



XXXVI. Notes on the use of Nicol's prism

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XXXVI. *Notes on the Use of Nicol's Prism.* By JAMES C. M'CONNEL, B.A., Assistant Demonstrator at the Cavendish Laboratory, Cambridge*.

1. *On the Error in the Measurement of a Rotation of the Plane of Polarization caused by the Axis, about which the Nicol turns, not being parallel to the Incident Light.*

SUPPOSE we have a beam of parallel light traversing a Nicol's prism mounted in a graduated circle. Unless we have taken special precautions, we shall find that, when the Nicol is rotated, the plane of polarization of the emergent light turns through an angle somewhat different from that measured by the circle. For instance, if the axis of rotation of the Nicol is inclined 3° to the direction of the incident or emergent light, as large an error as 1° may be made in measuring a rotation of 60° .

It is, however, tolerably well known that, as far as this error is proportional to the first power of the small angle of deviation, it may be eliminated by taking the mean of the readings in the two opposite positions of the Nicol circle separated from one another by nearly two right angles. It is of course only to a first approximation that this proportionality can be considered to hold good. And the main object of the present investigation is to determine, what is the outstanding error we are liable to in assuming this proportionality; or, in other words, with what accuracy we must adjust the Nicol circle, that this first approximation may be sufficient.

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The phenomenon in its simplest form may be described thus:—If the emergent ray be parallel to the long edges of the prism it is polarized in a plane perpendicular to the principal plane of the prism, *i. e.* to the plane containing the optic axis and the normal to the face. If the ray be inclined to this position but still lie in the principal plane, then the new plane of polarization is as nearly parallel to the old one as is consistent with its containing the new ray. This indeed is obvious from symmetry. But now let the ray lean out of the principal plane: there is a marked change in the plane of polarization. It has been twisted about the ray, and the angle of twist is nearly one third of the angle of deviation. From this explanation it is easy to see in a general way how the error arises, and also how it may be eliminated by taking readings in the two opposite positions of the Nicol.

There is another important point that is apparent on the face of the matter. If the direction of the ray coincide with the axis of rotation, the ray will not move relatively to the prism when the prism is turned, and the rotation measured by the circle will be identical with the actual rotation of the plane of polarization. We shall find, moreover, that to the second order of approximation the final error depends entirely on the angle between these two directions (see equation 8). It is independent of any reasonably small errors in the adjustment of the Nicol in the circle.

In a Nicol's prism used as a polarizer, it is the second half of the prism that determines the plane of polarization of the emergent light. If the optic axis of the spar in the first half be not accurately parallel to the axis in the second half, the effect is not to turn the plane of polarization, but merely to mix a little ordinary light with the emergent polarized light.

The figure represents a portion of the sphere of unit radius.

X is the optic axis. N the normal to the face.

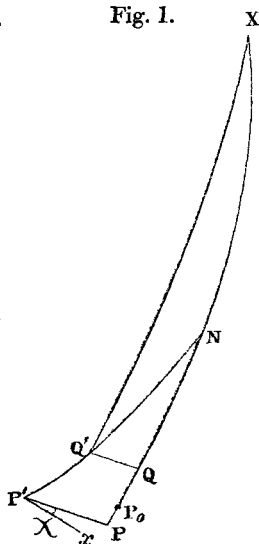
So XN is the principal plane.

P' is the wave-normal of the emergent light.

Q' is the wave-normal of the internally incident light.

Draw Q'Q, P'P perpendicular to XN produced. Join XQ'.

Fig. 1.



Let $P'x$ be the plane of polarization of P' and $PP'x=\chi$. Then χ is a function of $P'P$ and PN .

The exact function would be a very complicated expression, so we shall content ourselves with an approximation. $P'P$ is always small, and P lies near some fixed point, P_0 say, in the plane PN . Let

$$P'P=\alpha, \quad PP_0=\beta.$$

We shall reject the cubes and higher powers of α and β and limit ourselves to expanding χ as far as the squares. We know that when α is zero, χ is zero; so our expression can only contain the terms α , α^2 , and $\alpha\beta$.

In a Nicol's prism it is the extraordinary ray which emerges, so the plane of polarization of Q' is perpendicular to $Q'X$. Let us assume for the present that the plane of polarization of P' makes the same angle with the plane of incidence $P'Q'N$ as does the plane of polarization of Q' . The plane of polarization of Q' makes with $P'Q'N$ an angle $90^\circ - NQ'X$. Hence

$$\chi = 90^\circ - NQ'X - NP'P. \quad \dots \quad (1)$$

But in the triangle $NP'P$, P is a right angle; so we have

$$\tan(90^\circ - NP'P) = \cot NP'P = \frac{\sin P'P}{\tan NP} = \frac{\alpha}{\tan NP}, \quad \dots \quad (2)$$

neglecting cubes.

It remains to find $NQ'X$.

$$NQ'X = XQ'Q - NQ'Q,$$

$$\therefore \chi = 90^\circ - NP'P + NQ'Q - XQ'Q. \quad \dots \quad (3)$$

By spherical triangles,

$$\left. \begin{aligned} \tan(90^\circ - XQ'Q) &= \frac{\sin Q'Q}{\tan XQ}, \\ \tan(90^\circ - NQ'Q) &= \frac{\sin Q'Q}{\tan NQ}. \end{aligned} \right\}$$

Also

$$\left. \begin{aligned} \sin Q'Q &= \sin Q'N \sin N, \\ \sin P'P &= \sin P'N \sin N. \end{aligned} \right\}$$

But $\sin P'N = \mu \sin Q'N$, where μ is the extraordinary index of refraction for that wave, which we may take to be constant.

$$\therefore \sin Q'Q = \frac{1}{\mu} \sin P'P = \frac{\alpha}{\mu} \text{ to our order of approximation.}$$

Hence

$$\left. \begin{aligned} \tan(90^\circ - XQ'Q) &= \frac{\alpha}{\mu} \frac{1}{\tan XQ}, \\ \tan(90^\circ - NQ'Q) &= \frac{\alpha}{\mu} \frac{1}{\tan NQ}. \end{aligned} \right\} \quad \dots \quad (4)$$

Since $\tan XQ$ and $\tan NQ$ occur in small terms, we may reject squares in finding their values.

In NQ let us take a point Q_0 such that $\sin NP_0 = \mu \sin NQ_0$. Then, since $\sin(NP_0 + \beta) = \mu \sin(NQ_0 + QQ_0)$,

$$\beta \cos NP_0 = \mu QQ_0 \cos NQ_0,$$

and

$$\left. \begin{aligned} \frac{1}{\tan XQ} &= \frac{1}{\tan XQ_0} - \frac{QQ_0}{\sin^2 XQ_0} = \frac{1}{\tan XQ_0} - \frac{\beta \cos NP_0}{\mu \cos NQ_0 \sin^2 XQ_0}, \\ \frac{1}{\tan NQ} &= \frac{1}{\tan NQ_0} - \frac{QQ_0}{\sin^2 NQ_0} = \frac{1}{\tan NQ_0} - \frac{\beta \cos NP_0}{\mu \cos NQ_0 \sin^2 NQ_0}. \end{aligned} \right\} (5)$$

Now in the expansion of the tangent of a small angle the squares of the angle do not appear. So we have by (3), (2), and (4),

$$\chi = \frac{\alpha}{\tan NP} + \frac{\alpha}{\mu} \frac{1}{\tan XQ} - \frac{\alpha}{\mu} \frac{1}{\tan NQ},$$

while

$$\frac{1}{\tan NP} = \frac{1}{\tan NP_0} - \frac{\beta}{\sin^2 NP_0}.$$

So by (5),

$$\begin{aligned} \chi &= \alpha \left\{ \frac{1}{\tan NP_0} + \frac{1}{\mu \tan XQ_0} - \frac{1}{\mu \tan NQ_0} \right\} \\ &+ \alpha \beta \left\{ -\frac{1}{\sin^2 NP_0} - \frac{\cos NP_0}{\mu^2 \cos NQ_0} \left(\frac{1}{\sin^2 XQ_0} - \frac{1}{\sin^2 NQ_0} \right) \right\} (6) \\ &= l\alpha - m\alpha\beta \text{ say.} \end{aligned}$$

In Nicol's prism, as it is usually cut,

$$NX = 41\frac{1}{2}^\circ,$$

$$NP_0 = 22^\circ,$$

$$\mu = 1.54,$$

$$\therefore NQ_0 = 14^\circ.$$

Substituting these values, we obtain

$$\begin{aligned} l &= .32, \quad m = .84; \\ \chi &= .32\alpha - .84\alpha\beta. \quad . \quad . \quad . \quad . \quad (7) \end{aligned}$$

In the foregoing we have assumed that there is no rotation of the plane of polarization on refraction out of the spar, or, in other words, that the angle between the plane of polarization and the plane of incidence remains unchanged. There is of course a change, but it is merely due to the disturbing effect of reflection and is very small. If we treat the spar as

a homogeneous medium and use Fresnel's formulæ, we find the rotation is about $\cdot 005 \alpha$ in the case of an ordinary Nicol.

I have calculated from Neumann's theoretical formulæ for refraction at the surface of crystals what the rotation would be in the case of light entering the Nicol and exciting only the extraordinary wave, and find it is less than with a homogeneous medium. In the case of light leaving the Nicol there are two reflected waves, so the formulæ would probably be very complicated. I think we may safely assume that the rotation is less than $\cdot 01 \alpha$, and may therefore be neglected. In the case of a flat-ended Nicol the incidence is nearly direct, and the rotation may obviously be neglected.

Owing to the circumstance that this rotation is negligible, the whole of this investigation applies without alteration to the case when the Nicol is used as an analyzer, and the light is consequently incident on the spar. In this case we have to find the position of the plane of polarization that only the ordinary wave may be excited, and the formula for the rotation is the same as in an isotropic medium.

We are now in possession of a convenient formula for expressing the position of the plane of polarization in terms of the direction of the emergent light relative to fixed planes in the Nicol. We shall apply this formula to the discussion of the error in measuring a rotation. This part of the investigation would be very short if we confined ourselves to the first approximation. A number of complications are introduced by the necessity of retaining the squares and products of small quantities.

The figure represents a portion of the sphere of unit radius. Let

A be the axis of rotation,

N P the principal plane,

P' the emergent wave-normal,

P'x the plane of polarization.

Draw A P₀ and P' P perpendicular to the principal plane. This fixes P₀, which has hitherto been arbitrary within certain limits. A and P' are fixed in space, while N and P₀ are rotated round A.

For simplicity, suppose the circle-reading is zero when P' lies on A P₀. Then $\theta = \angle P_0 A P'$ is the reading at any time. Produce A P' to meet N P₀ in K.

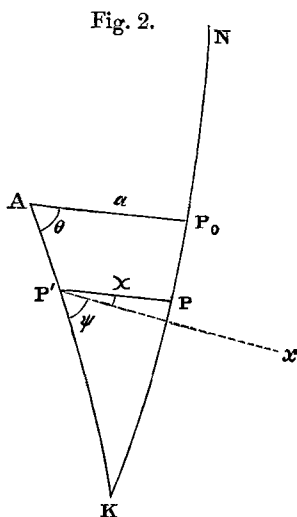


Fig. 2.

What we want to measure is the rotation of $P'x$ round P' ; so let $xP'K = \psi$. As before, let $P'P = a$, $PP_0 = \beta$, and let $AP_0 = \alpha$, $AP' = r$. Then α , β , a , r are all small. Since the figure AP_0PP' is small, we have to our order of approximation

$$\left. \begin{aligned} \alpha &= a - r \cos(\psi + \chi), \\ \beta &= r \sin(\psi + \chi). \end{aligned} \right\}$$

Let us now find the difference between θ and $\psi + \chi = PP'K$. By spherical triangles,

$$\left. \begin{aligned} \cos K &= \sin(\psi + \chi) \cos PP', \\ \cos K &= \sin \theta \cos a; \end{aligned} \right\}$$

$$\therefore \sin(\psi + \chi) \cos PP' = \sin \theta \cos a,$$

$$\sin \theta = \sin(\psi + \chi) \left(1 - \frac{a^2}{2} + \frac{a^2}{2}\right),$$

$$\sin(\psi + \chi) + (\theta - \psi - \chi) \cos(\psi + \chi) = \sin(\psi + \chi) \left(1 - \frac{a^2}{2} + \frac{a^2}{2}\right);$$

$$\therefore \theta - \psi - \chi = \frac{a^2 - a^2}{2} \tan(\psi + \chi)$$

$$= \frac{a^2 - a^2}{2} \tan \psi, \text{ neglecting cubes.}$$

But

$$a^2 - a^2 = 2ar \cos \psi - r^2 \cos^2 \psi,$$

and

$$\chi = l\alpha - m\alpha\beta$$

$$= l(a - r \cos(\psi + \chi)) - mr \sin \psi (a - r \cos \psi)$$

$$= l\{a - r \cos \psi + lr \sin \psi (a - r \cos \psi)\} - mr \sin \psi (a - r \cos \psi)$$

$$= la - lr \cos \psi + (l^2 - m)r \sin \psi (a - r \cos \psi);$$

$$\therefore \theta = \psi + la - lr \cos \psi + (l^2 - m + 1)ar \sin \psi - (l^2 - m + \frac{1}{2})r^2 \sin \psi \cos \psi.$$

Now let the plane of polarization be turned through 180° and the new reading of the Nicol circle be $180^\circ + \theta_1$. Instead of ψ we must write in the last formula $180 + \psi$. So

$$\theta_1 = \psi + la + lr \cos \psi - (l^2 - m + 1)ar \sin \psi - (l^2 - m + \frac{1}{2})r^2 \sin \psi \cos \psi;$$

$$\therefore \frac{\theta + \theta_1}{2} = \psi + la - (l^2 - m + \frac{1}{2})r^2 \sin \psi \cos \psi,$$

$$\frac{\theta_1 - \theta}{2} = lr \cos \psi - (l^2 - m + 1)ar \sin \psi.$$

Inserting numerical values, we have

$$\frac{\theta + \theta_1}{2} - \psi = .32a + .24r^2 \sin \psi \cos \psi, \quad . . . \quad (8)$$

$$\theta_1 - \theta = .64r \cos \psi, \text{ approximately.} \quad . . . \quad (9)$$

To render these formulæ intelligible to one who has not read through the investigation, we may remark that we have to deal with one plane fixed in space, viz. the plane containing the axis of rotation of the Nicol circle and the direction of the emergent light.

θ and $180^\circ + \theta_1$ are the readings of the circle in the two opposite positions, and $\theta = 0$ when the principal plane of the Nicol is at right angles to the fixed plane.

ψ is the angle between the plane of polarization and the fixed plane.

a is the angle between the axis of rotation and the principal plane.

r is the angle between the axis of rotation and the emergent light.

The angles are supposed to be expressed in circular measure in these as in all the other formulæ.

The first term on the right-hand side of equation (8) is a constant, and is therefore of no consequence in measuring a change of ψ . The second term is a measure of the outstanding error.

Let us suppose

$$r = 3^\circ, \quad \psi = 30^\circ.$$

Then

$$.24r^2 \sin \psi \cos \psi = 1';$$

but if $\psi = -30^\circ$,

$$\text{the same} = -1'.$$

So in measuring a rotation of 60° we may be subject to an error of $2'$ if the axis of rotation be inclined to the emergent light at an angle of 3° . To make sure that the error shall be less than $1'$, the last-mentioned angle must be kept within 2° .

Equation (9) affords the means of deducing the value of r from the difference of readings in the two opposite positions.

The second term of (8) is due to two main causes. One is that the rotation of the plane of polarization takes place about one axis, while the rotation is measured about another. The effect of this appears in the coefficient $-\frac{1}{2}$. The other part of the error is due to the values of χ in the two opposite positions not exactly neutralizing each other. This appears in the coefficient $m - l^2$. If these two causes had reinforced instead of counteracting one another, the resultant error would have been five times as large.

If the arrangements permit of taking readings when the plane of polarization is inclined at 90° to its two first positions, we can, by taking the mean of all four readings, get rid of the error depending on the squares of small quantities, as is evident from (8). This is what Lord Rayleigh has done in his recent measurements of the electromagnetic rotation in bisulphide of carbon.

The formula (6) applies to almost every mode of cutting a Nicol, except the important case when the ends are cut off square to the length, the case of the so-called "flat-ended Nicol." This case requires the investigation to be modified. The peculiarity consists in N lying very close to P'.

$$\chi = 90^\circ - NP'P + NQ'Q - XQ'Q,$$

as before;

and by (4) and (5),

$$\begin{aligned} \tan(90^\circ - XQ'Q) \\ = \frac{\alpha}{\mu} \frac{1}{\tan XQ_0} - \frac{\alpha\beta}{\mu^2 \sin^2 XQ_0}. \end{aligned}$$

If we take N as the point from which to measure β , P₀ and Q₀ will coincide with N.

We have now to find $NP'P - NQ'Q$. The process is precisely similar to one already performed in finding equation (8); and we obtain

$$\begin{aligned} NP'P - NQ'Q &= \frac{PP'^2 - QQ'^2}{2} \tan NP'P \\ &= \frac{\mu^2 - 1}{2\mu^2} \alpha\beta; \end{aligned}$$

$$\begin{aligned} \therefore \chi &= \frac{\alpha}{\mu \tan XN} - \frac{\alpha\beta}{\mu^2 \sin^2 XN} - \frac{\mu^2 - 1}{2\mu^2} \alpha\beta \\ &= l\alpha - m\alpha\beta, \text{ say.} \end{aligned}$$

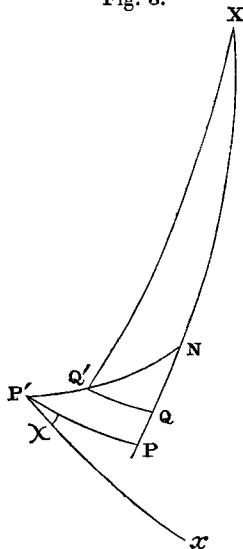
Taking $XN = 63^\circ$, $\mu = 1.52$, we obtain

$$l = .33, \quad m = .83.$$

So the numbers are almost exactly the same as in the case of the ordinary Nicol.

As a test of the accuracy of the above calculations, I made a few observations on a flat-ended Nicol. I found that, turning the Nicol through $30'$, about an axis in the principal plane

Fig. 3.



and at right angles to the incident light, turned the plane of polarization through $11' \pm 1'$.

In a paper on Polarizing Prisms (Phil. Mag. 1883) Mr. Glazebrook has suggested a new form of flat-ended prism in which the axis of the spar is at right angles to the length of the prism. In this case $XN=90^\circ$, and $\mu=1.49$.

$$\therefore l=0, \quad m=.73,$$

$$m-l^2-\frac{1}{2}=.23.$$

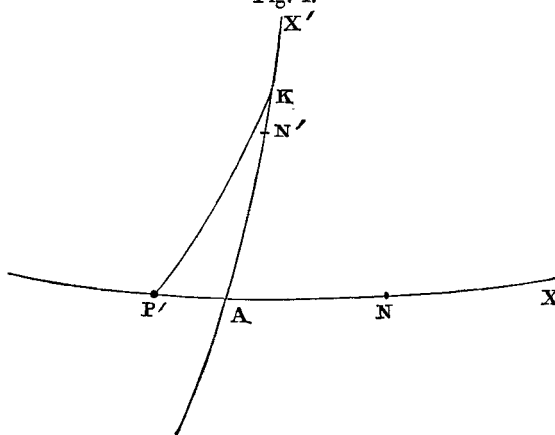
So instead of equation (8) we have

$$\frac{\theta+\theta_1}{2}-\psi=.23r^2 \sin \psi \cos \psi,$$

and the outstanding error is substantially the same as before.

Equation (9) appears to be inconsistent with a result obtained by Mr. Glazebrook (Phil. Mag. Oct. 1880, p. 252). For the sake of simplicity, he supposed the ordinary ray only to emerge from the crystal. He examined two positions of the Nicol. First the emergent ray lay in the principal plane, so that the plane of polarization coincided with the principal plane. Next he supposed the Nicol to be turned through 90° about an axis, lying in the principal plane, but inclined at 5° to the emergent ray, and he found that now the plane of polarization was inclined at $5^\circ 3'$ to the principal plane. *But this does not show that the plane of polarization has been turned through $90^\circ \pm 5^\circ 3'$ about the direction of the ray, as is evident on examination of the annexed figure.*

Fig. 4.



P' is, as before, the emergent light ; N, N', X, X' are the two positions of the normal to the face and the axis.

In the first position the plane of polarization is $P'ANX$.

In the second position let it be $P'K$.

Mr. Glazebrook finds $P'KA = 5^\circ 3'$.

The angle, however, that we wish to find, is the angle through which the plane of polarization has been turned about the direction of the ray, viz. $AP'K$, and this, of course, is not equal to $90^\circ - P'KA$.

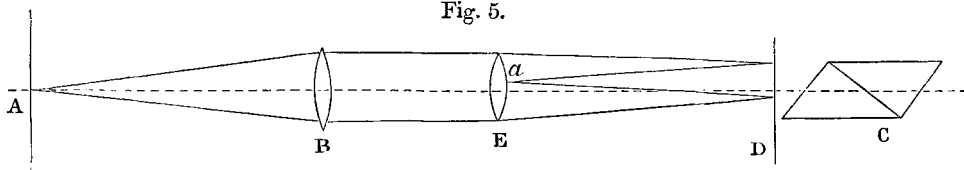
After I had written the greater part of this paper, I found that the ground had been already traversed by Sande Bakhuyzen (*Pogg. Ann.* cxlv. p. 259, 1872). His investigation is considerably longer than mine, and he only examines the case of the ordinary Nicol. He obtains results of the same general form; but he falls into at least one serious error; so his numerical values are quite different. Instead of assuming, as I have done, that the rotation of the plane of polarization on refraction out of the crystal is negligible, he uses a complicated formula, giving directly the position of the plane of polarization of the incident ray in which only the extraordinary ray is excited in the crystal. This formula he has apparently deduced from a formula given by Neumann (*Pogg. Ann.* xlii. p. 11). And it is here that the error occurs. He has not paid sufficient attention to Neumann's explanation of how the quantities in the formula are to be measured. He puts β for ω ; whereas he should put $\beta - \pi$ for ω . Making this correction in his equation (5), we find all the terms in the denominator are of one sign, and the value of $\tan A$ is considerably diminished. We find, too, that $90^\circ - A$ is practically the same as the angle between the plane of incidence and the plane of polarization in the crystal, which is what I have assumed.

In this very same operation Bakhuyzen falls into another mistake. He is considering the case of light emerging from the Nicol, while he uses the formula for light entering the crystal. Even with glass this would have been wrong; but with crystal the cases are totally distinct, for when light is refracted out of the crystal there are two reflected rays instead of only one.

It may not be out of place here to describe a simple method for testing the necessary adjustment, which I lately found convenient. In my arrangement (fig. 5) the source of light was an illuminated slit A, whence the light passed through a lens B, placed so that the slit was at its principal focus. Thus a parallel beam of light fell on the polarizing Nicol C. To the Nicol circle I attached a plane mirror D as nearly as possible at right angles to the axis of rotation. Between B and D, I interposed a lens E at a distance of half its focal length from D. An image of the slit is thus formed at a on the surface

of the lens E, or rather on a scrap of paper attached to it. The image *a* of course remains at the same point on the paper,

Fig. 5.



however the lens E is moved laterally in its own plane. On rotating the Nicol circle I found that *a* did not move; so I knew that the mirror was at right angles to the axis of rotation. I then adjusted the Nicol circle till the middle point of the image of the slit fell on the optical centre of the lens, which I had previously marked on the paper. If I had chosen a wrong point as the optical centre, the error would have been at once evident on turning the lens E through 180° in its own plane. This method gave without much difficulty an accuracy of 1° , with a lens E of only 4 inches focal length.

2. On a new Method of obtaining the Zero-reading of a Nicol Circle.

In a large class of experiments on polarized light, it is necessary to know the reading of the Nicol circle when the plane of polarization is parallel to the axis of rotation of some part of the apparatus, *e. g.* of the table of a spectrometer. The usual method of obtaining this reading depends on the polarizing power of a glass reflecting surface. It is easy to fix the surface on the spectrometer-table parallel to the axis of rotation. If the angle of incidence be made that of maximum polarization, and the Nicol turned till the reflected light is reduced to a minimum, the plane of polarization is then perpendicular to the plane of incidence. This method is simple, but it is not very sensitive. Even with poor illumination the minimum reflected light is, with glass at any rate, by no means evanescent; and as the Nicol is turned the intensity remains sensibly constant for some distance on either side of the minimum point. The sensitiveness, too, does not increase, but rather falls off with a more powerful light.

The method I am about to describe is nearly as simple; and, as it depends on the crossing of two Nicols, its sensitiveness is only limited by the power of the source of light. In its simplest form the process is as follows:—An auxiliary Nicol is fixed on the table of the spectrometer, the polarizer turned till the light is quenched, and the reading taken.

Then the table of the spectrometer is turned through two right angles—the axis of rotation having been previously set perpendicular to the incident light—the light quenched, and the reading taken again. *The plane of polarization now leans as much to one side of the axis as it did before to the other.* So the mean of the two readings is the reading when the plane of polarization is parallel to the axis of rotation of the table.

The above is of course only a general explanation. The statement in italics requires fuller examination, and we shall find that it is only strictly true when the Nicol is symmetrical. Let us, then, first suppose that the Nicol is perfectly symmetrical—that is, that the two faces are parallel and the optic axes of the two halves coincident. We shall assume throughout the investigation that the axis of rotation is accurately perpendicular to the incident light. In the previous note we have carefully determined the position of the plane of polarization in terms of the direction of the emergent light, and we have shown that, to a very close approximation, it is at right angles to that position of the plane of polarization of light incident along the same path which gives complete extinction in the Nicol. The discrepancy was shown to be negligible. So there is no ambiguity in speaking of the plane of polarization of either half of the Nicol for a particular direction of the light.

In the first position of our Nicol on the spectrometer, the emergent light is parallel to the incident, and the planes of polarization of the two halves are also parallel. The axis of rotation, too, is at right angles to the incident light. Imagine, then, a line fixed relatively to the Nicol to be drawn from the second face parallel to the emergent light. When the Nicol has been rotated through two right angles, this line becomes again parallel to the incident light, and the plane of polarization of the second half of the Nicol now leans exactly as much to one side of the axis of rotation as the plane of polarization of the first half leant before to the other. The process therefore is so far strictly accurate.

Secondly, let us suppose that the Nicol, though not symmetrical, yet produces no angular deviation on light traversing it. We have seen in the previous note that the plane of polarization of a Nicol is nearly at right angles to the principal plane, and that to a first approximation the small deviation is proportional to the angle between the external light and the principal plane.

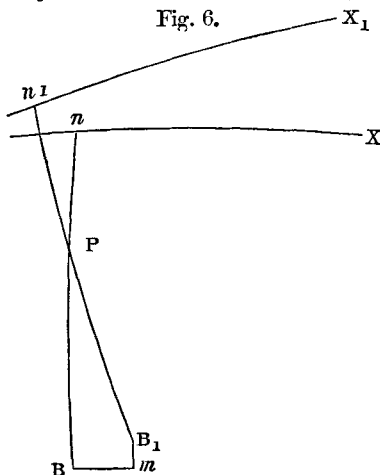
Let X_n , X_{n_1} be the principal planes of the two halves.

B , B_1 their poles.

P the direction of incident and emergent light.

Draw Bm at right angles to BP and B_1m at right angles to Bm . We may assume that BB_1 is very small and BP

Fig. 6.

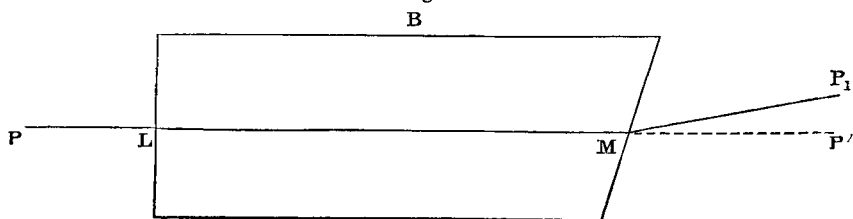


nearly a right angle. The angle between the planes of polarization of the two halves is composed of two parts, of which one, BPB_1 , is approximately equal to Bm , and the other is approximately proportional to the difference between Pn_1 and Pn and therefore to B_1m . So the angle is independent of small variations in the position of P , and is therefore a constant for each particular Nicol. In some Nicols, as we go from one half to the other, the plane of polarization is turned in the direction of a right-handed screw. These we may call right-handed Nicols with a certain rotation-angle. Supposing, then, our auxiliary Nicol is right-handed, the plane of the second half leans at first too much to the right, but, when the Nicol has been rotated through 180° , it leans too much to the left. The correct reading in the case of a right-handed Nicol is therefore that obtained by turning the Nicol circle from the mean reading through half the rotation-angle in the right-handed direction.

The question now arises, how we are to determine the rotation-angle. The following process explains itself. Place the auxiliary Nicol B on the path of the light from the polarizer A , which is mounted in a graduated circle. Turn A till the light is quenched and take a reading. Then turn A so as to allow the light to pass through B , and place in the path of the light from B a third Nicol C . Adjust C till the light from B is quenched. Remove B , cross C with A , and read again.

So far the matter is tolerably simple, but if the Nicol produce deviation of the light complications are introduced.

Fig. 7.



The figure represents a flat-ended Nicol B traversed by a ray of light $PLMP_1$. The argument, however, is perfectly general. A small error is due to the Nicol C having to deal first with light parallel to MP_1 and afterwards with light parallel to PP' . This may be eliminated by taking two readings, the Nicol B in the interval having been turned through two right angles or thereabouts about LM. The mean thus found gives us the position of the plane of polarization of PL, which is most nearly parallel to the plane of polarization of MP_1 . We know nothing about the plane of polarization of light which issues parallel to MP' . Clearly therefore, when our Nicol is mounted on the spectrometer-table, we ought not to turn it exactly through 180° , but we ought to turn it till the incident light is related to the face M in the same way as P_1M was at first.

We have then this practical rule. Place the auxiliary Nicol on the spectrometer in such a position that the deviation lies wholly in the plane of rotation. To get the second reading, turn the table in the direction of the deviation through an angle equal to 180° less the deviation. Correct the mean of the two readings by half the rotation-angle as before.

Thus we are able to get an accurate value of the zero-reading in spite of the dissymmetry of the Nicol and the deviation of the light, provided only the two faces of the Nicol are good planes. We have been compelled, however, to limit ourselves to first approximations. On this point it is to be noticed that the outstanding error is due to two causes, the inaccuracy of adjustment of the auxiliary Nicol, and the dissymmetry of the Nicol itself. The latter we may assume to be very small; while the former has very slight effect. It only comes in as a secondary cause, for if the dissymmetry were removed the outstanding error would be zero. Hence it is not necessary in general to take elaborate precautions about the setting of the Nicol in the determination either of the zero-reading or of the rotation-angle. If great accuracy be

required, it will be sufficient to attach permanently to the Nicol a small reflecting surface, and make this always perpendicular to the incident light. The planes of polarization of both halves will then be perfectly definite.

I have treated the method as applied to a Nicol used as polarizer, but it is by no means restricted to this case. The polarizer may be any kind of polarimeter, or if necessary the polarimeter may be the analyzer, while the auxiliary Nicol is used as polarizer. The method occurred to me when I was engaged on some observations on the refraction of polarized light at the surface of Iceland spar. I will describe the arrangements in so far as they bear on the matter in hand; for, as it happened, they gave a very high degree of accuracy in the determination of the zero-reading.

As my source of light I employed part of the filament of a Swan incandescent lamp. I selected a straight piece, and cut off the light from the rest of the filament with a diaphragm placed immediately in front of the lamp. The filament was placed at the principal focus of a lens, from which the light passed to the polarizing Nicol. Then came the spectrometer, from which the collimator had been dismantled; so only the telescope remained. When the telescope was focussed for infinity and directed towards the light, a sharply defined image of the selected portion of the filament was seen. The top and bottom limits of the image were not hard lines, owing to the diaphragm not being quite at the principal focus of the collimating lens. They were, however, sufficiently definite for practical purposes.

It was necessary for my other observations that the axis of rotation of the spectrometer should be set at right angles to the light coming from the middle of the filament. I accomplished this in the following manner. I first mounted a reflecting surface on the table of the spectrometer, and adjusted it to be parallel to the axis of rotation. This may be done with accuracy although the telescope be tilted. Next I adjusted the axis of the telescope to be at right angles to this surface, by getting the cross wires to coincide with their image formed by reflection at the surface. Then I removed the reflecting surface, directed the telescope towards the light, and tilted the spectrometer till the cross wires coincided with the middle of the image.

When the auxiliary Nicol was mounted on the spectrometer-table and the polarizer turned past the crossed position, the image of the filament by no means entirely disappeared; but a patch of nearly complete extinction moved down the image from the top to the bottom. This was of course to be expected, since different positions of the image correspond to different

directions through the Nicols, and therefore to different positions of the two planes of polarization. The reading was taken when the patch was halfway down the slit. This method of reading proved to be very sensitive. Without any special care I could get a number of successive readings whose greatest difference was $1'$. This was quite sufficient for my purposes. Indeed my verniers were only graduated to $1'$. But I am convinced that a far higher degree of sensitiveness could be reached if desired, by proceeding on the same lines.

In securing great accuracy two precautions become very important. The source of light should be of uniform brightness. Here a well-made carbon filament is nearly perfect. Again, as much light should reach the eye from the top of the filament as from the bottom, abstraction being made of course of what is stopped by the polarizing properties of the Nicols. For this it is convenient that the lateral throw of the Nicols should be as small as possible. Flat-ended Nicols are therefore to be preferred. But if a pair of Nicols were made for the purpose, it would be easy so to slant the faces that the extraordinary ray should go straight through, and there should be no lateral throw.

The incandescent lamp is a more powerful source of light for this purpose than is perhaps at first sight apparent. For it may be shown that with a given slit and object-glass the intensity of the illumination of the surface of the first Nicol depends solely on the intrinsic brightness of the source of light. The filament is practically a slit with the source of light brought into contact with it, so there is no difficulty about the whole of the surface of the Nicol being illuminated, and the illumination is just as powerful as would be given by a glowing sheet of the same brightness half an inch broad placed a little distance behind a slit. The intrinsic brightness of the filament of a Swan lamp is many times greater than that of a good gas-flame viewed edgewise. Besides, even if the gas-flame be placed very close to the slit, it is only the nearer portion of the flame that illuminates the whole surface of the Nicol. The further portion only lights up a narrow strip in the middle of the surface.

I used two flat-ended Nicols, belonging to Mr. Glazebrook, of about half an inch aperture. One of these, which gave an angular throw of $15'$, had a left-handed rotation-angle of $38'$.

I find that the plan of reading by means of the motion of the patch of extinction has been the subject of an elaborate paper by Lippich (*Sitzb. der kais. Akad. der Wissensch. Wien*, Febr. 1882).