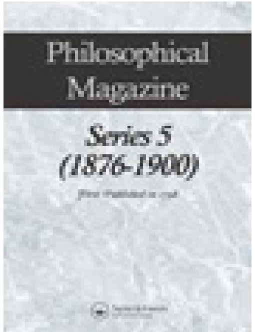

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VIII. On the relation between the notes of open and stopped pipes

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V. The effect of external pressure on a thermometer's bulb is directly proportional to the pressure as far as about 140 atmospheres. The ascent of the zero of a thermometer on keeping is consequent on a change of state in the glass, being the same whether the thermometer be open or closed, and therefore independent of atmospheric pressure.

VI. When all corrections are made, every individual thermometer has specific characters whereby it differs from all other thermometers.

VII. A number of bodies have been rigorously purified, and their fusion-points determined, with a good second place of decimals, in terms of the air-thermometer: these points range from about 35° to 121°. The possession of these bodies, which can always be preserved without risk, will enable any observer to obtain standard points within that distance, and save a vast amount of tedious experimentation.

Anderson's College, Glasgow.

VIII. *On the Relation between the Notes of Open and Stopped Pipes.* By R. H. M. Bosanquet, *Fellow of St. John's College, Oxford.*

To the Editors of the Philosophical Magazine and Journal.

Gentlemen,

It has long been known to practical men that, if an open pipe be stopped at one end, the note of the stopped pipe is not exactly the octave below the note of the open pipe, as it should be according to Bernouilli's theory, but the stopped pipe is somewhat less than an octave below the open pipe; in ordinary organ-pipes the difference is said to be about a major seventh instead of an octave. It has occurred to me lately that the theory of this phenomenon is not generally known; and the following account of it, with some of its applications, may be of interest. I should mention that the investigations were made some time ago, before the publication of my Notes on the Theory of Sound, in the Philosophical Magazine last year; and they were not mentioned there only because the methods depending on them proved of insufficient accuracy for the purpose then in view.

Consider a cylindrical tube open at both ends. Let its length be l, and its diameter 2R. Then the effective (or reduced) length of the pipe is $l+2\alpha$; where α is the correction for one open end, which formed the subject of the investigations contained in Nos. 5 and 6 of my "Notes" (Phil. Mag. [V.] vol. iv. pp. 25, 125, 216).

Now suppose a flat stopper, fitting airtight, to be applied at one end of the tube. It may then, according to the ordinary theory, be regarded as equivalent to the half of an open pipe whose middle point, or node, coincides with the face of the stopper, the effective length measured from the node being $l+\alpha$. The length of the corresponding open pipe would be double of this, or $2(l+\alpha)$. The ratio of the notes is consequently $(l+2\alpha) : 2(l+\alpha)$, which may be put in the form

$$\frac{1}{2} \times \frac{l+2\alpha}{l+\alpha};$$

that is to say, the interval in question differs from an octave by the interval whose ratio is $(l+2\alpha) : (l+\alpha)$.

The following experiment was made with an iron cylindrical tube, 4·9 inches in length and 2 inches in diameter. The notes were determined, as in my former investigations, by blowing short jets of air against the edges. The tube was stopped by standing it upright on a flat surface, and applying a little oil round the edge in contact with the surface. The notes of the pipe, open and stopped, made with one another the interval of a minor seventh; *i. e.* they deviated from the octave by a whole tone. The ratio (9 : 8) was determined with some slight accuracy by comparison with the notes of my enharmonic organ. The tuning of this instrument is not, however, sufficiently stable to base very accurate work on. Then

$$\frac{9}{8} = \frac{l+2\alpha}{l+\alpha},$$

or

$$\frac{1}{8} = \frac{\alpha}{l+\alpha};$$

$$\therefore\ l = 7\alpha.$$

And $l = 4{\cdot}9$ in.,

$$\therefore\ \alpha = {\cdot}7 \text{ in.};$$

and $R = 1$ in.,

$$\therefore\ \alpha = {\cdot}7\,R.$$

The value of α for this tube was formerly determined at ·635 R (Phil. Mag. vol. iv. p. 219). The tube has been shortened by about ·1 inch since; but this cannot affect the correction. It appears then that the present process presents general correspondence with the result of the former investigation; but the numerical values of α do not coincide very exactly.

When I originally investigated this subject some time ago, I anticipated that I should be able, by observation of the interval between open and stopped pipes, to determine α in an accurate manner. For this purpose I constructed many pipes, in which the interval in question was as nearly as possible of definite magnitude, generally a semitone less than the octave; but the method proved too inaccurate to be of any real use. An excellent and perfect tonometer is required to measure the intervals accurately; and if we have that, it can be applied to the solution of the problem with greater advantage in other ways. The present method, however, is quite sufficient for the approximate demonstration of the value of α.

There are difficulties in the way of the exact application of these principles to ordinary organ-pipes. First, it is impossible to blow an open and stopped pipe in a similar manner with the same mouthpiece. The pitch varies considerably with the force of the blowing; and the two notes produced with different blowing are not comparable. Again, there is a considerable correction of unknown amount to be taken account of, due to the closing-in of the mouth-end of the pipe.

We may, however, partly get over these difficulties. In the first place, it is possible to arrange a pipe so as to blow the fundamental when open and the twelfth when stopped, without variation of the wind. Secondly, the correction due to the closing-in of the parts round the mouth can be determined for pipes of given shape by sawing one of them across so as to leave a plain circular end. The correction due to the difference in pitch $+\alpha$ (correction for circular end) gives the total value of the correction for the mouth.

The following is an example:—Organ-pipe 9·5 inches from upper lip to open end; diameter ·95 inch. When arranged so as to blow the fundamental when open and twelfth when stopped, the twelfth was 2 commas of the enharmonic organ sharper than the note corresponding to the fundamental. Taking these to be true commas, which they are very nearly, we may take the resulting interval to be 40 : 41.

The correction for the mouth was determined by sawing across a similar pipe; it is roughly

$$\lambda = \frac{l}{6}.$$

Then

$$\frac{41}{40} = \frac{l+\lambda+\alpha}{l+\lambda},$$

$$l+\lambda = 40\alpha,$$

$$\alpha = \cdot 28 \text{ in.};$$

and since

$$2R = \cdot 95 \text{ in.},$$

$$\alpha = \cdot 59\,R \text{ nearly.}$$

This is closer than could be expected considering the extremely rough measurement of the two commas. It will be remembered that the value of α is known to be generally about ·55 R to ·6 R.

IX. *Notices respecting New Books.*

An Elementary Treatise on Spherical Harmonics and Subjects connected with them. By the Rev. N. M. Ferrers, *M.A., F.R.S., Fellow and Tutor of Gonville and Caius College, Cambridge.* London: Macmillan and Co. 1877. Crown 8vo, pp. 160.

THE author's object in this treatise is "to exhibit, in a concise form, the elementary properties of the expressions known by the name of Laplace's functions, or Spherical Harmonics." More than two fifths of it, comprised in chapters ii. and iii., are devoted to the discussion of the particular case in which the spherical Surface Harmonic (P_i) is a function of μ only. This function Mr. Ferrers calls a Zonal Surface Harmonic; it is the same function as that which Mr. Todhunter calls a "Legendre's Coefficient." The author investigates briefly and elegantly the chief properties of P_i, and then applies them to determine the potential of various forms of attracting matter. Of these the last which he considers is the following comprehensive case:—to find "the potential of a spherical shell of finite thickness whose density is any solid zonal harmonic." These investigations serve as a foundation for those contained in the following chapters. Thus, in the fourth chapter the subject of General, Tesseral, and Sectorial Spherical Harmonics is somewhat briefly treated. It is well known that the general Surface Harmonic of the degree i consists of $2i+1$ terms of the form

$$C \cos \sigma\phi \sin^\sigma \theta \frac{d^\sigma P_i(\mu)}{d\mu^\sigma}, \quad S \sin \sigma\phi \sin^\sigma \theta \frac{d^\sigma P_i(\mu)}{d\mu^\sigma}:$$

to these terms individually Mr. Ferrers gives the name of Tesseral Surface Harmonics of the degree i and order σ; and the last of these terms, viz. those for which $\sigma = i$, he calls Sectorial Surface Harmonics of the degree i. In the fifth chapter he notices very briefly the Spherical Harmonics "of the second kind;" and in the sixth chapter he treats of Ellipsoidal Harmonics, a name which he proposes to give to the functions called by Mr. Todhunter "Lamé's functions."

It is well known that one of the standing difficulties of this subject resides in the proof of the theorem that "any function which does not become infinite between the limits of integration can be expanded in a series of Spherical Harmonics." Thus, Mr. Todhunter notices four or five proofs, and is not, to all appearance, completely