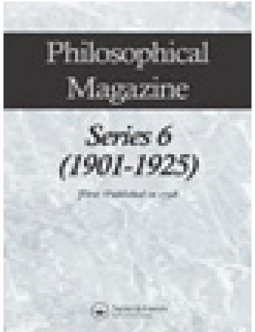

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XXV. *On a Compact Apparatus for determining Young's Modulus for Thin Wires.* *By* CHARLES H. LEES, *D.Sc.*, *and* ROGER E. GRIME, *B.Sc.**

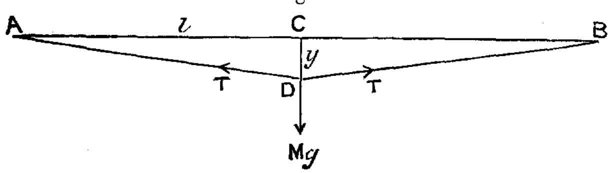
MANY who have determined Young's Modulus for a material both by the bending of a beam and by the stretching of a wire of the material, must have contrasted the compactness of the apparatus used in the former with the bulkiness of that used in the latter method. It was this contrast which led us to attempt to devise a simple and compact apparatus for testing wires. The initial difficulties met with in making the apparatus at the same time compact and reliable proved much less serious than we anticipated, and as the apparatus in its final form proved easy to use and satisfactory, it seemed to us worth while calling the attention of physicists to a method of determining Young's Modulus for thin wires which does not appear to have received in the past the attention it deserves.

Theory of the Method.

The method utilizes the depression produced in the middle of a straight horizontal length of thin wire †, supported rigidly at its ends, by a load applied at its middle point.

If AB be a wire whose resistance to bending may be neglected, of length $2l$, supported at A and B, and if a mass

Fig. 1.



M, suspended at its middle point C, depress that point to D, where CD $= y$, the downward force Mg at D is balanced by the vertical components of the pull T in DA and DB;

$$i.e.\ \ \mathrm{M}g = 2\mathrm{T}\cos \mathrm{C\hat{D}B} = 2\mathrm{T}\sin \mathrm{C\hat{B}D},$$

$$\text{or}\quad \mathrm{T} = \frac{\mathrm{M}g}{2}\operatorname{cosec}\theta \quad \text{where} \quad \theta = \mathrm{C\hat{B}D}.$$

When $\mathrm{C\hat{B}D} = 0$, and therefore M $= 0$, let $\mathrm{T} = \mathrm{T}_0$. Then the change of length of a length originally l, due to the increase of the pull of the wire from T_0 to T is equal to

* Communicated by the Authors.

† Up to about No. 27 S.W.G. in the apparatus used. For thicker wires the flexural rigidity renders it necessary, if an accuracy of 1 per cent. is required, to treat the wire as an *elastica*. It is neither straight nor is the tension in it constant, and the simple theory gives too high values for the modulus.

$l\frac{T-T_0}{a\epsilon}$, where a is the cross section of the wire and ϵ is the Young's modulus of its material, Hooke's law being supposed to hold up to the maximum value of T used.

Therefore

$$1+\frac{T-T_0}{a\epsilon}=\sec C\hat{B}D$$

$$\therefore \qquad 1+\frac{\frac{Mg}{2}\operatorname{cosec}\theta-T_0}{a\epsilon}=\sec\theta.$$

$$\frac{Mg}{2}\operatorname{cosec}\theta-T_0=a\epsilon(\sec\theta-1) \quad . \quad . \quad (1)$$

From equation (1) it is evident that T_0 should be taken as small as possible to secure the maximum accuracy of determination of ϵ.

If a second mass M_1 suspended from C produce a deflexion θ_1 at B,

$$\frac{M_1 g}{2}\operatorname{cosec}\theta_1-T_0=a\epsilon(\sec\theta_1-1).$$

Hence eliminating T_0,

$$\frac{g}{2}(M_1\operatorname{cosec}\theta_1-M\operatorname{cosec}\theta)=a\epsilon(\sec\theta_1-\sec\theta)$$

or

$$\epsilon=\frac{g}{2a}\cdot\frac{M_1\operatorname{cosec}\theta_1-M\operatorname{cosec}\theta}{\sec\theta_1-\sec\theta}.$$

Or, in terms of the half-length l of the wire and the depressions $y\ y_1$ at the centre

$$\epsilon=\frac{g}{2a}\frac{M_1\sqrt{1+\left(\frac{l}{y_1}\right)^2}-M\sqrt{1+\left(\frac{l}{y}\right)^2}}{\sqrt{1+\left(\frac{y_1}{l}\right)^2}-\sqrt{1+\left(\frac{y}{l}\right)^2}}.$$

or

$$\epsilon=\frac{gl}{2a}\cdot\frac{\frac{M_1}{y_1}\sqrt{\frac{1+(y_1/l)^2}{1+(y/l)^2}}-\frac{M}{y}}{\sqrt{\frac{1+(y_1/l)^2}{1+(y/l)^2}}-1} \quad . \quad . \quad . \quad . \quad (2)$$

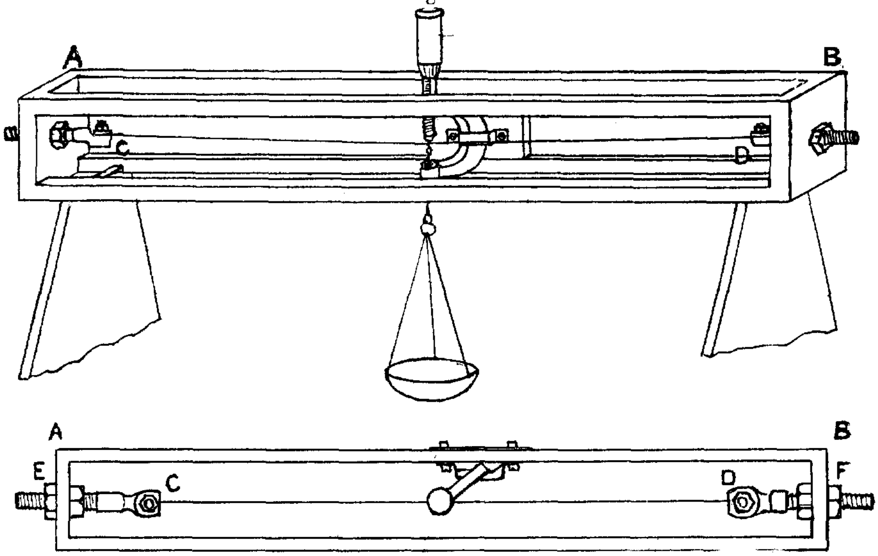
In all the experiments which follow $\frac{y}{l}$ is sufficiently small to allow the expression under the root to be written 1 in the numerator and $1+\frac{y_1^2-y^2}{2l^2}$ in the denominator; and the equation then takes the simpler approximate form

$$\epsilon=\frac{gl^3}{a}\cdot\frac{\frac{M_1}{y_1}-\frac{M}{y}}{y_1^2-y^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Description of the Apparatus.

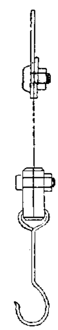
The wire was supported rigidly in an iron frame AB (fig. 2) by two clamps C, D whose distance apart could be adjusted

Fig. 2.



by means of the nuts E, F. These were adjusted till the tension in the wire was rather more than that necessary to straighten it when unloaded. In order to be able to reproduce this state of tension readily in successive experiments, a subsidiary wire half the length of the one experimented on, and cut from the same piece, was suspended vertically and loaded at the lower end (fig. 3) till the note it gave on being plucked with a pointed brass wire was identical with that given by the wire experimented on when held at its centre and plucked in the same way. The accuracy of the determination of T_0 by this method was not, however, sufficient to admit of its being used in equation (1) to determine ϵ, and equation (2) was therefore used throughout.

Fig. 3.



The load was applied at the centre of the length of the wire by means of a very small hook of steel wire, from which a scale-pan could be suspended. The vertical descent of the hook was measured by a Brown & Sharp micrometer screw-gauge with a pitch of ·5 mm., and head graduated in ·01 mm., admitting of estimation down to ·001 mm. The screw-stop in the lower jaw of the screw-gauge was removed to allow the wire suspending the scale-pan to pass downwards along the axis of the screw. The gauge was fixed rigidly in the iron frame by metal straps, and was placed obliquely to save space.

The end surface of the screw of the gauge was polished in order to give a reflexion of the hook with which it had to be brought into contact, and thus enable the point of contact to be determined with greater accuracy. A small magnifying-glass was used to increase still further the accuracy of the setting.

The depression due to the load was determined both while the load was being increased and while it was being diminished. The zeros at no load were taken before and after the loading, and the mean taken if they were in close agreement. If not, the observations were rejected.

Equation (2) is obtained on the assumption that the supports of the wire are rigid, and in some of the earlier experiments this was tested by placing a second wire in the clamps alongside the wire experimented on, loading it lightly, and observing whether on loading the experimental wire above the limit to which it was proposed to go in the actual experiment, any depression of the subsidiary wire was produced. This was found not to be the case.

The temperature throughout was that of the air of the room (16° C.). It was not found necessary to take any special precautions to maintain the apparatus at constant temperature. So long as the wire was not touched during an experiment, it came back at the end to the same position, showing that its temperature had remained sufficiently constant.

The following tables give the measurements made on a few wires, and show the degree of accuracy which can be readily obtained with the apparatus.

Iron wire, No. 34 S.W.G. $l=24{\cdot}05$ cms., mean diameter $\cdot 0230$ cm. $\therefore$ area a of cross-section $=\cdot 000415$ sq. cm., and $\frac{gl^3}{a}=3{\cdot}28\times 10^{10}$.

Initial stretching force in dynes.	M. gr.	y mean. cms.	$\frac{M}{y}$.	y^2.	$\frac{M_1}{y_1}-\frac{M}{y}$.	$y_1^2-y^2$.	$\frac{\frac{M_1}{y_1}-\frac{M}{y}}{y_1^2-y^2}$.	ϵ. dynes/sq. cm.
Not recorded.	15·50	·2461	62·98	·0605				
	25·50	·3759	67·84	·1413	4·86	·0808	60·2	$1{\cdot}97\times 10^{12}$
	35·50	·4845	73·27	·2347	10·29	·1742	59·1	$1{\cdot}94\times 10^{12}$

Mean $\epsilon=1{\cdot}95\times 10^{12}$.

Iron wire, No. 34 S.W.G. $l=26{\cdot}50$ cms., mean diameter ·02414 cm. ∴ area a of cross-section = ·0004577 sq. cm., and $\frac{gl^3}{a}=3{\cdot}99\times10^{10}$.

Initial stretching force in dynes.	M. gr.	y mean. cms.	$\frac{M}{y}$.	y^2.	$\frac{M_1}{y_1}-\frac{M}{y}$.	$y_1^2-y^2$.	$\frac{\frac{M_1}{y_1}-\frac{M}{y}}{y_1^2-y^2}$.	ϵ. dynes/sq. cm.
$440\times g$	15·0	·3755	39·95	·1410				
	25·0	·5310	47·08	·2820	7·13	·1410	50·6	$2{\cdot}02\times10^{12}$
	35·0	·6479	54·00	·4198	14·05	·2788	50·4	2·01

Mean $\epsilon=2{\cdot}01\times10^{12}$.

Iron wire, No. 29 S.W.G. $l=26{\cdot}50$ cms., mean diameter ·03228 cm. ∴ area a of cross-section = ·0008184 sq. cm., and $\frac{gl^3}{a}=2{\cdot}23\times10^{10}$.

Initial stretching force in dynes.	M. gr.	y mean cms.	$\frac{M}{y}$.	y^2.	$\frac{M_1}{y_1}-\frac{M}{y}$.	$y_1^2-y^2$.	$\frac{\frac{M_1}{y_1}-\frac{M}{y}}{y_1^2-y^2}$.	ϵ. dynes/sq. cm.
$790\times g$	15·0	·2251	66·37	·0507				
	25	·3446	72·54	·1187				
	35	·4412	79·33	·1947	12·96	·1440	90·0	$2{\cdot}00\times10^{12}$
	45	·5219	86·24	·2724	13·70	·1537	89·2	1·99
$950\times g$	15·0	·1992	75·30	·0397				
	25	·3112	80·33	·0968				
	35	·4053	86·36	·1643	11·06	·1246	88·80	$1{\cdot}98\times10^{12}$
	45	·4845	92·88	·2347	12·55	·1379	91·02	2·03

Mean $\epsilon=2{\cdot}00\times10^{12}$.

Steel wire (pianoforte-wire), No. 29 S.W.G. $l=26{\cdot}40$ cms., mean diameter ·03500 cm. ∴ area a of cross-section = ·000962 sq. cm., and $\frac{gl^3}{a}=1{\cdot}88\times10^{10}$.

Initial stretching force in dynes.	M. gr.	y mean. cms.	$\frac{M}{y}$.	y^2.	$\frac{M_1}{y_1}-\frac{M}{y}$.	$y_1^2-y^2$.	$\frac{\frac{M_1}{y_1}-\frac{M}{y}}{y_1^2-y^2}$.	ϵ. dynes/sq. cm.
$1830\times g$	25·0	·1712	146·02	·0293				
	35	·2339	149·63	·0547				
	45	·2919	154·16	·0852	8·14	·0559	146	$2{\cdot}74\times10^{12}$
	55	·3462	158·86	·1198	9·23	·0651	142	2·67

Mean $\epsilon=2{\cdot}70\times10^{12}$.

A second similar wire. $l=26{\cdot}40$ cms., mean diameter ·03550 cm. ∴ area a of cross-section $=·000990$, and $\frac{gl^3}{a}=1{\cdot}82\times10^{10}$.

Initial stretching force in dynes.	M. gr.	y mean. cms.	$\frac{M}{y}$.	y^2.	$\frac{M_1}{y_1}-\frac{M}{y}$.	$y_1^2-y^2$.	$\frac{\frac{M_1}{y_1}-\frac{M}{y}}{y_1^2-y^2}$.	ϵ. dynes/sq. cm.
$1870\times g$	25·0	·1614	154·90	·0260				
	35	·2208	158·51	·0487				
	45	·2767	162·63	·0765	7·73	·0505	153	$2{\cdot}78\times10^{12}$
	55	·3284	167·47	·1078	8·96	·0591	152	2·77

Mean $\epsilon=2{\cdot}78\times10^{12}$.

Nickel wire, No. 30 S.W.G. $l=26{\cdot}50$ cms., mean diameter = ·03184 cm. ∴ area a of cross-section = ·000796 sq. cm., and $\frac{gl^3}{a}=2{\cdot}29\times10^{10}$.

Initial stretching force in dynes.	M. gr.	y mean. cms.	$\frac{M}{y}$.	y^2.	$\frac{M_1}{y_1}-\frac{M}{y}$.	$y_1^2-y^2$.	$\frac{\frac{M_1}{y_1}-\frac{M}{y}}{y_1^2-y^2}$.	ϵ. dynes/sq. cm.
$330\times g$	15	·3909	38·37	·1528	10·09	·1134	88·9	$2{\cdot}04\times10^{12}$
	25	·5159	48·46	·2662	9·19	·1024	89·8	2·06
	35	·6071	57·65	·3686	19·28	·2158	89·3	2·05
$450\times g$	15	·3566	42·06	·1272	9·42	·1086	86·8	$1{\cdot}99\times10^{12}$
	25	·4856	51·48	·2358	9·00	·0991	90·8	2·08
	35	·5787	60·48	·3349	18·42	·2077	88·6	2·03
$530\times g$	15	·3033	49·46	·0920	8·30	·0953	87·1	$1{\cdot}99\times10^{12}$
	25	·4328	57·76	·1873	8·35	·0930	89·8	2·06
	35	·5294	66·11	·2803	16·65	·1883	88·4	2·02

Mean $\epsilon=2{\cdot}04\times10^{12}$.

Aluminium wires Nos. 25 and 30 S.W.G. were tried, but neither gave consistent results, owing to permanent set being produced by the small loads used.

Copper wire, No. 27 S.W.G. $l = 26{\cdot}50$ cms., mean diameter ·0412 cm. ∴ area a of cross-section = ·00133 sq. cm., and $\frac{gl^3}{a} = 1{\cdot}37 \times 10^{10}$.

Initial stretching force in dynes.	M. gr.	y mean. cms.	$\frac{M}{y}$.	y^2.	$\frac{M_1}{y_1} - \frac{M}{y}$.	$y_1^2 - y^2$.	$\frac{\frac{M_1}{y_1} - \frac{M}{y}}{y_1^2 - y^2}$.	ε. dynes/sq. cm.
$650g$	15	·2749	54·56	·0756	7·47	·0863	86·6	$1{\cdot}19 \times 10^{12}$
	25	·4024	62·03	·1619	7·50	·0915	82·0	1·12
	35	·5034	69·53	·2534	14·97	·1778	84·2	1·15
$380g$	15	·3630	41·32	·1318	9·44	·1108	85·2	1·17
	25	·4925	50·76	·2426	8·80	·1027	85·7	1·17
	35	·5876	59·55	·3453	18·24	·2135	85·4	1·17
$630g$	15	·2774	54·07	·0769	7·75	·0866	89·5	1·23
	25	·4044	61·82	·1635	7·83	·0890	88·0	1·21
	35	·5025	69·65	·2525	15·58	·1756	88·7	1·22

Mean $\epsilon = 1{\cdot}18 \times 10^{12}$.

TABLE OF RESULTS.

	YOUNG'S MODULUS at 16° C.
	dynes per sq. cm.
Iron wire, No. 34 A.	$1{\cdot}95 \times 10^{12}$
,, No. 34 B.	2·01
,, No. 29	2·00
Steel pianoforte-wire, No. 29 A.	2·70
,, ,, No. 29 B.	2·78
Nickel wire, No. 30	2·04
Copper wire, No. 27	1·18

XXVI. *On the Application of Legendre's Functions to the Theory of the Jacobian Elliptic Integrals.* By J. W. NICHOLSON, *B.Sc. (Lond. & Vict.), Trinity College, Cambridge* *.

THE object of this paper is to indicate a method of some generality, of expanding (1) functions expressible in terms of the complete elliptic integrals of Jacobi; and (2) definite integrals containing incomplete elliptic integrals in their integrand, in series of Legendre coefficients of increasing

* Communicated by the Author.