Review
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Lampe gives a collection of solutions to harder problems on maxima and minima; No. 6 is:—To draw through a tangent to the (circular) base of a right cone, an elliptic section whose area is a maximum or minimum. This problem deserves mention, because the solutions sometimes given are incomplete (*e.g.* Williamson's *Differential Calculus*, Art. 148: Frenet's *Exercises*, 216).

Schafheitlin gives a theorem on Bessel functions. It was suspected by Nielsen that two equations $J_m(x)=0$, $J_n(x)=0$ could not have a common root (other than $x=0$), supposing the difference $(m-n)$ to be an integer; it is here proved that with $m=n+3$, and with a special value for n (nearly equal to 4) the second root of J_n is the same as the first root of J_m. T. J. I'a. B.

Introduction to the Infinitesimal Calculus. By H. S. Carslaw. Pp. 103. (Longmans, Green & Co., 1905.)

Kurze Einleitung in die Differential- und Integral Rechnung. Von Irving Fisher. Pp. 72. (Teubner, 1904.)

These two little books are very much alike in character. Each is designed to serve as a short introduction to the Calculus sufficient for the purposes of a special class of students. Mr. Carslaw is thinking of engineering students, Dr. Fisher of students of economics, and the former naturally require a good deal more mathematics than the latter; and so, although the range of the subjects treated is much the same in the two books, Mr. Carslaw generally gives us a good deal more in the way of detailed applications. Each author cherishes a hope that his book may be useful as an introductory course even for mathematical students. In Mr Carslaw's case this hope is certainly justified. I have myself subjected his book to the best of all possible tests, actual use in teaching backward students, and find it admirably adapted for the purpose. But I am afraid there is hardly enough detail in Dr. Fisher's book to make it very useful in such cases.

A great merit of Mr. Carslaw's book is his treatment of the elements of "Conics" in the only sensible way, viz.: as an easy illustration of the processes of the Calculus. He only devotes one chapter to the Conic Sections, but a great many of their elementary properties for which there is no room in the text are introduced in admirably chosen examples. It is a very great help to have an easy book from which one can teach Analytical Geometry and the Calculus at the same time. I wish that Mr. Carslaw had seen his way to include the elements of the theory of e^x and $\log x$ as well. These functions ought only to be introduced after the notions of the Calculus have been mastered, and Mr. Carslaw might have done it all in a very few pages. He says "we assume a knowledge of the properties of the following series:—$e^x=\dots$, $a^x=\dots$, $\log(1+x)=\dots$." But the kind of person who will use this book has really no such knowledge at all: at least that is my experience—and there really is no book in which exactly what is wanted can be found. It is the treatment of the exponential and the logarithm which seems to me the least satisfactory feature in both books. Both authors assume too much, and neither makes it very clear precisely what he is assuming. On the other hand both authors handle the troublesome (though of course not really difficult) subject of "differentials" in a lucid and unobjectionable way. G. H. HARDY.

MATHEMATICAL NOTES.

183. [P. 3. b.] The name of the author of the note on Inversion in the *Gazette*, No. 47, p. 88, was accidentally omitted. The note was by Mr. V. Ramaswami Aiyar, of Arni, India. For want of space we were compelled to omit certain applications and a further development of the theorem.

184. [I. 1.] *The third approximation to the n^{th} root of a number.*

Mr. C. S. Jackson and Mr. F. J. W. Whipple have given investigations (in the May and December numbers of the *Gazette*) for the approximation*

$$(1+x)^{\frac{1}{n}}=(1+px)/(1+qx),$$

where x is small and $p=\frac{1}{2}(1+1/n)$, $q=\frac{1}{2}(1-1/n)$.

The following method follows the lines which I have suggested (*Gazette*, October, 1904) for dealing with some of the elementary power-series.

Write
$$y=(1+x)-[(1+px)/(1+qx)]^n,$$

then
$$\frac{dy}{dx}=1-n(p-q)\frac{(1+px)^{n-1}}{(1+qx)^{n+1}};$$

so that $\frac{dy}{dx}$ vanishes with x, if $n(p-q)=1$.

Then
$$\frac{d^2y}{dx^2}=\frac{(1+px)^{n-1}}{(1+qx)^{n+1}}\left[\frac{(n+1)q}{1+qx}-\frac{(n-1)p}{1+px}\right],$$

which also vanishes with x, if $(n-1)p=(n+1)q$.

Thus $p=\frac{1}{2}(1+1/n)$, $q=\frac{1}{2}(1-1/n)$, as given above; with these values we find

$$\frac{d^2y}{dx^2}=\frac{n^2-1}{2n^2}x\frac{(1+px)^{n-2}}{(1+qx)^{n+2}}.$$

If we differentiate $(1+px)^{n-2}(1+qx)^{-(n+2)}$, it will be found that the result is

$$-(1+px)^{n-3}(1+qx)^{-(n+3)}(1+4pqx).$$

Thus $(1+px)^{n-2}(1+qx)^{-(n+2)}$ decreases as x increases from 0; and consequently is less than 1 for positive values of x.

That is,
$$\frac{d^2y}{dx^2}<\frac{n^2-1}{2n^2}x \quad\text{or}\quad \frac{d}{dx}\left(\frac{dy}{dx}-\frac{n^2-1}{4n^2}x^2\right)<0, \ x \text{ being positive}$$

Now $\frac{dy}{dx}-\frac{n^2-1}{4n^2}x^2$ is zero for $x=0$, and so is negative for positive values of x.

That is,
$$\frac{d}{dx}\left(y-\frac{n^2-1}{12n^2}x^3\right)<0,$$

and by the same argument,

$$y-\frac{n^2-1}{12n^2}x^3<0.$$

Similarly, since $\frac{d^2y}{dx^2}$ is positive for positive values of x, we infer that y is positive; so that

$$0<y<\tfrac{1}{12}(1-1/n^2)x^3, \text{ if } x \text{ is positive.}$$

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* To reduce this to the form given, write $x=N/a^n$, and multiply both sides by a.