Review
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must be spoken of as lying in a straight line, for there is to be one such point on any arbitrary line; we shall have therefore in each plane one ideal line, and as this is determined by two ideal points, it is fitted to replace the idea of the aspect of a plane. By adjoining the ideal line we change the plane from an open surface to a closed one. All these ideal points and lines must be regarded as lying in one ideal plane, in order that there may be one ideal point on every line, one ideal line in every plane.

These considerations lead to the conclusion that the system of geometry will be most harmoniously developed if we throw aside the restriction of our domain to the visible universe. The region of our researches is now to include the visible universe, and as much more as can be formulated and is found convenient. For the present we adjoin simply one plane; the domain is the visible universe increased by one ideal plane.

To show that this ideal plane with its elements is of the same nature as any other plane with its elements, von Staudt examines and compares the constructions and proofs for the different possible combinations of actual and ideal elements. This involves some tedious detail; but it appears unavoidable with this method of handling the subject. The conclusion to which this minute investigation leads is that the geometry of this enlarged domain is the same as that of the restricted domain—the visible universe —without the exceptions; the laws of combination of the elements now hold universally.

We can picture this ideal plane on any arbitrary plane, P_1, of the visible universe (for example, by representing its points by the points in which its *Schein* from any vertex is cut by P_1); the plane P_1 must then be represented on some other plane P_2, P_2 on P_3, and so on; there will always be one plane left over, one plane not pictured on the planes of the universe; the domain is "the universe + one plane." Von Staudt does not state this explicitly, but his attitude towards imaginary elements suggests that something of this nature may have been in his mind.

C. A. Scott.

(*To be continued.*)

REVIEWS.

Leçons nouvelles sur les applications géométriques du calcul différentiel. Par W. de Tannenberg, Professeur à la Faculté des Sciences de l'Université de Bordeaux. Paris, Hermann, 1899; 8vo, 192 pp.

Opinions seem to differ as to how far the geometrical applications of the Calculus ought to be collected from the four corners of mathematics and presented to the student *en bloc*. Many people think it is a mistake to let them occupy so much space as is generally granted to them at present in the text-books. "If I were writing a book on the Differential

Calculus," said a well-known Cambridge man to the writer some time ago, "I would spend less time in juggling with curves; it smothers what really needs to be taught."

Most teachers will perhaps agree that the pedal-of-the-evolute-of-the-inverse and other remote relatives of a given curve get more attention than they deserve from text-book writers and Tripos examiners; one is tempted to wonder whether they are as important as some of the matters which are left out, "on account," as we learn from the text-book preface, "of the limitations of expense and space."

Once admitted to be unsatisfactory, the chapters devoted to geometrical applications in the books may be altered in one of two ways: either by shortening them until their length corresponds more nearly to their importance, or by substituting fresh matter for that at present given. Many considerations seem to us to point to the former of these plans as the best. In the first place, the geometrical applications of analysis take up at present more than their fair share of a student's mathematical course. He is expected to be familiar with the most trivial properties of the conic sections and the cycloids, and to work out difficult examples on curvature, evolutes, pedals, intrinsic equations, and parabolic asymptotes.

No doubt these are interesting in their way, but they are not on the high road to anywhere of importance, and they take time; and we cannot help feeling that some retrenchment in this region would be abundantly justified if it gave the student opportunity at a later period of his course to read more about infinite series, differential equations, harmonic analysis, and theory of functions. At present a low wrangler knows next to nothing about these latter subjects.

This position may be supported by considerations of a more pedagogical character. If mathematics is to be partitioned into "subjects," to each of which is to be devoted a text-book, a course of lectures, and an examination paper, then it is undesirable that one subject should grow by swallowing up parts of others: and we think that few people can handle a text-book like that of Edwards without feeling that this has happened to the Differential Calculus.

We have thought it well to bring up this question, because the author of the book under review—De Tannenberg's *Leçons Nouvelles sur les applications géométriques du calcul différentiel*—evidently takes a different view of the matter. He, in fact, proposes to add 190 pages of fresh applications to the heavy burden we already have to bear.

The field he has chosen is the geometry of three dimensions, which is left almost untouched in the usual English text-books on the Calculus. His work, in fact, is what we generally call the Theory of Surfaces; it developes the ideas to which most students are first introduced in the concluding chapters of Smith's *Solid Geometry*, and which they possibly continue in the treatises of Salmon and Frost. But the great book to which it must be referred is Darboux's *Theory of Surfaces*, to which the author is evidently considerably indebted. And it seems to us that the best way to treat it is first to enter an objection against its title, and then to estimate its value as a text-book on the Theory of Surfaces.

The book is philosophically divided into five parts, dealing respectively with the descriptive properties of twisted curves, the descriptive properties of surfaces, the metric properties of twisted curves, the metric properties of ruled surfaces, and the metric properties of surfaces in general. And here let it be said that partitions based on such principles as the distinction between descriptive and metric properties are in our opinion really useful, and ought to appear more prominently in elementary works than they do at present.

The first two parts, which deal with such matters as envelopes and osculating planes, do not call for much notice; the last chapter of the second part, however, in a few articles on complexes and congruences, introduces the reader to the elements of line-geometry.

In the third part, which is devoted to the metric properties of twisted curves, M. de Tannenberg's favourite method is a use of the formulae connected with the "Trihedron of Serret," *i.e.*, the system of moving axes constituted by the tangent, principal normal, and binormal, at a variable point of the curve.

The fourth part consists of two chapters, one devoted to skew and one to developable surfaces; the chief results of the theory of ruled surfaces are neatly and clearly proved, and applied to problems such as that of Bertrand, "To find the conditions which must be satisfied by a curve whose principal normals are also the principal normals of another curve."

The fifth part, which is concerned with the metric properties of surfaces in general, is much longer than the others, and occupies nearly half the book. A surface S is supposed to be defined by three equations, expressing the coordinates x, y, z, in terms of two variable parameters u, v; as u and v vary, the point x, y, z, traces out the surface. Any element of arc ds, traced on the surface, can be expressed in terms of the corresponding increments of the superficial coordinates u and v by an equation of the form

$$ds^2 = Edu^2 + 2Fdudv + Gdv^2,$$

where E, F, G, are functions of u and v; and these functions E, F, G, are invariant when the surface S is displaced in any manner in space. If now the direction-cosines of the normal to the surface be denoted by a, b, c, it is found that the differential form $\Sigma dadx$, or

$$-(E'du^2 + 2F'dudv + G'dv^2)$$

is likewise invariant. It is then shown that the six functions E, F, G, E', F', G', characterize the surface; in other words, if these functions are given, the surface is completely determinate, independently of its position in space. On this as basis, the author considers lines of curvature, geodesics, asymptotic lines, and the general theory of curves traced on a surface.

It is a matter for wonder that the results and methods of the Theory of Surfaces have not met with greater popularity in this country. They are interesting, divisible into pretty fragments, and more exciting than the plane geometry over which we lavish so much care. Perhaps M. de Tannenberg's book will bring this branch of mathematics into more general cultivation. If so, we shall rejoice. E. T. WHITTAKER.