements more accurately than individual guesses. We can measure radiant thermal energy of low degree with Langley's bolometer ; we can measure radiant actinic energy by the chemical action it produces ; and we can measure total radiant energy in a variety of ways ; but how can we isolate or measure radiant luminiferous energy ? If there were a high temperature thermometer or other instrument for accurately measuring high temperatures, we might attack this problem with some hope of results.

It is sufficient, however, for our present purpose to know what the eye is able to tell us, namely, that increasing the temperature increases the light in a greater ratio. From this it follows that the efficiency of an incandescent filament is some direct function of its temperature above that of the surrounding space.

Granting that the efficiency increases with increased temperature, we have now to determine whether the efficiency does or does not depend upon any other conditions. It has frequently been claimed that the efficiency depends upon the form of the filament, whether it be cylindrical, flat or square. At a meeting of this Institute of June 8, 1886, for instance, this was by general consent considered an established fact.<sup>2</sup> Others have asserted, but without giving any proof, that the efficiency depends upon the pressure ; some claiming that low-tension series lamps and others that high-tension multiple lamps are the more efficient. Others, again, have claimed that at the same pressure and with the same shaped filament, lamps of greater candle-power have a different efficiency from lamps of smaller power.

The trouble seems to have been that we are not always careful to distinguish between efficiency and convenience or adaptability. Each individual finds that a certain form of filament or a certain

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<sup>2</sup> Trans. Am. Inst. E. E., vol. IV, p. 20-23.

method of distribution gives better satisfaction than another or is more suited to his purpose, and soon persuades himself that it is "more efficient" than any other.

In order to study the effects of these various conditions upon the efficiency of a filament, it will be convenient to eliminate the effects which we know will be produced by variations in temperature. We assume, therefore, in this discussion that all the filaments and all parts of the filaments under consideration are at the same temperature.

Let  $T$  represent this temperature and  $T'$ , the temperature of the supposed vacuous space surrounding the filament. We must assume further that the material of which the filaments are constructed is perfectly homogeneous in itself and that it is uniform in all the various forms of filaments considered.

Let  $S$  denote the specific radiating power of the material at the temperature,  $T$ , and  $S'$ , its specific resistance at the same temperature.

Any variation in  $S$  or  $S'$  might affect the efficiency, and for this very reason experimental proof is very difficult to obtain. Comparisons of filaments made by different processes are entirely worthless in determining the effect of form or length of a filament on its efficiency.

It is a difficult matter to produce by the same process two carbons of different sizes and shapes that shall have the same specific radiating power and specific resistance at the same temperature, and to produce them by different processes is entirely out of the question. Any comparison, therefore, of short series filaments with long, multiple filaments of a different manufacture is of no value whatever in settling this question.

Let  $C$ , represent the candle-power of any filament at temperature,  $T$ .

$c$ , its current,

$E$ , the potential difference at its terminals,

$R$ , its resistance,

$r$ , its radius (considered as a cylinder),

$L$ , its length,

$H$ , the energy received per unit of time,

$H'$ , the energy radiated per unit of time.

Since the energy developed in any portion of an electric circuit is proportional to the current and pressure, we have

$$H = K E c, \quad \dots\dots(1)$$

in which  $K$  is a constant depending upon the units employed.

If we neglect the small quantity of heat lost by conduction from the ends of the filament to the conducting wires, and assume that the space surrounding the filament is a perfect vacuum, then the entire energy of the current will be expended in radiation, and we have

$$H = H'. \quad \dots\dots(2)$$

Let  $K'$  represent a constant depending on  $(T - T')$ .

The rate of radiation of any surface depends only upon the elevation of its temperature above that of the surrounding space and the specific radiating power of the surface at that temperature. We have, therefore,

$$H = H' = K' S \times 2 \pi r L = K E c. \quad \dots\dots(3)$$

The resistance of any conductor is proportional to its length and inversely proportional to its cross section, and we have

$$R = S' \frac{L}{\pi r^2} \quad \dots\dots(4)$$

From Ohm's law and (4)

$$c = \frac{E}{R} = \frac{E \pi r^2}{S' L} \quad \dots\dots(5)$$

By eliminating successively  $E$ ,  $c$  and  $r$  from (3) and (5) we get

$$\frac{K' S \times 2 \pi r L}{K c} = \frac{c S' L}{\pi r^2} \text{ or, } \frac{c^2}{r^3} = \frac{2 \pi^2 K' S}{K S'} \dots\dots(6)$$

$$\frac{K' S \times 2 \pi r L}{K E} = \frac{E \pi r^2}{S' L}, \text{ or } E^2 r = \frac{2 K' S L^2 S'}{K} \dots\dots(7)$$

and

$$\frac{K E c}{K' S \times 2 \pi L} = \sqrt{\frac{c S' L}{\pi E}} \quad \dots\dots(8)$$

From (6)

$$K' S \times 2 \pi^2 r^3 = K S' c^2. \text{ Hence,}$$

$$r = \sqrt[3]{\frac{K S' c^2}{2 K' S \pi^2}} \quad \dots\dots(9)$$

and

$$c = \pi \sqrt{\frac{2 K' S r^3}{K S'}} \quad \dots\dots(10)$$

showing that the square of the current is proportional to the cube of the radius or diameter of the filament for all lamps at constant temperature; and that this relation is independent of the pressure, of the length and resistance of the filament and independent of the candle-power;

From (7) we have

$$K' S \times 2 L^2 S^2 = K E^2 r. \quad \text{Hence,}$$

$$L = \sqrt{\frac{K E^2 r}{2 K' S S'}} = E \sqrt{r} \sqrt{\frac{K}{2 K' S S'}} \quad \dots (11)$$

$$\text{and} \quad r = \frac{L^2}{E^2} \times \frac{2 K' S S'}{K} \quad \dots (12)$$

showing that the ratio of the square of the length to the diameter depends only upon the pressure and is independent of candle-power and resistance;

From (8)

$$K^2 E^3 c = 4 \pi K'^2 S^2 S' L^3. \quad \text{Hence,}$$

$$E = \frac{L}{\sqrt[3]{c}} \sqrt[3]{\frac{4 \pi K'^2 S^2 S'}{K^2}} \quad \dots (13)$$

$$\text{and} \quad c = \left( \frac{L}{E} \right)^3 \times \frac{4 \pi K'^2 S^2 S'}{K^2} \quad \dots (14)$$

showing that the current is proportional to the cube of the ratio of the length to the pressure.

Since  $K'$ ,  $S$  and  $S'$  are constant, both the light and the heat radiated will be proportional to the surface and we have

$$C = K'' L \times 2 \pi r \quad \dots (15)$$

in which  $K''$  is a constant depending upon the units employed and upon  $T$ .

$$\text{Hence,} \quad L = \frac{C}{2 \pi K'' r} \quad \dots (16)$$

$$\text{and} \quad r = \frac{C}{2 \pi K'' L} \quad \dots (17)$$

Combining (15) with (3)

$$\frac{K' S C}{K''} = K E c, \text{ or}$$

$$\frac{C}{K E c} = \frac{K''}{K' S} \quad \dots (18)$$

which shows that the candle-power is proportional to the energy,  $K E c$ ; or that the efficiency,  $\frac{C}{K E c}$ , is constant and independent of the pressure, length, radius, resistance, current and candle-

power of the filament. This means that the energy required to produce a given candle-power will be proportional to the candle-power and will be the same, whether it is expended in driving a large current through a short, thick filament, or a small current through a long, slender filament; provided the temperature or state of incandescence is the same and provided no heat is lost by conduction through the terminal wires.

Equation (18) shows also that the efficiency does depend upon  $K'$ ,  $K''$  and  $S$ , that is, upon the temperature of the filament, the temperature of the surrounding space and the specific radiating power, and upon them only.

From (18)

$$C = E c \times \frac{K K''}{K' S} \quad \dots\dots(19)$$

$$E = \frac{C}{c} \times \frac{K' S}{K K''} \quad \dots\dots(20)$$

and

$$c = \frac{C}{E} \times \frac{K' S}{K K''} \quad \dots\dots(21)$$

From (4)

$$r = \sqrt{\frac{S' L}{\pi R}} = \sqrt{\frac{S'}{\pi}} \times \sqrt{\frac{L}{R}} \quad \dots\dots(22)$$

and

$$L = \frac{\pi r^2 R}{S'} \quad \dots\dots(23)$$

From (5)

$$E = R c \quad \dots\dots(24)$$

From (21) and (24)

$$c = \frac{C}{c R} \times \frac{K' S}{K'' K} = \sqrt{\frac{C}{R}} \times \sqrt{\frac{K' S}{K'' K}} \quad \dots\dots(25)$$

We have thus far confined ourselves to the consideration of cylindrical filaments varying in length and diameter. It remains now to show that the efficiency is independent of the form of cross section.

Let us suppose we have a number of lamps, all made of the same material, having the same specific resistance and radiating power, all burning at the same temperature and all giving the same amount of light. Let them be of any pressure and let the cross-section be of any form—circular, elliptical, square, triangular, etc. The filaments will all have equal radiating surfaces; since unequal surfaces of the same character at the same temperature

could not radiate equal amounts of light. But equal surfaces of the same character at the same temperature must radiate equal amounts of heat of all wave lengths. Hence, the total amounts of radiant energy are equal. Since the energy received is equal to the energy radiated and the amounts of light are equal, we have

$$\dot{Q} = Q E c \quad \dots\dots(26)$$

in which  $Q$  is a constant, and from (18) we find its value to be

$$Q = \frac{K K''}{K' S} \quad \dots\dots(27)$$

This shows that the efficiency is independent of the form of cross-section of the filament and depends only upon its temperature, the specific radiating power of its surface, and the temperature of the surrounding space.

When we find, therefore, that one lamp is more efficient than another we must infer, not that it is on account of larger or smaller size, not because it is on a series or on a multiple circuit, not that it is because it has high or low resistance, or a certain form of cross-section, but that it is at a higher temperature, or is made of material having a different specific radiating power.

Of all possible forms of cross-section the circular has the largest area for the same radiating surface per unit of length, and consequently, has the advantage of great strength. The current density will also be less in this than any other form; and hence, any "disintegrating" or "electrolytic effect" of the current, if there is any such, will be least in the cylindrical filament. This will be made clear by supposing a number of lamps of equal candle-power to be burning at the same temperature and pressure, but having various forms of filament, cylindrical, flat, square, etc. We have already seen that these lamps all consume equal amounts of energy, and since they have the same pressure, they must take the same current. Therefore, the filament which has the greatest area of cross section will have the least current density. The fact that the cylindrical filament has the greatest cross-section does not signify that it will require a greater current to keep it at the same temperature. Its form is better adapted for retaining heat than any other. Again, the several lamps will have equal resistances; otherwise they could not take equal currents with the same pressure. From this it follows that the cylindrical filament will be longer and will have less surface and greater mass per unit of length than any other.

A tubular filament would have a greater external diameter, but smaller cross-section of material, and would be shorter than the cylindrical.

The advantages and disadvantages of the various forms may be summed up as follows:—

The cylindrical has the advantage of greater strength and less current density on account of greater cross-section. It has the disadvantage of greater length and fragility.

The tubular filament is strongest in form, being shorter and of greater external diameter than the cylindrical. It has the disadvantage of greater current density than the cylindrical. Both the tubular and the cylindrical have the advantage of uniform illumination in all directions except in and near the plane of the filament.

Flat, or angular filaments have the disadvantage of great current density, and of being unequally heated, since the edges or projecting corners will always be a little cooler<sup>3</sup> than the other parts.

A short, thick filament has the advantage of being stronger and more durable than a long, slender one; but the disadvantage of wasting a greater percentage of heat by conduction through the terminal wires. In this latter respect, it is true, that the long filament is slightly more efficient than the short. A 10-ampere filament, a foot long, would waste no more energy by conduction than a 10-ampere filament an inch long.

Aside from the difficulty of obtaining lamps of different types from different makers which have the same specific resistance and radiating power, there is the difficulty, or rather impossibility, of getting them all to burn at the same temperature, and the difficulty of knowing, even approximately, what that temperature is, and whether it is the same in two lamps or not. For these reasons no experimental comparisons yet published are of any value, either in corroborating or refuting these conclusions. There is one method, however, of testing the formulæ. This method the writer has tried upon some twenty-five different

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<sup>3</sup> It was asserted before this Institute, June 8, '86, that the edges or corners of a square filament are hotter than the remainder of the surface; but no reason was given. Such a statement scarcely needs refutation, being contrary, not only to established laws, but to the most commonplace and every-day experience. We need only observe the cooling of a square bar of red-hot iron to convince ourselves that the sharp corners are the coolest parts. And they would remain the coolest parts even if heat were continually supplied by an electric current to the interior of the bar.



types with the most satisfactory results. The method is as follows:—

A lamp is constructed with a filament of known dimensions, treated to the required process and exhausted to a certain pressure, as determined by a McLeod vacuum gauge. It is then placed on a circuit of a certain pressure, which gives it a certain desired temperature or efficiency, and the candle-power and current are measured. From the data thus obtained, we calculate by the above equations all the data for lamps of other types. The lamps when made according to this data from the same material, by the same process, and exhausted to the same pressure, should give the calculated candle-power at the calculated pressure.

It must be remembered that the formulæ take no account of the small amount of heat that escapes by conduction through the wires. With lamps of high resistance this may be safely ignored in practice, but in changing from a long to a very short and thick filament it must be taken into account.

The following examples may serve to better illustrate the method—

Let us suppose that a lamp has been constructed to give a certain candle-power on a circuit of given pressure, and it is desired to construct other lamps having the same efficiency (temperature) and life, but of different candle-power, for the same or for different pressure. We will call this lamp for convenience the zero lamp, and denote its particular values of the variables by the subscript,  $o$ .

$$\text{Let } C = C_o = 16 \text{ candles.}$$

$$c = c_o = .6 \text{ ampere.}$$

$$E = E_o = 100 \text{ volts.}$$

$$R = R_o = 167 \text{ ohms.}$$

$$L = L_o = 170 \text{ millimeters.}$$

$$r = r_o = .07 \text{ millimeter.}$$

The constants  $K$ ,  $K'$ ,  $K''$ ,  $S$  and  $S'$  will have the same value for all values of  $C$ ,  $c$ ,  $E$ ,  $R$ ,  $L$  and  $r$ , and hence, will be found by substituting for these quantities the simultaneous values,  $C_o$ ,  $c_o$ ,  $E_o$ ,  $R_o$ ,  $L_o$  and  $r_o$ , respectively.

If  $H$  is measured in horse-power, we have from (1)  $K = \frac{1}{\pi 4.5}$ .  
From (4)

$$R_o = S' \frac{L_o}{\pi r_o^2} \text{ or}$$

$$S' = \frac{\pi R_o r_o^2}{L_o} = \frac{\pi \times 167 \times (.07)^2}{170} = .015122.$$

from (3)

$$K' S = \frac{K E_o c_o}{2 \pi r_o L_o} = \frac{100 \times .6}{746 \times 2 \pi (.07) \times 170} = .0010757,$$

from (15)

$$K'' = \frac{C_o}{2 \pi r_o L_o} = \frac{16}{2 \pi (.07) \times 170} = .21399.$$

Substituting these values of  $K$ ,  $K''$ ,  $S'$  and  $K' S$  in equations (4) to (25), we are enabled to solve any problem involving  $R$ ,  $E$ ,  $C$ ,  $r$ ,  $c$ , and  $L$ , when any two of the quantities except  $c$  and  $r$  are given. The equations apply only to lamps of the same material and at the same temperature as the zero lamp. By starting with a lamp of different material, or by burning the same lamp at a different temperature, we obtain a different set of constants, which substituted in equations (4) to (25) adapt them to the changed conditions.

Suppose we wish to construct a lamp of 20 candle-power for a 90-volt circuit:—

$$C = 20.$$

$$E = 90.$$

From (21)

$$c = 3.75 \frac{20}{90} = .8333 \text{ ampere}$$

from (9)

$$r = .098466 \sqrt[3]{(.8333)^2} = .087196 \text{ millimeters}$$

from (11)

$$L = 6.4190 \times 90 \sqrt{.087196} = 170.59 \text{ millimeters}$$

and from (4)

$$R = .0048134 \frac{170.59}{(.087196)^2} = 108.00 \text{ ohms.}$$

The calculations may be verified by substitution in some of the other formulæ.

In constructing the lamp we take a filament 170.59 millimeters long, and of such a radius that, when treated to the given process until at the proper exhaustion it takes .833 ampere with 90 volts, it will have a radius of .0872 mm. If the treatment of the carbon does not increase its diameter, the initial and

final radius will be the same; but the radius of the *finished carbon* is to be .0872 mm. When these conditions are fulfilled, we know with certainty that the lamp has the proper temperature if it gives 20 candles. If any single condition is not fulfilled we know with the same certainty that the lamp has not the proper temperature. If two or more of the required conditions are simultaneously not fulfilled, then we know nothing about the resulting temperature.

We are thus enabled to construct lamps of uniform life for varying conditions of current and candle-power; and also to regulate the length of life and the efficiency, increasing or diminishing either the life or the efficiency at pleasure to suit the requirements of particular conditions, remembering always that the life and efficiency are inverse functions of each other.

The fact that the life of most commercial lamps is very irregular, shows that either the material or the temperature is not uniform, and hence, that there is lack of uniformity either in the process or in the dimensions. The writer has examined a number of different makes of lamps, and found that they generally vary in length and cross-section enough to produce serious differences in temperature.

Suppose, again, we wish to construct a 50-candle lamp for a series circuit of 5 amperes:

$$C = 50.$$

$$c = 5.$$

From (20)

$$E = 3.75 \times \frac{50}{5} = 37.5 \text{ volts.}$$

from (9)

$$r = .098466 \sqrt[3]{(5)^2} = .28792 \text{ millimeter}$$

from (11)

$$L = 6.419 \times 37.5 \sqrt{.28792} = 129.16 \text{ millimeters}$$

from (4)

$$R = .0048144 \frac{129.16}{(.28792)^2} = 7.5 \text{ ohms.}$$

In this case the loss of heat by conduction through the connecting wires is considerable, and must be taken into account.

The above equations are not limited to incandescent filaments, but apply equally to any conductor which may be kept at a cer-

tain fixed temperature by an electric current. By using the proper constants, useful formulæ may be derived for calculating conductors, so that under given conditions they will be heated to a required temperature.

#### DISCUSSION.

PROF. EDWARD L. NICHOLS:—Every one, I think, who is interested in the study of incandescent lamps will be very glad to hear reiterated the fact, which I think none of us will desire to question, that efficiency is a function of the temperature and not a function of the shape of the carbon and of the various other things to which it has often been ascribed. I think there can be no question on this point. It seems to me the clearest point that we have in reference to radiant energy—that the thing which we must get in order to get increased efficiency is higher temperature, and that any method that will enable us to do this will enable us to increase the efficiency. That the temperature of the lamp cannot be measured to-day is probably true. I have seen, however, within a few weeks, a paper by H. F. Weber, of Zurich, which leads me to believe that the day is much nearer than we thought when we shall be able to express the efficiency of any incandescent body in terms of its temperature directly. Mr. Weber has worked out a formula which he has applied to a great variety of examples where the temperature could be measured; that is, he has gone over the literature of incandescence and has selected all those experimental investigations which can be said to be quantitative in any fair sense of the word. Almost all the experiments deal with platinum, because platinum is a substance of which we know quite accurately the law of expansion under change of temperature, and also the law of resistance with change of temperature. These various methods, however, have not been regarded as directly comparable; but Mr. Weber seems to have been able to get out a formula which can be applied to each of them separately. They are found to fall into line beautifully, so that he has put to a very severe test the formula which he proposes; and in the article to which I referred which appeared in the *Repertorium der Physik*, and which I believe has not been translated into English, he claims that he has applied this formula to a variety of incandescent lamps, with great success. I hope this may be true. I am sorry to say I cannot give the

formula itself from memory. It contains two or three constants which are to be determined, but comparatively speaking it is a simple formula. I am in hopes, therefore, that we shall before very long be in a position to modify Mr. Reed's statement, which at the present time is undoubtedly justified—that we do not have means of measuring the temperature of incandescent lamps. The thing, of course, which has led to much confusion in this matter, is the fact that lamps behave very differently towards different observers and at different times, and at different times towards the same observer. I think the point there is largely a matter of vacuum. The formula which we have seen developed this evening assumes either a perfect vacuum or a vacuum which is equally good in all cases, and it is a matter of fact, well known, of course, to all those who work with incandescent lamps, that an apparently slight difference in vacuum—really a very large difference—is the thing which changes altogether the incandescence of the filament. In other words, while conduction is an almost negligible factor in this discussion, convection, even where the amount of air remaining in the bulb is small, is by no means a negligible factor, and the failure to recognize more promptly on all sides this simple relationship is due to the fact that we have to deal with lamps which may be identical in every other respect, and yet which vary quite widely in the matter of exhaustion. I must confess that it is one of the chief difficulties which lie in the way of any one, who will experiment with lamps of this kind. It can be overcome by the use of a pressure gauge measuring the vacuum in each case, but ordinarily as we know, this is not done. I should like to call attention to a very well known method by means of which any one can detect slight differences in temperature in two incandescent lamps, even though he may not be able to express either one of those in degrees centigrade, and that is by means of a very old form of photometer known as the Rumford or shadow photometer. Let any one take two lamps made with the very greatest care by our methods of to-day, and of the same type and marked to give 16 or 20 or whatever candle power it may be, at the same voltage. Set those up so that the light from them will shine upon a sheet of white paper, and interpose a block so as to get partial shadow, there being one portion in complete shadow and two portions in partial shadow. An inspection will show that one of those shadows is always a shade bluer than the other.

Now, this method, I presume, many of you have used. I have used it with very much satisfaction in attempting oftentimes to get two lamps as nearly alike as possible. This method is one which gives a very delicate means of getting relative differences of temperature or at least of determining when two lamps are of precisely the same temperature; so that while we are not in position to express that temperature, we are in position to determine equality of temperature with a considerable degree of accuracy. It seems to me that a formula of the kind presented in this paper, which enables one to calculate, as it does evidently with great readiness, just what the dimensions of a lamp should be, will be of great value.

MR. GEO. B. PRESCOTT, JR.—Among the points that I understood Mr. Reed to make, and I am rather afraid I may be in error about it, was this—that one of the advantages of a cylindrical filament or one of circular cross-section over any other form was that the current density would be less, and to illustrate this I understood him to select a number of carbons of all sorts of cross-section, all sorts of shapes of cross-section at the same temperature or rather of the same difference of potential and the same current flowing, showing that the resistance was the same. Now, if they were of the same resistance and the same length, the cross-section must have been alike and I should suppose that the current density would be the same in all events.

MR. REED:—They are not the same length. I have stated that the E. M. F. and resistance were the same—the same current at the same potential. Now, they have different areas of cross-section and different lengths, which you will find to be true by inspecting this formula. Of course, if the lengths were the same the formula could not be true. The length bears a certain relation to the other constants.

THE CHAIRMAN (Vice-President T. C. Martin):—As you will have noted, Mr. Reed in his paper mentioned the fact that the paper itself was called out by a statement made before the Institute in 1886. When Mr. Reed first brought this to my attention it appeared to me that we were not at all wishful to make the publications of the Institute a vehicle for any kind of errors, or heresies, or heterodoxies, and it therefore seemed to me well that he should bring his paper before us on that subject, so that it might be cleared up, no matter how well there might be an understanding on the point in the better informed circles and

where the investigation had been pursued to any length, and I think Mr. Reed has certainly rendered us a service in giving us so admirable a paper in so brief and succinct a form as he has given it to us this evening.

There being no other discussion, we will proceed to the next paper upon the programme—that by another member who is well known to you, Mr. Delany. I have much pleasure in asking Mr. Delany to give us his paper on his New Line Adjusting System.