The very rapid increase of electromotive force with diminished resistance at the lowest speed, seems to show that the speed is very considerably overrated when stated as 800 or 533, with the resistance between 1 and 2 B. A. units. I hope soon to have the means of accurately measuring the speed realised, and shall then repeat these experiments for a scientific, and not a mere practical purpose.

(2.) On the Law of Extension of India-rubber at Different Temperatures.

To fill the vacancies in Foreign Honorary Fellowships caused by the deaths of Claude Bernard, Elias Magnus Fries, Henri Victor Regnault, Angelo Secchi, the following Gentlemen were elected:—

Frank Cornelius Donders, Utrecht.
Asa Gray, Cambridge, U.S.
Jules Janssen, Paris.
Johann Benedict Listing, Gottingen.

The following Gentlemen were duly elected Fellows of the Society:—

Thomas H. Cockburn Hood, F.G.S., Junior Carlton Club, Pall Mall.
Thomas Gilray, M.A., 6 Carlung Place, Edinburgh.
Alex. Bennett M'Grigor, LL.D., 19 Woodside Terrace, Glasgow.
James Blaikie, M.A., 14 Viewforth Place, Edinburgh.

Monday, 17*th March* 1879.

Professor KELLAND, President, in the Chair.

The following Communications were read:—

1. On Gravitational Oscillations of Rotating Water. By Sir William Thomson.

(*Abstract.*)

This is really Laplace's subject in his Dynamical Theory of the Tides; where it is dealt with in its utmost generality except one important restriction,—the motion of each particle to be infinitely nearly horizontal, and the velocity to be always equal for all particles in the same vertical. This implies that the greatest depth must be small in comparison with the distance that has to be travelled to find the deviation from levelness of the water-surface altered by a sensible fraction of its maximum amount. In the present short communication I adopt this restriction; and farther, instead of supposing the water to cover the whole or a large part of the surface of a solid spheroid as does Laplace, I take the simpler problem of an area of water so small that the equilibrium-figure of its surface is not sensibly curved. Imagine a basin of water of any shape, and of depth, not necessarily uniform, but, at greatest, small in comparison with the least diameter. Let this basin and the water in it rotate round a vertical axis with angular velocity ω so small that the greatest equilibrium slope due to it may be a small fraction of the radian: in other words, the angular velocity must be small in comparison with $\sqrt{\frac{g}{\frac{1}{2}A}}$, where g denotes gravity, and A the greatest diameter of the basin. The equations of motion are

$$\left.\begin{aligned} \frac{du}{dt} - 2\omega v &= -\frac{1}{\varrho}\frac{dp}{dx} \\ \frac{dv}{dt} + 2\omega u &= -\frac{1}{\varrho}\frac{dp}{dy} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where u and v are the component velocities of any point of the fluid in the vertical column through the point (xy), relatively to horizontal axes Ox Oy revolving with the basin; p the pressure at any point x, y, z, of this column; and ϱ the uniform density of the liquid. The terms $\omega^2 x$, $\omega^2 y$, which appear in ordinary dynamical equations referred to rotating axes represent components of centrifugal force, and therefore do not appear in these equations. Let now D be the mean depth and D + h the actual depth at any time t in the position (xy). The "equation of continuity" is

$$\frac{d(\mathrm{D}u)}{dx} + \frac{d(\mathrm{D}v)}{dy} = -\frac{dh}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Lastly, by the condition that the pressure at the free surface is constant, and that the difference of pressures at any two points in the fluid is equal to $g \times$ difference of levels, we have

$$\left.\begin{aligned} \frac{dp}{dx} &= g\varrho\,\frac{dh}{dx} \\ \frac{dp}{dy} &= g\varrho\,\frac{dh}{dy} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Hence for the case of gravitational oscillations (1) become

$$\left.\begin{aligned} \frac{du}{dt} - 2\omega v &= -g\,\frac{dh}{dx} \\ \frac{dv}{dt} + 2\omega u &= -g\,\frac{dh}{dy} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (4)$$

From (1) or (4) we find by differentiation, &c.

$$\frac{d}{dt}\left(\frac{dv}{dx} - \frac{du}{dy}\right) + 2\omega\left(\frac{du}{dx} + \frac{dv}{dy}\right) = 0 \quad . \quad . \quad . \quad (5)$$

which is the equation of vortex motion in the circumstances.

These equations reduced to polar coordinates, with the following notation,—

$$x = r\cos\theta\,,\quad y = r\sin\theta$$
$$u = \zeta\cos\theta \quad \tau\sin\theta\,,\quad v = \zeta\sin\theta + \tau\cos\theta\,,$$

become

$$\frac{D\zeta}{r} + \frac{d(D\zeta)}{dr} + \frac{d(D\tau)}{rd\theta} = -\frac{dh}{dt} \quad . \quad . \quad . \quad . \quad (2')$$

$$\left.\begin{aligned} \frac{d\zeta}{dt} - 2\omega\tau &= -g\,\frac{dh}{d\zeta} \\ \frac{d\tau}{dt} + 2\omega\zeta &= -g\,\frac{dh}{rd\theta} \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (4')$$

$$\frac{d}{dt}\left(\frac{\tau}{r} + \frac{d\tau}{dr} - \frac{d\zeta}{rd\theta}\right) + 2\omega\left(\frac{\zeta}{r} + \frac{d\zeta}{dr} + \frac{d\tau}{rd\theta}\right) = 0 \quad . \quad (5')$$

In these equations D may be any function of the coordinates. Cases of special interest in connection with Laplace's tidal equations are had by supposing D to be a function of r alone. For the present, however, we shall suppose D to be constant. Then (2) used in (5) or (2′) in (5′) gives after integration with respect to t

$$\frac{dv}{dx}-\frac{du}{dy}=2\omega\,\frac{h}{\mathrm{D}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

or in polar coordinates

$$\frac{\tau}{r}+\frac{d\tau}{dr}-\frac{d\zeta}{rd\theta}=2\omega\,\frac{h}{\mathrm{D}} \quad . \quad . \quad . \quad . \quad . \quad (6')$$

These equations (6) (6′) are instructive and convenient though they contain nothing more than is contained in (2) or (2′), and (4) or (4′).

Separating u and v in (4), or ζ and τ in (4′), we find

$$\left.\begin{aligned}&\frac{d^2u}{dt^2}+4\omega^2u=-g\left(\frac{d}{dt}\,\frac{dh}{dx}+2\omega\frac{dh}{dy}\right)\\ \text{and}\quad&\\ &\frac{d^2v}{dt^2}+4\omega^2v=\quad g\left(2\omega\frac{dh}{dx}-\frac{d}{dt}\,\frac{dh}{dy}\right)\end{aligned}\right\} \quad . \quad . \quad (7)$$

or in polar coordinates

$$\left.\begin{aligned}&\frac{d^2\zeta}{dt^2}+4\omega^2\zeta=-g\left(\frac{d}{dt}\,\frac{dh}{dr}+2\omega\frac{dh}{rd\theta}\right)\\ &\frac{d^2\tau}{dt^2}+4\omega^2\tau=\quad g\left(2\omega\frac{dh}{dr}-\frac{d}{dt}\,\frac{dh}{rd\theta}\right)\end{aligned}\right\} \quad . \quad . \quad (7')$$

Using (7) (7′), in (2) (2′), with D constant, or in (6) (6′) we find—

$$g\mathrm{D}\left(\frac{d^2h}{dx^2}+\frac{d^2h}{dy^2}\right)=\frac{d^2h}{dt^2}+4\omega^2h \quad . \quad . \quad . \quad (8)$$

and

$$g\mathrm{D}\left(\frac{d^2h}{dr^2}+\frac{1}{r}\,\frac{dh}{dr}+\frac{d^2h}{rd\theta^2}\right)=\frac{d^2h}{dt^2}+4\omega^2h \quad . \quad . \quad (8')$$

It is to be remarked that (8) and (8′) are satisfied with u or v substituted for h.

I. Solutions for Rectangular Coordinates.

The general tpye-solution of (8) is $h=\epsilon^{\alpha x}\epsilon^{\beta y}\epsilon^{\gamma t}$ where α, β, γ, are connected by the equation

$$\alpha^2+\beta^2=\frac{\gamma^2+4\omega^2}{g\mathrm{D}} \quad . \quad . \quad . \quad . \quad . \quad (9)$$

For waves or oscillations we must have $\gamma=\sigma\sqrt{-1}$ where σ is real.

I*a*. *Nodal Tesseral Oscillations.*

For nodal oscillations of the tesseral type we must have $\alpha = m\sqrt{-1}$, $\beta = n\sqrt{-1}$ where m and n are real, and by putting together properly the imaginary constituents we find

$$h = \mathrm{C} \frac{\sin}{\cos} \sigma t \frac{\sin}{\cos} mx \frac{\sin}{\cos} ny \quad . \quad . \quad . \quad . \quad (10),$$

where m, n, σ are connected by the equation

$$m^2 + n^2 = \frac{\sigma^2 - 4\omega^2}{g\mathrm{D}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11).$$

Finding the corresponding values of u and v, we see what the boundary conditions must be to allow these tesseral oscillations to exist in a sea of any shape. No bounding line can be drawn at every part of which the horizontal component velocity perpendicular to it is zero. Therefore to produce or permit oscillations of the simple harmonic type in respect to form, water must be forced in and drawn out alternately all round the boundary, or those parts of it (if not all) for which the horizontal component perpendicular to it is not zero. Hence the oscillations of water in a rotating rectangular trough are not of the simple harmonic type in respect to form, and the problem of finding them remains unsolved.

If $\omega = 0$, we fall on the well-known solution for waves in a non-rotating trough, which are of the simple harmonic type.

I*b*. *Waves or Oscillations in an endless Canal with straight parallel sides.*

For waves in a canal parallel to x, the solution is

$$h = \mathrm{H}\epsilon^{-ly} \cos(mx - \sigma t) \quad . \quad . \quad . \quad . \quad . \quad (12),$$

where l, m, σ satisfy the equation

$$m^2 - l^2 = \frac{\sigma^2 - 4\omega^2}{g\mathrm{D}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (13),$$

in virtue of (9) or (11).

Using these in (7) we find that v vanishes throughout if we make

$$l = \frac{2\omega m}{\sigma} \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14);$$

and with this value for l in (12), we find, by (7),

$$u = \mathrm{H}\,\frac{gm}{\sigma}\,\epsilon^{-ly}\cos(mx - \sigma t) \qquad . \quad . \quad . \quad (15):$$

and using (14) and (13) we find

$$m^2 = \frac{\sigma^2}{g\mathrm{D}} \qquad . \quad . \quad . \quad . \quad . \quad . \quad (16),$$

from which we infer that the velocity of propagation of waves is the same for the same period as in a fixed canal. Thus the influence of rotation is confined to the effect of the factor $\epsilon^{-2\omega m/\sigma . y}$. Many interesting results follow from the interpretation of this factor with different particular suppositions as to the relation between the period of the oscillation $\left(\frac{2\pi}{\sigma}\right)$, the period of the rotation $\left(\frac{2\pi}{\omega}\right)$, and the time required to travel at the velocity $\frac{\sigma}{m}$ across the canal. The more approximately nodal character of the tides on the north coast of the English Channel than on the south or French coast, and of the tides on the west or Irish side of the Irish Channel than on the east or English side, is probably to be accounted for on the principle represented by this factor, taken into account along with frictional resistance, in virtue of which the tides of the English Channel may be roughly represented by more powerful waves travelling from west to east, combined with less powerful waves travelling from east to west, and those of the southern part of the Irish Channel by more powerful waves travelling from south to north combined with less powerful waves travelling from north to south. The problem of standing oscillations in an endless rotating canal is solved by the following equations—

$$\left.\begin{aligned} h &= \mathrm{H}\,\{\epsilon^{-ly}\cos(mx - \sigma t) - \epsilon^{ly}(\cos mx + \sigma t)\} \\ u &= \mathrm{H}\frac{gm}{\sigma}\{\epsilon^{-ly}\cos(mx - \sigma t) + \epsilon^{ly}\cos(mx - \sigma t)\} \\ v &= 0 \end{aligned}\right\} \quad . \quad (17)$$

If we give ends to the canal we fall upon the unsolved problem referred to above of tesseral oscillations. If instead of being rigorously straight we suppose the canal to be circular and endless, provided the breadth of the canal to be small in comparison with the radius of the circle, the solution (17) still holds. In this case, if c denote the circumference of the canal, we must have $m = \frac{2i\pi}{c}$, where i is an integer.

II. Oscillations and Waves in Circular Basin (Polar Coordinates).

Let

$$h = \mathrm{P} \cos(i\theta - \sigma t) \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

be the solution for height, where P is a function of r. By (8′) P must satisfy the equation

$$\frac{d^2\mathrm{P}}{dr^2} + \frac{1}{r}\frac{d\mathrm{P}}{dr} - \frac{i^2\mathrm{P}}{r^2} + \frac{\sigma^2 - 4\omega^2}{g\mathrm{D}}\mathrm{P} = 0 \quad . \quad . \quad . \quad . \quad (19)$$

and by (7′) we find

$$\left.\begin{aligned} \zeta &= \frac{g}{\sigma^2 - 4\omega^2} \sin(i\theta - \sigma t)\left(\sigma\frac{d\mathrm{P}}{dr} - 2\omega i\,\frac{\mathrm{P}}{r}\right) \\ \tau &= \frac{-g}{\sigma^2 - 4\omega^2} \cos(i\theta - \sigma t)\left(2\omega\frac{d\mathrm{P}}{dr} - \sigma i\,\frac{\mathrm{P}}{r}\right) \end{aligned}\right\} \quad . \quad . \quad (20)$$

This is the solution for water in a circular basin, with or without a central circular island. Let a be the radius of the basin, and if there be a central island let a' be its radius. The boundary conditions to be fulfilled are $\zeta = 0$, when $r = a$, and when $r = a'$. The ratio of one to the other of the two constants of integration of (19), and the speed σ of the oscillation, are the two unknown quantities to be found by these two equations. The ratio of the constants is immediately eliminated, and the result is a transcendental equation for σ. There is no difficulty, only a little labour, in thus finding as many as we please of the fundamental modes, and working out the whole motion of the system for each. The roots of this equation, which are found to be all real by the Fourier-Sturm-Liouville-theory, are the speeds * of the successive fundamental modes, corresponding to the different circular nodal subdivisions of the i diametral divisions implied by the assumed value of i. Thus, by giving to i the successive values 0, 1, 2, 3, &c., and solving the transcendental equation so found for each, we find all the fundamental modes of vibration of the mass of matter in the supposed circumstances.

If there is no central island, the solution of (19) which must be taken, is that for which P and its differential coefficients are all finite when $r=0$. Hence P is what is called a Bessel's function of the first kind and of order i; and according to the established notation † we have

$$P = J_i\left(r\sqrt{\frac{\sigma^2-\omega^2}{gD}}\right) \quad . \quad . \quad . \quad (21)$$

The solution found above for an endless circular canal is fallen upon by giving a very great value to i. Thus, if we put $\frac{2\pi r}{i}=\lambda$ so that λ may denote wave-length, we have $\frac{i}{r}=\frac{2\pi}{\lambda}$, which will now be the m of former notation. We must now neglect the term $\frac{1}{r}\frac{dh}{dr}$ in (19), and thus the differential equation becomes

$$\frac{d^2h}{dr^2}+\left(\frac{\sigma^2-4\omega^2}{gD}-m^2\right)h=0\,,$$

or

$$\frac{d^2h}{dr^2}-l^2h=0 \quad . \quad . \quad . \quad . \quad (22),$$

where l^2 denotes $m^2-\frac{\sigma^2-4\omega^2}{gD}$. A solution of this equation is $h=c\epsilon^{-ly}$ where $y=a-r$, and using this in (20) above, we find $\zeta=\frac{-g}{\sigma^2-4\omega^2}\,C\sin(mx-\sigma t)(\sigma l-2\omega m)\epsilon^{-ly}$, where $mx=i\theta$. Hence, to make $\zeta=0$ at each boundary, we have $\sigma l=2\omega m$, which makes

* In the last two or three tidal reports of the British Association the word "speed," in reference to a simple harmonic function, has been used to designate the angular velocity of a body moving in a circle in the same period. Thus, if T be the period $\frac{2\pi}{T}$ is the speed; *vice versa*, if σ be the speed $\frac{2\pi}{\sigma}$ is the period.

† Neumann, "Theorie der Bessel'schen Functionen" (Leipzig, 1867), § 5; and Lommel, "Studien über die Bessel'schen Functionem" (Leipzig 1868), § 29.

$\zeta = 0$, not only at the boundaries, but throughout the space for which the approximate equation (22) is sufficiently nearly true. And, putting for l^2 its value above, we have

$$4\omega^2 m^2 = \sigma^2\left(m^2 - \frac{\sigma^2 - 4\omega^2}{gD}\right);$$

whence

$$m^2 = \frac{\sigma^2}{gD},$$

which agrees with (16) above.

I hope in a future communication to the Royal Society to go in detail into particular cases, and to give details of the solutions at present indicated, some of which present great interest in relation to tidal theory, and also in relation to the abstract theory of vortex motion. The characteristic differences between cases in which σ is greater than 2ω, or less than 2ω, are remarkably interesting, and of great importance in respect to the theory of diurnal tides in the Mediterranean, or other more or less nearly closed seas in middle latitudes, and of the lunar fortnightly tide of the whole ocean. It is to be remarked that the preceding theory is applicable to waves or vibrations in any narrow lake or portion of the sea covering not more than a few degrees of the earth's surface, if for ω we take the component of the earth's angular velocity round a vertical through the locality, that is to say, $\omega = \gamma \sin l$, where γ denotes the earth's angular velocity, and l the latitude.

2. On the Effects of Chloroform, Ethidene Dichloride, and Ether on Blood-Pressure. By Joseph Coats, M.D., William Ramsay, Ph.D., and John G. M'Kendrick, M.D., the University of Glasgow. Communicated by Professor M'Kendrick.

Abstract.

Dr Coats stated that this communication referred to part of an investigation on the physiological action of anæsthetics, undertaken, at the request of the British Medical Association, by Dr Ramsay, Dr M'Kendrick, and himself. After describing the method of obtaining accurate tracings of variations in blood-pressure by means of a kymograph, he stated that the facts obtained from these re-