

InSAR observations and models of crustal deformation due to a glacial surge in Iceland

A. Auriac^{1,5*}, F. Sigmundsson¹, A. Hooper², K. H. Spaans², H. Björnsson¹, F. Pálsson¹,
V. Pinel³, K. L. Feigl⁴

¹ *Nordic Volcanological Center, Institute of Earth Sciences, University of Iceland, Sturlugata 7, 101 Reykjavík, Iceland*

² *School of Earth and Environment, University of Leeds, LS2 9JT, United Kingdom*

³ *ISTerre, Université de Savoie, IRD, 73376 Le Bourget du Lac, France*

⁴ *Department of Geosciences, University of Wisconsin-Madison, 1215 West Dayton Street, Madison WI 53706, USA*

⁵ *Now at: Department of Geography, Durham University, South Road, Durham, DH1 3LE, UK*

SUMMARY

Surges are common at all the major ice caps in Iceland. Ice masses of gigatons may shift from the upper part of the outlet glacier towards the terminus in a few months, advancing the glacier front by up to several kilometres. The advancing ice front may be up to 100 m thick, increasing the load on crustal rocks correspondingly. We use the observed change in crustal loading during a surge of the western part of the Vatnajökull ice cap, Iceland, during 1993–1995 and the corresponding elastic crustal deformation, surveyed with interferometric synthetic aperture radar, to investigate the material properties of the solid Earth in this region. Crustal subsidence due to the surge reaches ~ 75 mm at the edge of the Síðujökull outlet glacier. This signal is mixed with a broad uplift signal of ~ 12 mm/yr, relative to our reference area, caused by the ongoing retreat of Vatnajökull in response to climate change. We disentangle the two signals by linear inversion. Finite element modelling is used to investigate the elastic Earth response of the surge, as well as

19 to confirm that no significant viscoelastic deformation occurred as a consequence of the
20 surge. The modelling leads to estimates of the Young's modulus and Poisson's ratio of
21 the underlying Earth. Comparison between the observed and modelled deformation fields
22 is made using a Bayesian approach that yields the estimate of a probability distribution
23 for each of the free parameters. Residuals indicate a good agreement between models and
24 observations. One-layer elastic models result in a Young's modulus of 43.2–49.7 GPa
25 (95% confidence) and Poisson's ratio of 0–0.27, after removal of outliers. Our preferred
26 model, with two elastic layers, provides a better fit to the whole surge signal. This model
27 consists of a one-kilometre-thick upper layer with an average Young's modulus of 12.9–
28 15.3 GPa and Poisson's ratio of 0.17, overlying a layer with an average Young's modulus
29 of 67.3–81.9 GPa and Poisson's ratio of 0.25.

30 **Key words:** Satellite Radar Interferometry - InSAR – Glaciology – Elastic deformation
31 – Glacial surge – Numerical solutions – Young's modulus – Poisson's ratio.

32 1 INTRODUCTION

33 Glaciers cover 11% of Iceland (Fig. 1) (Björnsson 1978). Since they are currently retreating, widespread
34 uplift induced by their melting occurs over a large area of Iceland. This uplift signal, reaching up to
35 20–25 mm/yr around the Vatnajökull ice cap, has been studied in detail over the past 20 years, to infer
36 some of the properties of the underlying Earth, such as the thickness of the elastic crust and the viscos-
37 ity of the underlying material (e.g., Pagli et al. 2007; Árnadóttir et al. 2009; Auriac et al. 2013). How-
38 ever, the Young's modulus E and Poisson's ratio ν remain uncertain. Since crustal behaviour is mostly
39 elastic at short time scales, these two elastic parameters control the upper Earth's layer deformation in
40 response to sudden stress perturbations. Quantitative estimates of E and ν are thus required to infer
41 stress variations from surface deformations, e.g. due to fault unloading or magma pressurization. Most
42 of the available estimates of the Young's modulus and Poisson's ratio are derived from seismic wave
43 velocities (e.g., Allen et al. 2002). The parameters, inferred from the rapid dynamic response to pass-
44 ing seismic waves, are called dynamic values. Seismic studies provide detailed maps of the spatial
45 variation of the Young's modulus, and how it increases with depth (Pálmason 1971; Gudmundsson
46 1988; Allen et al. 2002; Currenti et al. 2007; Hooper et al. 2011). In contrast, the static values of

* E-mail: ama3@hi.is / a.m.auriac@durham.ac.uk

47 the parameters correspond to a static load. They can be measured in laboratory experiments (Cheng
48 & Johnston 1981; Eissa & Kazi 1988; Asef & Najibi 2013) for a given range of confining pressure.
49 They can also be estimated from modelling of the deformation signal induced by well-constrained
50 surface loading perturbations, such as annual ice thickness variations (Grapenthin et al. 2006; Pinel
51 et al. 2007). Comparative studies have shown that there is a difference between the dynamic and the
52 static estimates of the Young's modulus, with a static-to-dynamic ratio (E_s/E_d) in the range 0.4–1.0
53 (Cheng & Johnston 1981; Asef & Najibi 2013). This ratio is highly dependent on the heterogeneity of
54 microscopic structures of the rock material and its porosity, such that the difference tends to decrease
55 with confining pressure. It follows that the estimate of static parameters from the dynamic ones is not
56 straightforward and there is a need to provide good static in-situ estimates.

57

58 The aim of this study is to use interferometric synthetic aperture radar (InSAR) measurements to
59 measure surface deformation associated with a glacial surge, and to model the observed deformation
60 to constrain the elastic properties of the Earth. Surges are common at the outlet glaciers of all the
61 major ice caps in Iceland (e.g., Thorarinsson 1969; Björnsson et al. 2003). Ice-flow at surge-type out-
62 let glaciers is generally too slow to remain in balance with their accumulation rates. As a result, the
63 glacier thickens in its upper part, thins and steepens in the lower part, and the terminus draws back.
64 After several years of glacier surface steepening, the basal sliding velocity increases in a zone centred
65 in the upper ablation area where crevasses are formed. Downstream from this zone of enhanced veloc-
66 ity, a step-like thickening of the glacier develops and a bulge, usually tens of metres high, advances at
67 rates of 20–80 m per day. Propagation of the bulge to the glacier terminus generally requires less than
68 a year. Once the bulge reaches the terminus, the glacier begins to advance as a vertical front, usually
69 20–50 m high. The maximum advance rate measured during a surge in Iceland was 100 m in 24 hours
70 at the ice front of Brúarjökull outlet glacier (located in the northern part of Vatnajökull ice cap) in
71 1963. The large outlets of Vatnajökull typically advance about 1 km. The advance of the terminus may
72 take several months. Surges alter the geometry of the ice caps, typically thinning the accumulation
73 area by 25–100 m, reducing ice-surface slopes, and increasing glacier surface area and ice thickness at
74 the terminus. Lingering effects of a surge can often be detected in the accumulation area in the form
75 of crevassing and surface lowering several years after the terminus has stopped advancing. Following
76 that, a quiescent phase takes over, building up to a new surge. Major surges, with return intervals of
77 several decades, have occurred in all the large lobate outlets of Vatnajökull.

78

79 In this study, we map crustal deformation using InSAR data, which provide deformation obser-
80 vations with high spatial resolution. SAR acquisitions from May–October, 1993–2002 are used to

81 measure the crustal deformation induced by a surge that occurred in 1993–1995 at the four major
82 outlet glaciers of western Vatnajökull (Fig. 1). We use the finite element method to model the surge-
83 induced crustal deformation and compare it to the InSAR observations. This allows us to estimate the
84 effective Young’s modulus, E , and Poisson’s ratio, ν , of the Icelandic crust/mantle.

85 **2 GLACIAL SURGE HISTORY**

86 The glacial surge we study took place in 1993–1995 at neighbouring outlet glaciers of western Vat-
87 najökull: Síðujökull, Tungnaárjökull, Skaftárjökull and Sylgjujökull (Fig. 1). The first indications of
88 a surge of Síðujökull were the formation of crevasses in 1990 in the accumulation area. In January
89 1994, a ~ 70 m high bulge was observed moving down-glacier, and 4 months later, the surge was over,
90 affecting an area of 500 km^2 and resulting in an advance of the glacier terminus by 1,150 m. On Tung-
91 naárjökull, increased ice velocities were first detected in 1992–1993 and in late 1994 a bulge started
92 to propagate downwards. The surge was finished in mid-1995, moving the terminus forward by about
93 1,200 m. The surface drawdown in the reservoir area extended 30 km up-glacier from the terminus.
94 On both outlets the reservoir area lowered by 10–80 m, and the terminus thickened in excess of 100 m
95 (more details in Björnsson et al. 2003). Skaftárjökull and Sylgjujökull surged in 1994–1995.

96
97 The redistribution of the ice mass during the surges of western Vatnajökull (Fig. 2) was estimated
98 by differencing surface maps of the glaciers from 1993 and 1995. Digital elevation models (DEMs)
99 for 1993 and 1995 were constructed by adjusting four basic maps available prior to and after the surge
100 (from 1980, 1990, 1995 and 1998) with the help of observed spatial surface elevation changes, i.e. a
101 time series of annual in situ GPS surveys at several scattered points over the glaciers in the 1980s and
102 1990s. We assumed that the main topographic forms of the glacier surface, shown in the 1980’s and
103 1990/91 DEMs, remained unchanged until the surges in 1993. Likewise, we assume the maps of 1995
104 and 1998 display the shape of the glacier surface after the surges in 1995. The 1980 DEM was created
105 from digitized elevation contour lines of the DMA series 1:500,000 paper maps (DMA series C761
106 produced by the Defence Mapping Agency Hydrographic/Topographic Center (DMAHTC), Washing-
107 ton DC) constructed from aerial photographs. The point elevation accuracy in this DEM is estimated
108 to be ~ 5 m. The 1990–91 map was produced from precision barometric altimetry profiles about 1 km
109 apart, with point accuracy of 2 m (Björnsson & Pálsson 1991; Björnsson et al. 1992). A DEM of
110 the terminus and lowest part of Tungnaárjökull was extracted from aerial photography survey in late
111 summer 1995, point elevation accuracy ~ 2 m. Finally, a DEM was derived by an airborne EMI-SAR
112 survey in 1998 (Magnússon et al. 2004), with estimated 1 m accuracy. We estimate uncertainty in the
113 regional elevation difference between the DEMs from 1993 and 1995 to be 2–5 m. The volume of

114 ice transferred in the surges, calculated as the difference between the 1993 and 1995 DEMs is esti-
 115 mated at $16 \pm 1 \text{ km}^3$, corresponding to $\sim 15 \text{ Gt}_{we}$ (water equivalent) assuming an average ice density
 116 of 917 kg/m^3 . We assume here that there were no changes in the snow and firn layers on the ice cap
 117 and that the ice density remained constant before, during and after the surge.

118 3 INSAR OBSERVATIONS

119 We used 27 acquisitions from the European Space Agency's ERS-1 and ERS-2 synthetic aperture radar
 120 satellites, descending track 9, captured over the southwestern part of Vatnajökull ice cap between 1993
 121 and 2002 (Fig. 1). We processed the SAR acquisitions in a similar way as Auriac et al. (2013), using
 122 the Repeat Orbit Interferometry PACkage (ROI-PAC) (Rosen et al. 2004) to focus the raw data, and
 123 the Delft Object-oriented Radar Interferometric Software (DORIS) (Kampes & Usai 1999) to form
 124 the interferograms. The small baseline approach from the Standard Method for Persistent Scatterers
 125 (StaMPS) (Hooper 2008) package was used to form interferograms from various pairs of images for
 126 which the differences in perpendicular and temporal baselines are small. From these, we selected 65
 127 highly coherent interferograms (Fig. 3), formed from 24 of the 27 original SAR acquisitions (Table 1).
 128 Finally, we cropped the scene to keep only the region surrounding the outlet glaciers, and resampled
 129 the coherent pixels to a 500 m grid. We also removed the points located on the ice cap and outliers
 130 (noisy points located along the lake and rivers in the west of the scene), leaving 2455 data points in
 131 total.

132
 133 The deformation observed in the interferograms is in the line-of-sight (LOS) direction between
 134 the satellite and the ground, which deviates $\sim 23^\circ$ from vertical, as the radars are side-looking. The
 135 LOS unit vector, in the direction from ground to satellite, is approximately $(-0.35, -0.10, +0.90)$ in
 136 east, north and up components. As the surge-induced crustal deformation is dominated by vertical
 137 movement (see Section 7), and InSAR is most sensitive to the vertical direction, the signal observed
 138 in the interferograms relates mostly to a vertical change corresponding to a subsidence. It is generally
 139 possible to separate the horizontal east-west deformation component from the vertical one by using
 140 SAR images acquired in both ascending and descending mode. However, due to the lack SAR data
 141 acquired in ascending configuration over the study area, this could not be achieved here. Only the
 142 crustal deformation from the surge at Síðujökull, Skaftárjökull and the southern part of Tungnaárjökull
 143 outlet glaciers is observed, as the InSAR data we use do not cover the margins of Sylgjujökull and the
 144 northern part of Tungnaárjökull outlet glaciers.

3.1 Surge signal and time series of interferograms

Interferograms spanning the year 1994 reveal a clear LOS lengthening signal associated with the glacial surge. Fig. 4 shows an example of such an interferogram, both wrapped and unwrapped, with maximum subsidence of $\sim 70\text{--}80$ mm observed at the ice margin, relative to a reference area located at a distance of ~ 15 km from the ice edge, in the bottom right corner of the InSAR scene. The reference area was chosen far away from the ice cap not to be influenced by the surge-induced crustal deformation. The surge signal decays rapidly away from the ice cap, with only ~ 15 mm subsidence observed at ~ 6 km from the ice edge. We inverted the 65 small baseline interferograms using least-squares to give a single-master time series of 23 unwrapped interferograms using StaMPS (Fig. 5). It shows cumulative displacement through time with respect to the first image on 26 June 1993, relative to the reference area. Two signals are observed: (i) the LOS lengthening signal related to the surge, appearing in the first image after the surge (19 June 1995) and all subsequent images, and (ii) the LOS shortening deformation due to glacial isostatic adjustment (GIA), as described by Auriac et al. (2013) (i.e. the ground deformation occurring around Vatnajökull due to the general retreat of the ice cap over the past 120 years and seasonal changes in snow and ice cover), most clearly visible over the eastern half of the scene as time increases.

Contrary to the observations made by Sauber & Molnia (2004) for the surge of Bering Glacier, Alaska, in 1993–1995, the deformation signal associated with the drawdown of the reservoir area on Vatnajökull ice cap could not be observed by GPS due to a lack of GPS measurements on the nunataks at the time of the surge. The deformation of these nunataks could not be retrieved by InSAR data as it is nearly impossible to reliably unwrap between the stable points outside the ice cap and the clusters of isolated stable points on nunataks.

3.2 Disentangling surge and GIA signals

The GIA and surge signals are both present in the 1993–2002 time series of interferograms. In order to model the surge separately, we first estimated the contributions of both signals for each pixel. Each signal has its own time frame, the GIA spanning the whole time series and the surge being a singular event spanned completely by a single pair of consecutive images, assuming the response of the surge is purely elastic (see Section 7). For a given pixel, the displacement as a function of time may therefore be modelled as a constant velocity (GIA) plus a step function (surge). Separating the two processes is achievable through least-squares inversion of the single-master time series data. The equation to solve

177 for each pixel is

$$178 \quad \Delta\phi_{\mathbf{i}} = \mathbf{A}\mathbf{x}_{\mathbf{i}} \quad (1)$$

179 where $\Delta\phi_{\mathbf{i}}$ is a vector with the phase value of the i^{th} pixel in each interferogram, \mathbf{A} is a design matrix
 180 and $\mathbf{x}_{\mathbf{i}}$ is a vector of unknown parameters we invert for. In our case, the vector of unknowns includes,
 181 for each pixel, two parameters of interest: (i) the estimation of the ongoing GIA signal through time,
 182 $v_{GIA,i}$, which is assumed constant before and after the surge (see Section 7), and (ii) the estimate of
 183 the step displacement caused by the surge, $d_{surge,i}$. The vector also includes two nuisance parameters
 184 that need to be evaluated for each pixel: the estimate of atmospheric component from the master
 185 acquisition, $a_{m,i}$, and the estimate of the DEM error, which is related to the perpendicular baseline,
 186 $c_{topo,i}$. For the i^{th} pixel, equation (1) can be rewritten as

$$187 \quad \begin{bmatrix} \Delta\phi_{i,1} \\ \vdots \\ \Delta\phi_{i,k} \\ \Delta\phi_{i,k+1} \\ \vdots \\ \Delta\phi_{i,n} \end{bmatrix} = \begin{bmatrix} \Delta t_1 & 1 & 0 & B_{perp_1} \\ \vdots & \vdots & \vdots & \vdots \\ \Delta t_k & \vdots & 0 & B_{perp_k} \\ \Delta t_{k+1} & \vdots & 1 & B_{perp_{k+1}} \\ \vdots & \vdots & \vdots & \vdots \\ \Delta t_n & 1 & 1 & B_{perp_n} \end{bmatrix} \begin{bmatrix} v_{GIA,i} \\ a_{m,i} \\ d_{surge,i} \\ c_{topo,i} \end{bmatrix} \quad (2)$$

188 where k is the index of the last interferogram before the surge, n is the total number of interferograms
 189 in the single-master time series, Δt is the time between the master and slave acquisitions, and B_{perp}
 190 the perpendicular baseline between the two acquisitions. We solved these equations for all the pixels
 191 and derived a vector with an estimate for the GIA and surge-induced crustal displacements, \mathbf{v}_{GIA} and
 192 \mathbf{d}_{surge} , respectively.

193
 194 We solved for the vector of unknown parameters using least-squares weighted by the inverse
 195 variance-covariance matrix of the data. To estimate the variance of each interferogram, we deramped
 196 a 34×19 km area at the southwest corner of the full InSAR scene, considered far from any signal,
 197 and calculated the variance of the phase of the selected pixels in this area for each interferogram of
 198 the single-master time series. We assumed the variance of this background signal to be representative
 199 of the complete scene. As the residual phase of the pixels in the interferogram is assumed to be uncor-
 200 related in time, off-diagonal elements of the variance-covariance matrix were set to zero.

201
 202 Fig. 6 shows the results of the least-squares inversion for the GIA and surge estimate, relative to a
 203 reference area at (-17.67°E, 63.97°N). The GIA signal, \mathbf{v}_{GIA} , has a maximum LOS shortening rate of
 204 up to 10–12 mm/yr at the ice margin east of Síðujökull outlet glacier, similar to the observations from

205 Auriac et al. (2013), while a maximum LOS lengthening of 70–75 mm is estimated for the surge step
 206 function, \mathbf{d}_{surge} .

207 **4 MODELLING**

208 **4.1 General set-up**

209 We modelled the elastic ground deformation caused by the surge with the finite element method, us-
 210 ing the Abaqus commercial software (ABAQUS 2009). This method also allowed us to investigate
 211 the possibility of a viscoelastic response of the Earth to the glacial surge, and thus test the assump-
 212 tions applied in the least-squares inversion (see Sections 3.2 and 7). We built the models following the
 213 same approach as Auriac et al. (2013), using a volume of $2000 \times 2000 \times 1000$ km in the east-west,
 214 north-south and depth dimensions, respectively. The same assumptions as mentioned by Auriac et al.
 215 (2013) stand, i.e. flat Earth, isotropic material, horizontal layering, and no plate spreading. The do-
 216 main is large enough so that the fixed boundary conditions at the vertical and lower boundaries do not
 217 significantly affect the modelled displacements. Even though our model configuration approximates
 218 a half-space, we prefer the term layer to refer to each finite volume with similar elastic properties.
 219 A model where the entire volume has uniform properties will thus be called a one-layer model, and
 220 a model with two different uniform properties within the total volume will be called a two-layer model.

222 The ice model is based on the ice mass changes described in Section 2 and Fig. 2. In order to
 223 account for the large variations over short distances in the surge model, we modified the original mesh
 224 (of the Earth model) used by Auriac et al. (2013) at the surface such that, in the load region, nodes are
 225 located every ~ 250 m. The mesh then becomes coarser with distance. More than 210,000 nodes are
 226 present at the surface. To implement the ice model in Abaqus, we searched for the surge model point
 227 closest to the centre of the mesh element’s face at the surface, and assigned it the corresponding value,
 228 defined as a pressure load.

230 Two series of models were created. We first created one-layer elastic models with Poisson’s ratio,
 231 ν , ranging from 0.025–0.500 with steps of 0.025, and Young’s modulus, E , of 20 GPa. In a purely
 232 elastic model, according to Hooke’s law, the displacement X induced by a surface load F is inversely
 233 proportional to E , and can be expressed as

$$234 \quad X = F/E \quad (3)$$

235 Since the same load (surge model) was applied in all our models, we can consider F as constant.

Using equation (3) and the predicted displacement for one value of E (20 GPa) from our modelling, we can calculate the surface deformation for any value of E by scaling. In our case, we calculated the deformation to E ranging from 5–100 GPa, for each value of ν . To verify the numerical modelling, we ran a few extra models with $\nu=0.25$ and $E=60$ GPa and 90 GPa, and compared the displacements to those calculated by scaling. Fig. 7 shows the results of this comparison for one randomly chosen node of the mesh, indicating full consistency. In addition, we calculated analytical solutions for the surge displacement using the half-space Green’s functions, by discretising the surge into point loads, applied to the centre of each element from the finite element mesh. This solution is based on the same ice model as the finite element models. The displacements (horizontal, U_r , and vertical, U_z) for a point surface load are

$$U_r(r) = -\frac{g}{2\pi} \frac{(1+\nu)(1-2\nu)}{E} \frac{1}{r} \quad (4)$$

and

$$U_z(r) = \frac{g}{\pi} \frac{1-\nu^2}{E} \frac{1}{r} \quad (5)$$

where r is the distance from the load, g is the acceleration of gravity, ν the Poisson’s ratio, and E the Young’s modulus (e.g. Pinel et al. 2007). The total displacement at each of the mesh points is estimated by considering the total ice mass and adding up the displacement induced by each of the point loads, using $\nu=0.25$ and $E=40$ GPa. Model displacements for other values of E were found by scaling. The predicted displacement with this method was compared to those obtained from the finite element models (Fig. 7).

The second series of models corresponds to two-layer elastic models with a 1-km-thick upper layer. The mesh and ice model are the same as used for the one-layer elastic models. We used different values for the Poisson’s ratio of each layer (ν_1 for the top layer and ν_2 underneath), using the best-fit value provided by the one-layer elastic models and more commonly used values for crustal rocks. The Young’s moduli (E_1 for the top layer and E_2 underneath) were varied from 10–18 GPa with steps of 2 GPa for E_1 , and from 55–90 GPa with steps of 5 GPa for E_2 .

4.2 Estimating the Young’s modulus and Poisson’s ratio

We solved for the best-fitting values of E and ν by comparing the deformation field calculated from the finite element models to the surge-induced LOS change estimated from the InSAR data. This was achieved using a statistical method based on Bayes’ rule, similar to that used by Hooper et al. (2013) and Auriac et al. (2013). The approach used here though is simpler, because no GPS data are used in the comparison between the observed surge-induced and modelled deformation fields. We calculated

268 the weighted residual sum of squares, WRSS, as

$$269 \quad WRSS = (\mathbf{d} - \mathbf{G}(\mathbf{m}))^T \mathbf{Q}^{-1} (\mathbf{d} - \mathbf{G}(\mathbf{m})) \quad (6)$$

270 where \mathbf{d} is the vector of observations, \mathbf{m} is the vector of model parameters, $\mathbf{G}(\cdot)$ is the model function
 271 that maps the model parameters to the observations, and \mathbf{Q} is the variance-covariance matrix of the
 272 InSAR observations, which are highly correlated in space.

273
 274 The variance-covariance matrix accounts for residual atmospheric, decorrelation and unwrapping
 275 errors. It was estimated by a bootstrapping approach based on the one described by Auriac et al. (2013)
 276 but accounting for the following improvements. We ensured here that interferograms from both before
 277 and after the surge were sampled during each realisation of the bootstrap. To ensure the estimate of
 278 the covariance includes the background noise only, we removed our estimate of d_{surge} (calculated
 279 using weighted least-squares) from each estimate of the surge obtained during bootstrapping. For
 280 500,000 random pairs of points, we then calculated the semi-variogram as the variance of the differ-
 281 ence of value of the residual d_{surge} between the two points in each pair. The semi-variogram was then
 282 binned according to the distance between the points and fitted with an exponential variogram func-
 283 tion, from which the covariance function was calculated. The diagonal elements were set to a constant
 284 ($\sim 20.7 \text{ mm}^2$), corresponding to the zero lag covariance which includes a nugget value (estimated as
 285 the semi-variogram value at zero lag).

286
 287 Residuals, $\mathbf{d} - \mathbf{G}(\mathbf{m})$, were calculated for each discrete value of the model parameters and inter-
 288 polated in between to derive the posterior probability distribution of the model parameters. For each
 289 set of residuals, we estimated and removed a plane which accounts for orbital effects (residual orbit
 290 signals resembling a bilinear ramp) and for the systematic offset between the relative LOS InSAR
 291 observations and the absolute model displacements.

292
 293 From equation (6) and according to Bayes' rule, the posterior probability can be estimated using

$$294 \quad p(\mathbf{m}|\mathbf{d}) = K \frac{\sigma^{-n}}{\sqrt{(2\pi)^n |\mathbf{Q}|}} \exp \left[-\frac{WRSS}{2\sigma^2} \right] \quad (7)$$

295 where K is a constant, σ is a scaling factor of the variance-covariance matrix to account for model
 296 errors, and n is the number of pixels. We set K in such a way that the total probability equals unity.
 297 We then determined the uncertainty region of our parameters as the area containing 95% of the total
 298 probability. The dimensionless scaling factor σ is constant for all combinations of model parameters
 299 within one series of models (one-layer elastic or two-layer elastic models), and was independently

300 calculated from the best WRSS estimate for each model series such that $WRSS/\sigma^2 = n$. It varied
 301 from 2.2 to 2.7 depending on the model used.

302 5 ALTERNATIVE APPROACH USING GIPHT

303 The General Inversion for Phase Technique, GIPhT, (Feigl & Thurber 2009; Ali & Feigl 2012) has
 304 also been applied to the surge data from western Vatnajökull ice cap. The approach used two wrapped
 305 interferograms created from four SAR acquisitions from the ERS-1 and 2 satellites, track 9. They span
 306 similarly long time intervals over 1993–1995 and 1998–2000. Assuming that the GIA signal is con-
 307 stant with time (see Section 7), we subtracted the later interferogram from the first one to remove the
 308 GIA deformation, providing an estimate of the surge displacement. The observed subsidence is more
 309 than one fringe (more than 28 mm of range change) in most areas and nearly two fringes close to the
 310 eastern edge of Síðujökull outlet glacier (~ 56 mm of range change) within ~ 10 km from the ice edge.
 311 This is consistent with what is observed in Fig. 6b.

312
 313 We modelled this estimate of surge deformation with the Green’s function approach (Eqs. 4 and
 314 5). For each pixel, the calculation convolves the map of the inferred mass redistribution of ice from
 315 the surge (Fig. 2) with the Green’s function.

316
 317 Consequently, we can estimate the Young’s modulus E and Poisson’s ratio ν of the rocks around
 318 the glacier by minimizing the residual between the observed and modelled values of the InSAR phase.
 319 To solve this inverse problem, we applied the GIPhT method as described by Feigl & Thurber (2009)
 320 and Ali & Feigl (2012).

321 6 RESULTS

322 Results of the comparison between the surge displacement field estimated from the least-squares in-
 323 version (\mathbf{d}_{surge}) and our finite element models (both one-layer and two-layer elastic) are presented in
 324 Figs. 8 to 11 and Table 2. The deformation patterns from the InSAR observations and the models are
 325 very similar. The magnitude of crustal deformation around the surging outlet glaciers, as well as the
 326 extent and decay of the signal away from the ice margin are well reproduced by the models, indicating
 327 high quality of the ice model and applicability of the Earth models.

328
 329 Comparison between the one-layer elastic models and the surge-induced crustal LOS displace-
 330 ment (\mathbf{d}_{surge}) are displayed in the top row of Fig. 8, showing \mathbf{d}_{surge} from which the ramp and offset

331 estimated during the Bayesian approach have been removed, the best-fit model, and the residuals be-
 332 tween the two. The residual plot shows that, although the best one-layer elastic model manages to
 333 predict quite well the pattern of deformation, it does not accurately reproduce the deformation within
 334 5 km of the ice edge, where residuals can reach 26–28 mm. The model cannot simultaneously repro-
 335 duce both the gradient of deformation in the near-field (1–2 km from the edge) and far-field, which
 336 requires a higher value. This compromise model results in the relatively low estimate of the Young’s
 337 modulus, $E=46.4^{+3.3}_{-3.2}$ GPa, shown in the probability estimate in Fig. 9 and in Table 2. The maximum
 338 posterior probability estimate for the Poisson’s ratio is 0.17, but the probability distribution function
 339 (Fig. 9) shows that this parameter is barely constrained by these data, and the 95% confidence interval
 340 spans 0–0.27. The GIPhT method, solving for the average value of the free parameters over a half-
 341 space, finds a Young’s modulus of $E=64.0\pm 6$ GPa and a Poisson’s ratio of $\nu=0.36\pm 0.06$. It seems
 342 this methods finds a good fit to the far-field deformation, explaining the difference with the Bayesian
 343 approach. From the results of our one-layer elastic models, we conclude that the crustal deformation
 344 pattern from the glacial surge cannot be adequately fit with a simple one-layer model.

346 In order to fit both the near- and the far-field displacements, a more complex model is needed.
 347 For this purpose, we ran the two-layer elastic models, solving for the best-fit Young’s modulus of
 348 each layer (E_1 for the top layer and E_2 underneath). The 1-km-thick top layer, with a relatively low
 349 Young’s modulus, is used to account for the large subsidence observed in the near-field region, while
 350 the underlying layer, with an overall higher Young’s modulus, is needed to accommodate the far-field
 351 deformation. The 1 km thickness of the top layer was chosen according to the fact that the near-field
 352 gradient of deformation, outlined with the one-layer elastic models, is only observed with 1–2 km
 353 from the ice edge. We used three different combinations of the Poisson’s ratios (ν_1 for the top layer
 354 and ν_2 underneath): (i) both ν_1 and ν_2 are set to 0.25, as it is a commonly assumed value for the Pois-
 355 son’s ratio of crustal rocks; (ii) we use $\nu_1=0.17$, as predicted by the one-layer models, and $\nu_2=0.25$;
 356 and (iii) both ν_1 and ν_2 are set to 0.17. Results from the Bayesian approach are presented in Fig. 10
 357 and best-fit estimates of E_1 and E_2 are displayed in Table 2 for all three settings. The figure shows
 358 that E_1 is overall better constrained than E_2 . The probability distributions for the second ($\nu_1=0.17$ and
 359 $\nu_2=0.25$) and third ($\nu_1=\nu_2=0.17$) combinations are quite similar as they have a large part of their 95%
 360 confidence regions in common. The residual plots obtained with each solution are presented in Fig.
 361 8. All three combinations provide a better fit to the near- and far-field deformation than the one-layer
 362 elastic models, but combinations two and three clearly provide the best-fit models. However, since
 363 a Poisson’s ratio of 0.17 is not realistic for the deeper part of the crust/mantle (see Section 7), our
 364 preferred model corresponds to the two-layer elastic model using $\nu_1=0.17$ and $\nu_2=0.25$. Its good fit is

365 also confirmed by the displacement along the two profiles, as discussed below. Our preferred model
 366 estimates the Young's moduli to be $E_1=13.9^{+1.4}_{-1.0}$ GPa and $E_2=73.9^{+8.0}_{-6.6}$ GPa. Residuals for this model
 367 lie mostly between -2 and 6 mm in absolute value. East of Síðujökull outlet glacier, some larger resid-
 368 uals occur. A plausible cause for those in the near-field region would be local inaccuracies in the ice
 369 model. For the far-field area, the residuals (ranging from -12 to -14 mm) are likely related to atmo-
 370 spheric signal adding some noise to the InSAR observations in this region.

371

372 Figure 11 shows the deformation along two profiles (shown in Fig. 8) going from the ice edge
 373 at Síðujökull outlet glacier towards the edge of the InSAR scene to the south (profile A) and to the
 374 southwest (profile B). The top panels compare the surge-induced displacement (\mathbf{d}_{surge}) to the four
 375 best-fit model predictions (one one-layer elastic models and three two-layer elastic models) to which
 376 we added the ramp and offset estimated during the Bayesian procedure. The lower panels of the figure
 377 give the residual displacement along each of the profiles for the four best-fit models. This figure shows
 378 that the best prediction of the surge-induced displacement in the near- and far-fields comes from our
 379 preferred model with $\nu_1=0.17$ and $\nu_2=0.25$.

380 7 DISCUSSION

381 The time series of interferograms show in detail the crustal deformation at the southwestern edge
 382 of Vatnajökull between 1993 and 2002. The signals observed are due to two different processes: the
 383 glacial surge that occurred in 1994 at Síðujökull, Skaftárjökull, Tungnaárjökull and Sylgjujökull outlet
 384 glaciers, causing LOS lengthening, and the GIA driven by the general retreat of the ice cap over the
 385 past 120 years which induces broad LOS shortening. Sources of uncertainty in the InSAR observa-
 386 tions include the effects of atmospheric artefacts, unwrapping errors and orbital effects. The first two
 387 sources are greatly reduced during the StaMPS analysis and least-squares inversion, with any remain-
 388 ing error considered in the Bayesian approach. The latter uncertainty related to orbits is reduced by
 389 estimating and removing a bilinear ramp from the residuals obtained after the comparison between
 390 InSAR observations and model results.

391

392 Using least squares inversion, we are able to disentangle the signals induced by the surge and
 393 the GIA. The method however relies on a number of assumptions. The first assumption is that the
 394 34×19 km area we use at the southwestern corner of the full InSAR scene to estimate the background
 395 noise of each interferogram is representative of the full scene. This assumption is reasonable because
 396 the area used represents a good portion of the full scene and should sample enough points to ob-
 397 tain a reliable variance of the background noise of the interferograms. Another assumption is that the

398 surge-induced crustal deformation is almost purely elastic. We have validated this assumption through
399 model tests, by comparing outputs from a model with an elastic layer underlain by a viscoelastic layer
400 to those from a one-layer elastic model. The models have identical elastic parameters ($E=60$ GPa and
401 $\nu=0.25$). The viscoelastic test model we used consists of a 20 km thick elastic layer and a viscosity
402 beneath this of 9.3×10^{18} Pa s, according to the best-fit model for the InSAR observations of the ERS
403 track 9 from Auriac et al. (2013). Outputs from the viscoelastic model were taken at different times
404 to evaluate both the short- and long-term responses from the surge. They were then compared to the
405 purely elastic response of the surge. After 6 months, the viscoelastic effect represents less than 1% of
406 the elastic component. On a short-term basis, the results thus show that the influence of the viscoelas-
407 tic response from the surge is negligible. The crustal response to the surge can therefore be modelled
408 as a step function in time.

409
410 The surge-induced crustal deformation signal appears clearly in the LOS deformation map ob-
411 tained from the least-squares inversion (Fig. 6), reaching a maximum of 75 mm LOS lengthening at the
412 margins of Síðujökull outlet glacier. Our finite element modelling gives three-dimensional displace-
413 ments and shows that horizontal displacements are nowhere more than 10% of the vertical component,
414 with a maximum near the ice edge. The model LOS change is formed by multiplying the displacement
415 at each pixel with the LOS unit vector. Therefore, the LOS deformation map mostly shows vertical
416 motion of the ground. The observed signal from the surge decays rapidly from the ice cap. Each of the
417 outlet glaciers mapped by our InSAR scene has a specific surge deformation signature, the displace-
418 ments at Síðujökull and Skaftárjökull outlet glaciers being up to 50 mm greater than those observed
419 on the southern part of Tungnaárjökull. This result is consistent with the ice model (Fig. 2), which
420 predicts less ice being transported to the terminus area of Tungnaárjökull than for Síðujökull and
421 Skaftárjökull. Moreover, the region where ice has been added extends over a larger area at Síðujökull
422 than Tungnaárjökull, increasing the extent of the surge-induced crustal deformation at the margins of
423 Síðujökull compared to Tungnaárjökull.

424
425 The GIA uplift rate over the 1993–2002 period estimated from the least-squares inversion reaches
426 12 mm/yr at the edge of the ice cap east of Síðujökull outlet glacier, relative to the reference area.
427 This result is consistent with those of Auriac et al. (2013) from a 1995–2002 time series. The GIA
428 uplift rate we estimate is assumed to be insensitive to the surge. Two effects linked to the surge could,
429 however, influence the GIA estimate. The first one corresponds to an eventual viscoelastic response of
430 the Earth following the surge. This possibility has been investigated as described above. We found that
431 the viscoelastic response induced by the surge reaches a maximum of 0.9 mm/yr (decreasing away

432 from the ice cap in a similar pattern as the elastic response from the surge), which corresponds to
 433 7.5% of the velocity estimated for the GIA uplift rate in this area during the inversion. Neglecting this
 434 effect causes a small underestimate of the uplift velocities induced by the GIA process around Síðu-
 435 jökull, Skaftárjökull, and Tungnaárjökull after the surge. Second, the ice model, with a step advance
 436 of ice during the surge, is an oversimplification. Consequently, the GIA uplift rate may be affected by
 437 increased ice melting after the surge, as observed after the surge of Bering Glacier, Alaska, in 1993–
 438 1995 (Sauber & Molnia 2004). The average summer melting on the highly crevassed ablation areas
 439 of the surging outlet glaciers of Vatnajökull ice cap has been observed to increase by $\sim 30\%$ over the
 440 2–3 years after the surge (Björnsson et al. 2003). The resulting deformation, because of the relatively
 441 short duration of the increased melting, will be mostly reflected in the elastic response of the crust to
 442 the unloading, and therefore has only a limited effect on the long-term GIA uplift. Moreover, since
 443 1995, the mass balance of glaciers in Iceland has been on average negative by $\sim 1 \text{ m}_{we}/\text{yr}$, after having
 444 been close to zero in the 1980s to the mid-1990s (Björnsson et al. 1998, 2002, 2013). The effect of
 445 this increase in melt rate would counteract the small underestimation of the GIA velocities caused by
 446 the viscoelastic response from the surge.

447
 448 Some inaccuracies in the estimate of Poisson’s ratio and Young’s modulus from our Bayesian ap-
 449 proach may be caused by assumptions made in the modelling and the statistical method itself. Since
 450 we built up our models according to Auriac et al. (2013), the same assumptions stand, i.e. flat Earth,
 451 isotropic material, horizontal layering, and no plate spreading. The flat Earth is a reasonable assump-
 452 tion regarding the relatively small size of the surging outlet glaciers. The other assumptions are a
 453 simplification of the real Earth. The fact that we assume a uniform value for the Young’s modulus,
 454 E , and Poisson’s ratio, ν , in one or two layers, means that the estimates we obtain from the Bayesian
 455 approach correspond to the average of these parameters for the Icelandic crust/upper mantle. For the
 456 two-layer elastic models, we assume a 1-km-thick top layer with a lower value of E than in the under-
 457 lying layer. This also represents a simplification of the real Earth, which should be better represented
 458 by a gradual increase in E with depth, as indicated by seismic studies (Allen et al. 2002). However,
 459 results from the comparison between model and observations show that our two-layer models suffice
 460 to fit the surge-induced crustal deformation in both the near- and far-field areas. Our results also de-
 461 pend on the assumption made during the Bayesian approach stating that the measurement errors have
 462 a multivariate Gaussian distribution. The 95% confidence area obtained with the Bayesian procedure
 463 should be interpreted as a formal uncertainty, i.e. a lower estimate of the true uncertainties, as it does
 464 not consider eventual model errors.

466 Uncertainties in the value of E and ν also stem from the ice model used in this study, which de-
467 pends largely on the assumption that the large-scale topographic features on the ice cap did not change
468 shape between the time of acquisitions used to create the DEMs. Comparison of recent surface DEMs
469 of western Vatnajökull (1998 (EMISAR), 2003 and 2010 from SPOT5 HRG and HRS images and
470 LiDAR survey 2010-2012) shows that this assumption is valid for almost all changes in elevation over
471 length scales of 10 km whereas features of less than 1 km in radius are almost randomly scattered.
472 Our ice model may also be influenced by the fact that we assume only a change in ice thickness with
473 no variations in the snow or firn layers or in ice density. When doing DEM differencing over an ice
474 cap, it is common to assume that the snow and firn layers are the same at both times of DEM acquisi-
475 tions. This is a fair assumption as the snow layer gets renewed by new snow every year and the lower
476 boundaries of the snow and firn layers are constantly transformed into firn and ice, respectively.

477
478 As an alternative way to validate our results, we ran the General Inversion for Phase Technique,
479 GIPhT, developed by Feigl & Thurber (2009) and extended by Ali & Feigl (2012), on the surge event
480 that occurred on western Vatnajökull outlet glaciers. We extracted from this method an estimate of the
481 Young's modulus, $E=64.0\pm 6$ GPa, and Poisson's ratio, $\nu=0.36\pm 0.06$. Comparison between the best-fit
482 E and ν estimated from our one-layer models, our two-layer elastic models, GIPhT approach, and the
483 values found in the literature are summarized in Tables 2 and 3.

484
485 The value of Young's modulus estimated from our Bayesian approach with the one-layer models
486 is different to the one inferred from the GIPhT method, likely because each approach tries to fit a dif-
487 ferent part of the surge-induced signal (see Section 6). The estimates of the static value of the Young's
488 modulus (E_s) we obtain with our two-layer elastic models are however in good agreement with what
489 was inferred by Pinel et al. (2007) and Grapenthin et al. (2006), considering that the values estimated
490 are all averages of the true values over the modelled crustal thickness, and that the Young's modulus is
491 increasing with depth, as demonstrated by seismic studies (Allen et al. 2002) and experimental results
492 (Heap et al. 2011; Asef & Najibi 2013). The values of the Earth parameters estimated by surface load
493 studies are restricted to the volume of Earth significantly influenced by the load variation. To a first
494 approximation, the effects of a surface load depend mostly on Earth properties at depth shallower than
495 the lateral extent of the surface load. It follows that a smaller extent load variation will sample the
496 Young's modulus at shallower levels, which can partly explain the small differences between various
497 studies. Comparison with Young's moduli values derived in Iceland from seismic studies (Pálmason
498 1971; Gudmundsson 1988; Allen et al. 2002; Hooper et al. 2011) reveals that the dynamic Young's
499 moduli appear larger than the static values, with a smaller difference at larger depth, as expected from

500 experimental studies (e.g., Jizba 1991). This effect has also been observed in other places such as in
 501 Hawaii where, at shallow depth (~ 2.7 km), E_s was estimated to be five times smaller than E_d (Hooper
 502 et al. 2002). Values estimated in Iceland are close to those found at Mount Etna and Hawaii (see Tables
 503 2 and 3). They are, however, much larger than the small value found by Beauducel et al. (2000) for
 504 a local study at Merapi volcano. This can be explained by the local estimation performed by these
 505 authors by running a model for a very shallow depth.

506
 507 The estimate of the Poisson's ratio, $\nu=0.17^{+0.10}_{-0.17}$, inferred from our one-layer models is lower than
 508 the $\nu=0.36\pm 0.06$ obtained with the GIPhT approach. This can be explained by the differences in each
 509 approach: the different ways to obtain the surge-induced LOS displacements by removing the GIA
 510 signal, and the different ways in dealing with the covariance between pixels. The Bayesian approach
 511 however shows that the Poisson's ratio parameter is not well constrained by the data, as shown by the
 512 95% confidence region. The low value of $\nu=0.17^{+0.10}_{-0.17}$ can be partly explained by the fact that Poisson's
 513 ratios are highly influenced by the presence of fluids in pores, cracks and fissures in the crust, varying
 514 from $\nu=0.27$ in drained conditions to $\nu=0.31$ for undrained conditions, as estimated by Jónsson et al.
 515 (2003). We argue that the surge takes place over a long enough time interval to obtain a drained value
 516 of the Poisson's ratio from our results, in which case a Poisson's ratio of $\nu=0.27$ falls at the edge of
 517 our uncertainties. Moreover, the choice of our preferred model using a Poisson's ratio of $\nu=0.17$ for
 518 the top layer and $\nu=0.25$ underneath has been motivated by the fact that the uppermost kilometre of
 519 the Icelandic crust is most likely highly fractured. Although a value of $\nu=0.17$ might be too low for
 520 the Poisson's ratio of the top layer, we argue that it should be lower than the Poisson's ratio at larger
 521 depth. The residual plots demonstrate that such a model manages to resolve most of the surge-induced
 522 signal in both near- and far-field areas (Fig. 8).

523 8 CONCLUSIONS

524 InSAR has proved to be a powerful tool for mapping the crustal deformation associated with glacial
 525 surges. The crustal subsidence signal induced by the studied surge, reaching up to 75 mm in LOS at
 526 the edge of Síðujökull outlet glacier, is well resolved right up to the ice margin. The high spatial reso-
 527 lution provided by the InSAR observations also shows the full extent of the surge signal, which decays
 528 fast over a ~ 10 km distance away from the ice cap. The pattern is well reproduced by the finite ele-
 529 ment modelling. The results show that the surge-induced crustal subsidence signal is composed of two
 530 zones: the far-field area, and the near-field area (~ 0.5 – 1 km wide band at the ice margin) which expe-
 531 riences higher deformation. Results from the finite element modelling demonstrate that the one-layer
 532 elastic models cannot fully explain both the near- and far-field deformation. The Bayesian approach

533 used to evaluate these models shows that the Poisson's ratio is poorly constrained, with $\nu < 0.27$. Our
534 preferred model come from the two-layer elastic models, where we use a Poisson's ratio of $\nu_1 = 0.17$ for
535 the upper layer and a Poisson's ratio of $\nu_2 = 0.25$ for the lower layer. As discussed above, these values
536 would indicate drained conditions and a highly fractured top part of the crust around Vatnajökull ice
537 cap. Inferring for the Young's modulus of each layer, we find best-fit values of $E_1 = 12.9\text{--}15.3$ GPa and
538 $E_2 = 67.3\text{--}81.9$ GPa for the upper and lower layers, respectively (95% confidence intervals). Residuals
539 are small and demonstrate that the models can accommodate for both the near- and far-field deforma-
540 tion. Our results are consistent with other studies, given that the depth at which it is possible to resolve
541 for the Earth parameters is dependent on the spatial extent of the load at the surface.

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638

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640

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643

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 647 Eastern Volcanic Zone (EVZ) is displayed in blue. Main ice caps names are indicated in red (D.: Drangajökull,
 648 S.: Snæfelsjökull, L.: Langjökull, M.: Mýrdalsjökull, H.: Hofsjökull, and V.: Vatnajökull). The color boxes show
 649 the area spanned by our InSAR data: red for the full scene and blue for the cropped one. The black box gives
 650 the area shown in (b). (b) Zoom in the southwestern region of Vatnajökull, with the names of the four surging
 651 outlet glaciers studied here (Sy.: Sylgjujökull, Tu.: Tungnaárjökull, Sk.: Skaftárjökull and Sí.: Síðujökull) and
 652 the cropped InSAR scene outlines (blue box).

653

654 **Fig. 2** Surface elevation change at Sylgjujökull (Sy.), Tungnaárjökull (Tu.), Skaftárjökull (Sk.) and Síðu-
 655 jökull (Sí.) outlet glaciers between 1993 and 1995. Negative values indicate an ice loss while positive values
 656 indicate a gain in ice.

657

658 **Fig. 3** Connections (black lines) between individual InSAR acquisitions (red dots) forming the 65 highly
 659 coherent small-baseline interferograms used in the study. The y-axis displays the perpendicular baseline be-
 660 tween each image and an arbitrary master image on 17 september 1996.

661

662 **Fig. 4** Interferograms spanning 31 July 1993 to 19 June 1995, showing the surge at Tungnaárjökull (Tu.),
 663 Skaftárjökull (Sk.) and Síðujökull (Sí.) outlet glaciers. The black and grey arrows show the azimuth of the satel-
 664 lite and the look direction, respectively. (a) Wrapped interferogram showing the deformation in fringes between
 665 $\pm\pi$. One full fringe (2π) equals 28.3 mm deformation. (b) Unwrapped interferogram. The black star designates
 666 the reference area and negative values indicate LOS lengthening.

667

668 **Fig. 5** Single-master time series created from the 65 small baseline interferograms, spanning 1993 to 2002.
 669 The deformation shown is in LOS (negative values for LOS lengthening), relative to the reference area indicated

670 by the black star. Each panel shows the cumulative change from the first interferogram on 26 June 1993, where
 671 Tu., Sk. and Sí. indicate Tungnaárjökull, Skaftárjökull and Síðujökull, respectively. The color scale has been
 672 modified such that points from -80 mm to -120 mm appear in the same color, to enhance the viewing of the
 673 surge signal.

674

675 **Fig. 6** Inferred deformation signals from the linear inversion ran on the single-master time series: (a) GIA
 676 signal estimated as a continuous velocity, in mm/yr, (b) surge displacement estimated as a step function, in mm.
 677 Both results are shown in LOS and with respect to the reference area, where negative values stand for LOS
 678 lengthening (note the difference in color scaling). The black and grey arrows show the azimuth of the satellite
 679 and the look direction, respectively. Tu., Sk. and Sí. indicate Tungnaárjökull, Skaftárjökull and Síðujökull outlet
 680 glaciers, respectively.

681

682 **Fig. 7** Vertical deformation observed at a randomly chosen mesh node as a function of Young's modulus.
 683 The red circles show results from the finite element models run with $E=20, 60$ and 90 GPa. The blue line gives
 684 the vertical deformation calculated with the finite element model result using $E=20$ GPa and scaling it for the
 685 different values of E , according to Hooke's law. The dashed green line, superimposed on the blue one, represents
 686 the deformation calculated with the Green's function approach using $E=40$ GPa, and scaled to other values of E
 687 using Hooke's law.

688

689 **Fig. 8** Top row: (a) Referenced LOS surge displacement estimated from the InSAR data (output from the
 690 least-squares inversion minus the ramp and offset estimated from the Bayesian approach), (b) best-fit one-layer
 691 elastic model ($E=46.4$ GPa and $\nu=0.17$) converted to LOS, and (c) residual between (a) and (b), respectively.
 692 Rows 2–4 show similar set of panels for the other models. (d), (e) and (f) Same as above but with the two-layer
 693 elastic best-fit model with $\nu_1=\nu_2=0.25$, $E_1=12.9$ GPa, and $E_2=70.5$ GPa. (g) (h) and (i) Same as above with
 694 $\nu_1=0.17$, $E_1=12.9$ GPa, $\nu_2=0.25$ and $E_2=73.9$ GPa. (j), (k) and (l) Same as above with $\nu_1=\nu_2=0.17$, $E_1=12.8$ GPa,
 695 and $E_2=76.2$ GPa. Tu., Sk. and Sí. indicate Tungnaárjökull, Skaftárjökull and Síðujökull outlet glaciers, respec-
 696 tively. The black and grey arrows show the azimuth of the satellite and the look direction, respectively. The
 697 black lines locate the profiles A and B presented in Fig. 11. Note the difference in scale between plots (a) to (c)
 698 from the one-layer elastic models and plots (d) to (l) from the two-layer elastic models.

699

700 **Fig. 9** Probability distribution estimate of the Young's modulus (E) and Poisson's ratio (ν) for one-elastic
 701 layer models. The best model (white cross) predicts $E=46.4$ GPa and $\nu=0.17$. The black outline shows the 95%
 702 confidence region, located between 43.2 – 49.7 GPa for E and 0 – 0.27 for ν , the black dashed line gives the 68%
 703 confidence region.

704

705 **Fig. 10** Probability distribution estimates of the Young's moduli for the upper (E_1) and lower (E_2) lay-
 706 ers for the two-layer elastic models. The plus symbols indicate the best-fit models in each case, the contin-

707 uous outlines the 95% confidence regions, and the dashed lines the 68% confidence regions. In green, we
 708 show the distribution for the models with $\nu_1=\nu_2=0.25$, indicating a best-fit model of $E_1=12.9^{+1.3}_{-1.0}$ GPa and
 709 $E_2=70.5^{+7.0}_{-6.0}$ GPa. In red, we show the results for our preferred model with $\nu_1=0.17$ and $\nu_2=0.25$, giving a
 710 best estimate of $E_1=13.9^{+1.4}_{-1.0}$ GPa and $E_2=73.9^{+8.0}_{-6.6}$ GPa. Results for the models with $\nu_1=\nu_2=0.17$ are shown in
 711 blue and predict a best-fit model of $E_1=13.8^{+1.3}_{-1.0}$ GPa and $E_2=76.2^{+8.4}_{-6.9}$ GPa. The uncertainties given here corre-
 712 spond to the 95% confidence regions. The color scale shows the probability distribution for our preferred model.

713

714 **Fig. 11** Plots showing the deformation along two profiles (location on Fig. 8). Results from profile A are
 715 displayed on the left side panels and results from profile B are shown in the right side panels. (a) and (b)
 716 Comparison between the surge displacement field (corresponding to \mathbf{d}_{surge}) and the best-fit models (where we
 717 added the ramp and offset estimated by the Bayesian approach), in black and coloured symbols respectively.
 718 (c) and (d) Residual displacement along each profile for each of the best-fit models. In all four panels, purple
 719 circles indicate the results obtained with the best-fit one-layer model, the green triangles are used for the best-fit
 720 two-layer model with $\nu_1=\nu_2=0.25$, the red squares correspond to the best-fit two-layer model with $\nu_1=0.17$ and
 721 $\nu_2=0.25$ (preferred model), and the blue inverted triangles show the best-fit two-layer model with $\nu_1=\nu_2=0.17$.

722

Table 1.

Acquisition date [yyyy-mm-dd]	Perpendicular baseline [m]
1993-06-26	-318
1993-07-31	-88
1993-09-04	174
1993-10-09	318
1995-06-19	-184
1995-08-28	152
1995-08-29	151
1995-10-02	269
1995-10-03	506
1996-06-04	-231
1996-07-09	394
1996-08-13	202
1996-09-17	0
1997-06-24	16
1997-07-29	109
1997-09-02	491
1998-07-14	-511
1998-08-18	-333
1998-09-22	125
1999-08-03	342
1999-09-07	-601
2000-08-22	119
2002-07-23	92

Table 2.

Poisson's ratio ¹	Static Young's modulus E_s [GPa]	Dynamic Young's modulus E_d [GPa]	Elastic Depth [km]	Source
$0.17^{+0.10}_{-0.17}$	$46.4^{+3.3}_{-3.2}$		~half-space	This study (one-layer elastic model)
0.36 ± 0.06	64 ± 6		half-space	This study using GIPhT method (Feigl & Thurber 2009; Ali & Feigl 2012)
$(\nu_1=0.25 / \nu_2=0.25)$	$E_1=12.9^{+1.3}_{-1.0} / E_2=70.5^{+7.0}_{-6.0}$		1 km / ~half-space	This study (two-layer elastic model)
$(\nu_1=0.17 / \nu_2=0.25)$	$E_1=13.9^{+1.4}_{-1.0} / E_2=73.9^{+8.0}_{-6.6}$		1 km / ~half-space	This study (two-layer preferred model)
$(\nu_1=0.17 / \nu_2=0.17)$	$E_1=13.8^{+1.3}_{-1.0} / E_2=76.2^{+8.4}_{-6.9}$		1 km / ~half-space	This study (two-layer elastic model)
(0.25)	29 ± 5		half-space	Pinel et al. (2007)
(0.25)	40 ± 15		half-space	Grappenthin et al. (2006)
(0.27)		45.7	0–1	Hooper et al. (2011), derived from seismic data by Allen et al. (2002)
		58.4	1–3	
		76.2	3–5	
		94.0	5–7	
		111.8	7–	
(0.25)		14.4	0–0.5	Gudmundsson (1988), derived from seismic data by Pálmason (1971)
		37.1	0.5–1	
		57.4	1–2.2	
		102	2.2–5.5	
		134	5.5–	

¹Values in brackets indicate an assumed value for this parameter, instead of inferred ones.

Table 3.

Poisson's ratio ²	Static Young's modulus E_s [GPa]	Dynamic Young's modulus E_d [GPa]	Elastic thickness [km]	Source
0.26	17.9–21.1	25.5		Heap et al. (2011)
(0.25)		11.5	0–1	Currenti et al. (2007)
		28.8	1–5	
		63	5–8	
		86	8–15	
		101	15–23	
		133	23–50	
(0.25)	11.25		2.7	Hooper et al. (2002)
(0.25)	0.7 ± 0.2			Beauducel et al. (2000)

²Values in brackets indicate an assumed value for this parameter, instead of inferred ones.

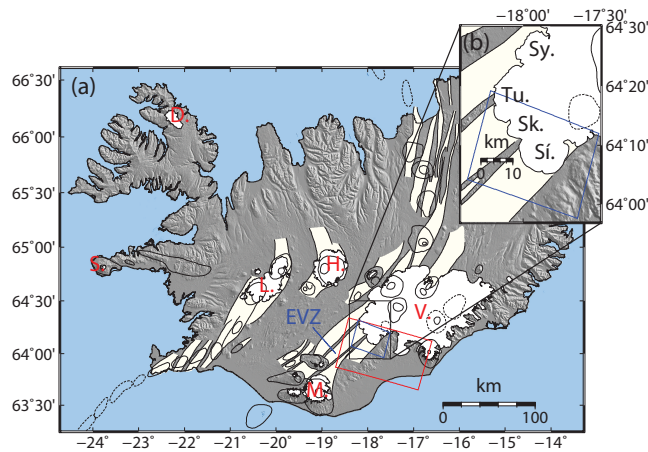


Figure 1. (a) Ice caps and tectonic setting of Iceland. Fissure swarms are shown in light yellow and central volcanoes with their associated calderas are represented by oval outlines (after Einarsson & Saemundsson 1987). The Eastern Volcanic Zone (EVZ) is displayed in blue. Main ice caps names are indicated in red (D.: Drangajökull, S.: Snæfelsjökull, L.: Langjökull, M.: Mýrdalsjökull, H.: Hofsjökull, and V.: Vatnajökull). The color boxes show the area spanned by our InSAR data: red for the full scene and blue for the cropped one. The black box gives the area shown in (b). (b) Zoom in the southwestern region of Vatnajökull, with the names of the four surging outlet glaciers studied here (Sy.: Sylgjujökull, Tu.: Tungnaárjökull, Sk.: Skaftárjökull and Sí.: Síðujökull) and the cropped InSAR scene outlines (blue box).

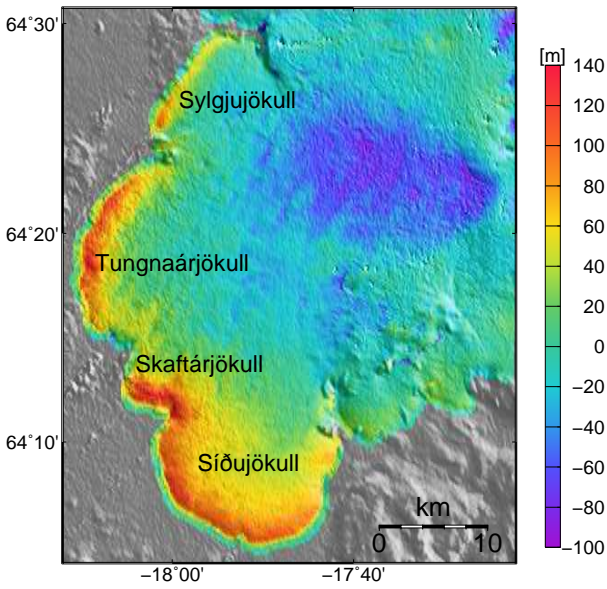


Figure 2. Surface elevation change at Sylgjujökull (Sy.), Tungnaárjökull (Tu.), Skaftárjökull (Sk.) and Síðujökull (Sí.) outlet glaciers between 1993 and 1995. Negative values indicate an ice loss while positive values indicate a gain in ice.

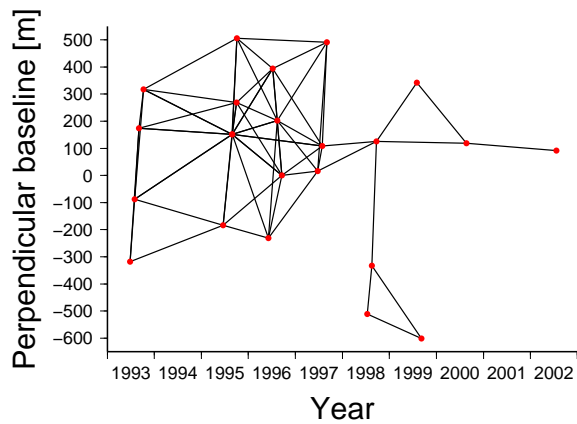


Figure 3. Connections (black lines) between individual InSAR acquisitions (red dots) forming the 65 highly coherent small-baseline interferograms used in the study. The y-axis displays the perpendicular baseline between each image and an arbitrary master image on 17 september 1996.

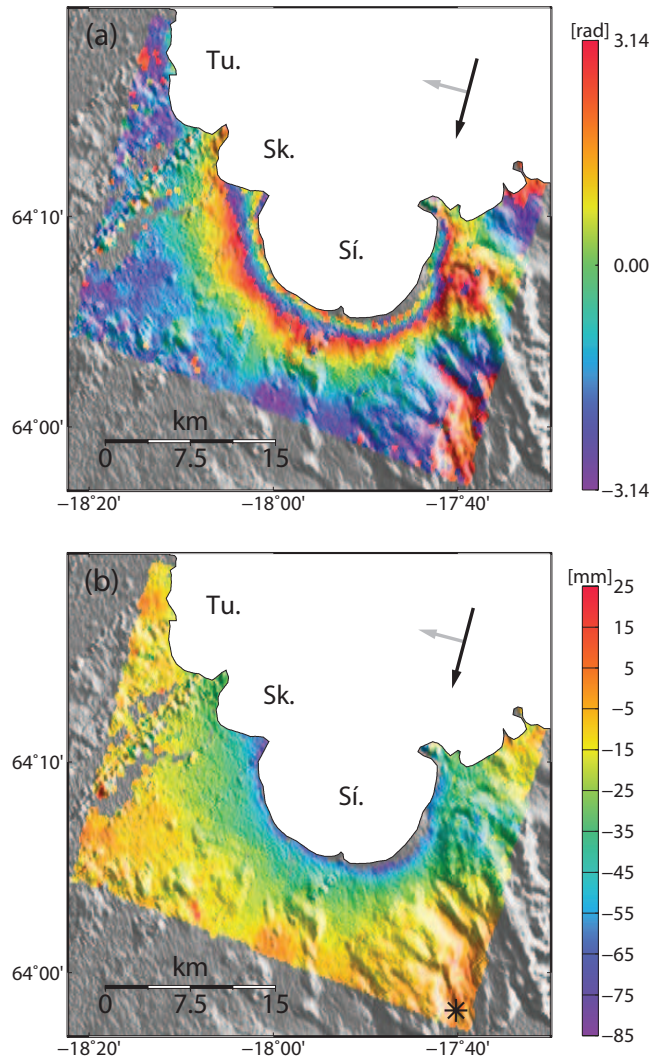


Figure 4. Interferograms spanning 31 July 1993 to 19 June 1995, showing the surge at Tungnaárjökull (Tu.), Skaftárjökull (Sk.) and Síðujökull (Sí.) outlet glaciers. The black and grey arrows show the azimuth of the satellite and the look direction, respectively. (a) Wrapped interferogram showing the deformation in fringes between $\pm\pi$. One full fringe (2π) equals 28.3 mm deformation. (b) Unwrapped interferogram. The black star designates the reference area and negative values indicate LOS lengthening.

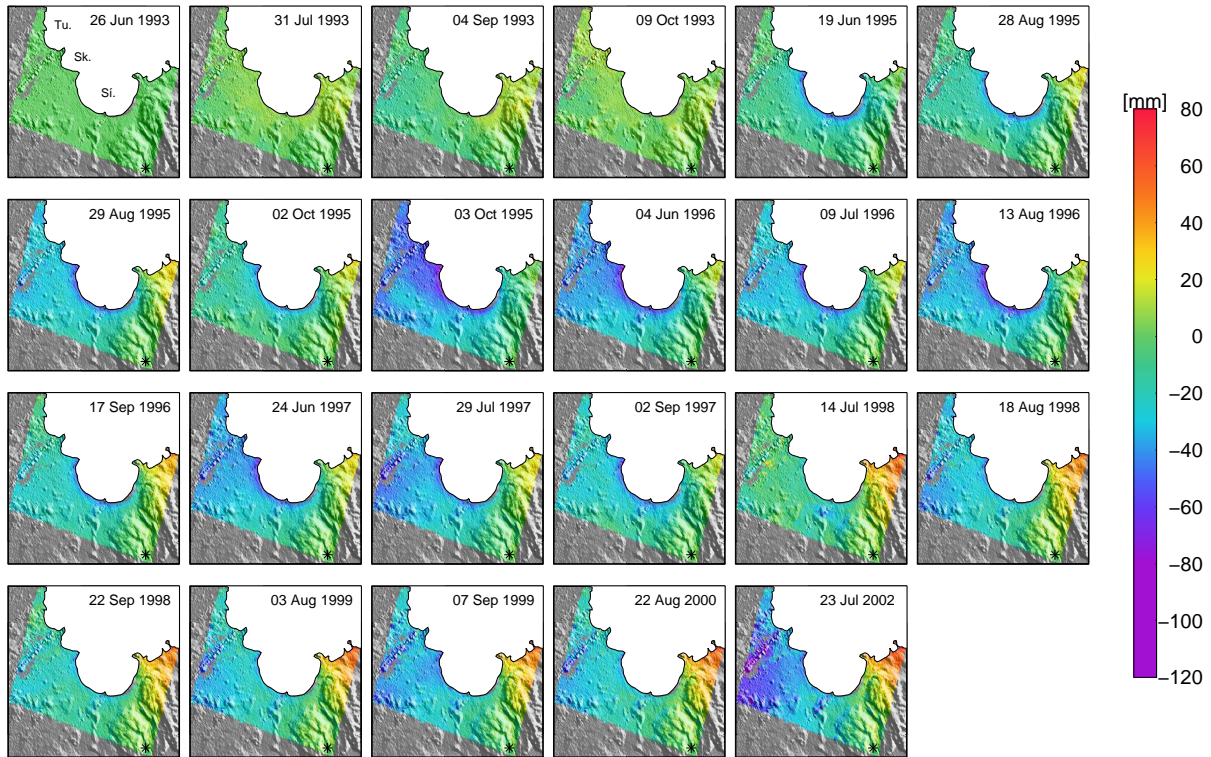


Figure 5. Single-master time series created from the 65 small baseline interferograms, spanning 1993 to 2002. The deformation shown is in LOS (negative values for LOS lengthening), relative to the reference area indicated by the black star. Each panel shows the cumulative change from the first interferogram on 26 June 1993, where Tu., Sk. and Sí. indicate Tungnaárjökull, Skaftárjökull and Síðujökull, respectively. The color scale has been modified such that points from -80 mm to -120 mm appear in the same color, to enhance the viewing of the surge signal.

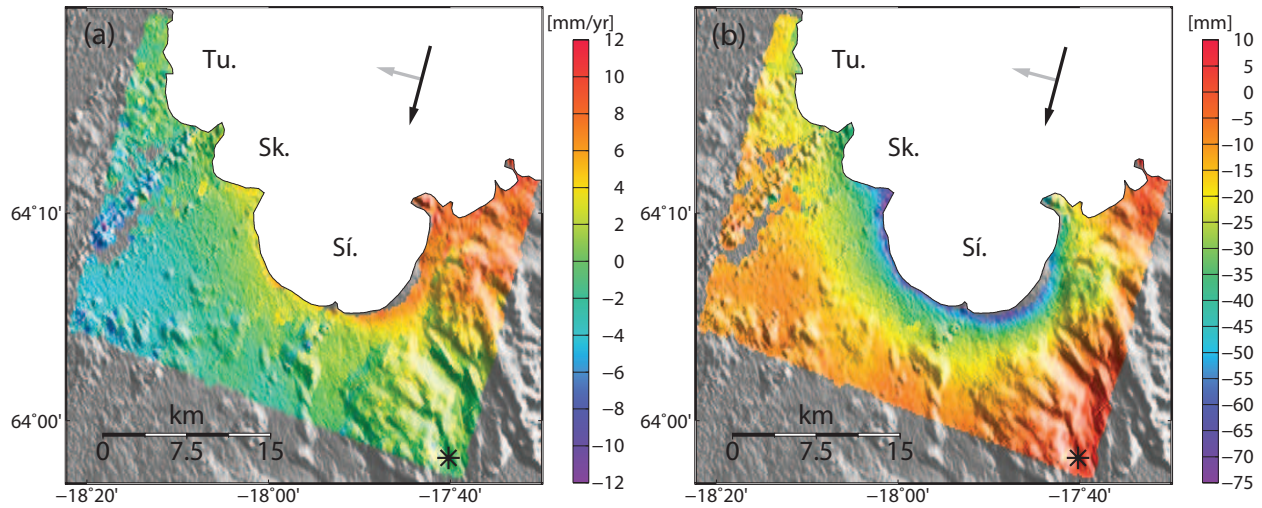


Figure 6. Inferred deformation signals from the linear inversion ran on the single-master time series: (a) GIA signal estimated as a continuous velocity, in mm/yr, (b) surge displacement estimated as a step function, in mm. Both results are shown in LOS and with respect to the reference area, where negative values stand for LOS lengthening (note the difference in color scaling). The black and grey arrows show the azimuth of the satellite and the look direction, respectively. Tu., Sk. and Sí. indicate Tungnaárjökull, Skaftárjökull and Síðujökull outlet glaciers, respectively.

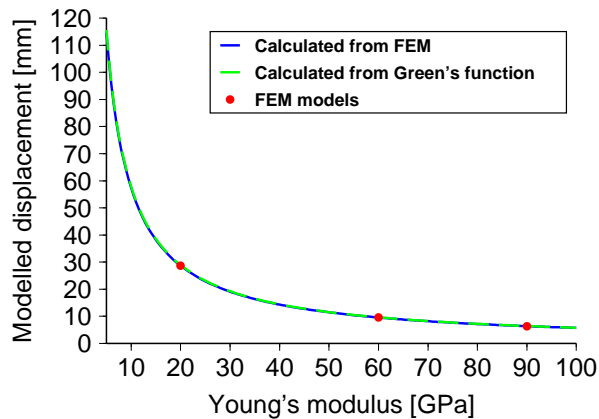


Figure 7. Vertical deformation observed at a randomly chosen mesh node as a function of Young’s modulus. The red circles show results from the finite element models run with $E=20$, 60 and 90 GPa. The blue line gives the vertical deformation calculated with the finite element model result using $E=20$ GPa and scaling it for the different values of E , according to Hooke’s law. The dashed green line, superimposed on the blue one, represents the deformation calculated with the Green’s function approach using $E=40$ GPa, and scaled to other values of E using Hooke’s law.

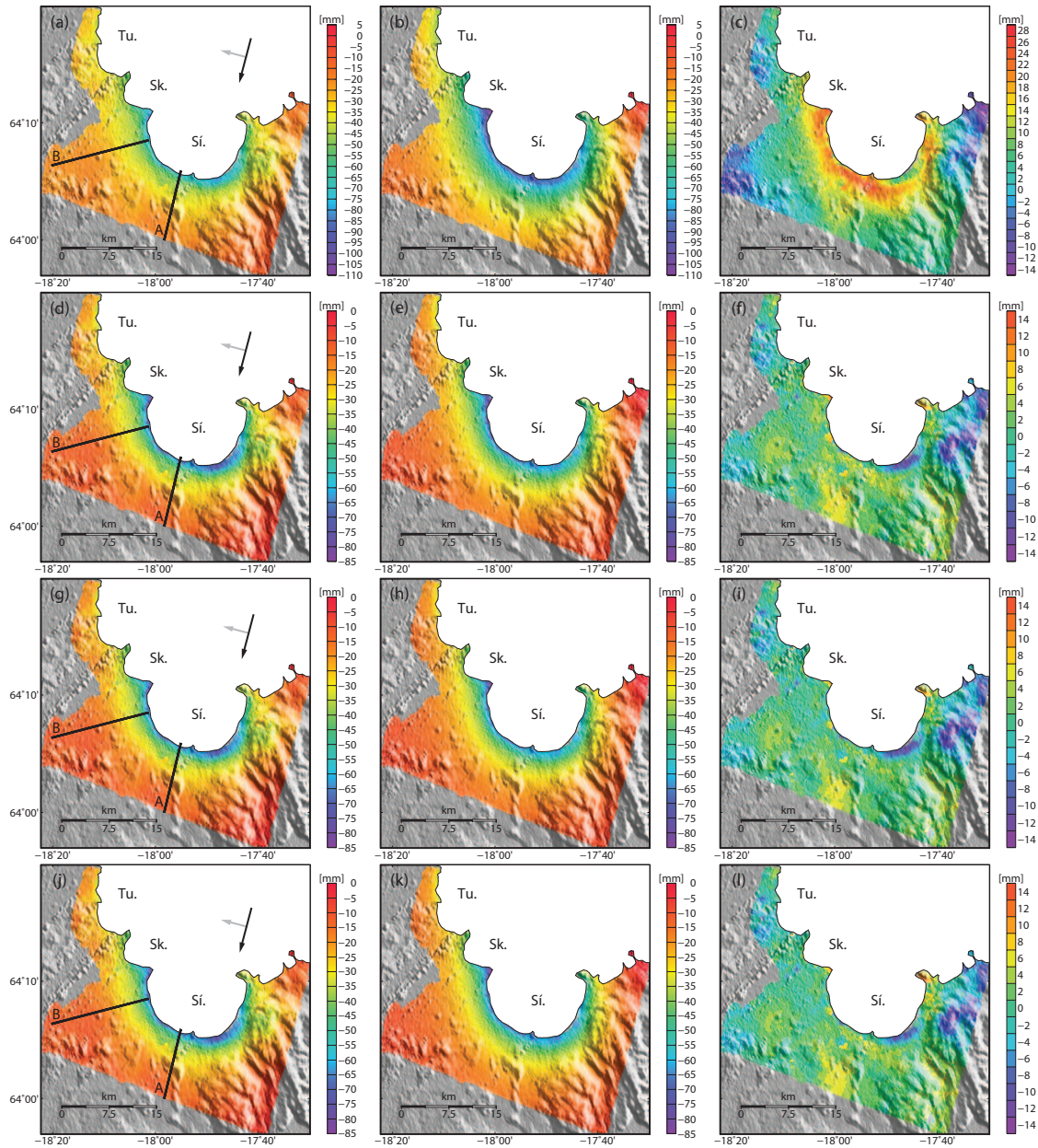


Figure 8. Top row: (a) Referenced LOS surge displacement estimated from the InSAR data (output from the least-squares inversion minus the ramp and offset estimated from the Bayesian approach), (b) best-fit one-layer elastic model ($E=46.4$ GPa and $\nu=0.17$) converted to LOS, and (c) residual between (a) and (b), respectively. Rows 2–4 show similar set of panels for the other models. (d), (e) and (f) Same as above but with the two-layer elastic best-fit model with $\nu_1=\nu_2=0.25$, $E_1=12.9$ GPa, and $E_2=70.5$ GPa. (g) (h) and (i) Same as above with $\nu_1=0.17$, $E_1=12.9$ GPa, $\nu_2=0.25$ and $E_2=73.9$ GPa. (j), (k) and (l) Same as above with $\nu_1=\nu_2=0.17$, $E_1=12.8$ GPa, and $E_2=76.2$ GPa. Tu., Sk. and Sí. indicate Tungnaárjökull, Skaftárjökull and Síðujökull outlet glaciers, respectively. The black and grey arrows show the azimuth of the satellite and the look direction, respectively. The black lines locate the profiles A and B presented in Fig. 11. Note the difference in scale between plots (a) to (c) from the one-layer elastic models and plots (d) to (l) from the two-layer elastic models.

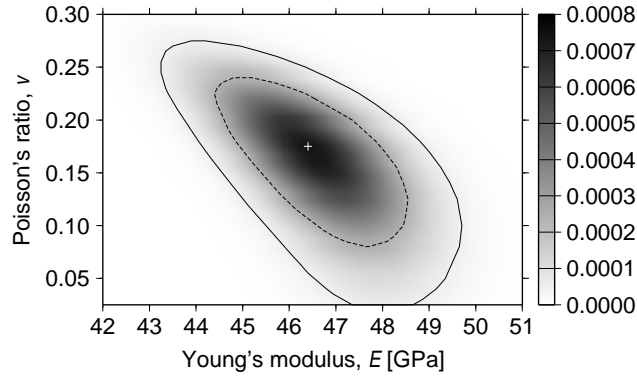


Figure 9. Probability distribution estimate of the Young's modulus (E) and Poisson's ratio (ν) for one-elastic layer models. The best model (white cross) predicts $E=46.4$ GPa and $\nu=0.17$. The black outline shows the 95% confidence region, located between 43.2–49.7 GPa for E and 0–0.27 for ν , the black dashed line gives the 68% confidence region.

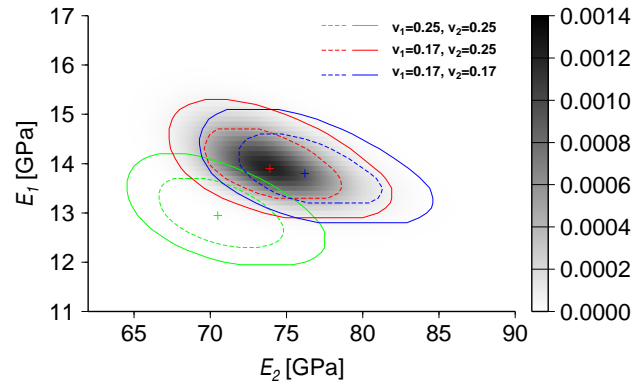


Figure 10. Probability distribution estimates of the Young's moduli for the upper (E_1) and lower (E_2) layers for the two-layer elastic models. The plus symbols indicate the best-fit models in each case, the continuous outlines the 95% confidence regions, and the dashed lines the 68% confidence regions. In green, we show the distribution for the models with $\nu_1=\nu_2=0.25$, indicating a best-fit model of $E_1=12.9^{+1.3}_{-1.0}$ GPa and $E_2=70.5^{+7.0}_{-6.0}$ GPa. In red, we show the results for our preferred model with $\nu_1=0.17$ and $\nu_2=0.25$, giving a best estimate of $E_1=13.9^{+1.4}_{-1.0}$ GPa and $E_2=73.9^{+8.0}_{-6.6}$ GPa. Results for the models with $\nu_1=\nu_2=0.17$ are shown in blue and predict a best-fit model of $E_1=13.8^{+1.3}_{-1.0}$ GPa and $E_2=76.2^{+8.4}_{-6.9}$ GPa. The uncertainties given here correspond to the 95% confidence regions. The color scale shows the probability distribution for our preferred model.

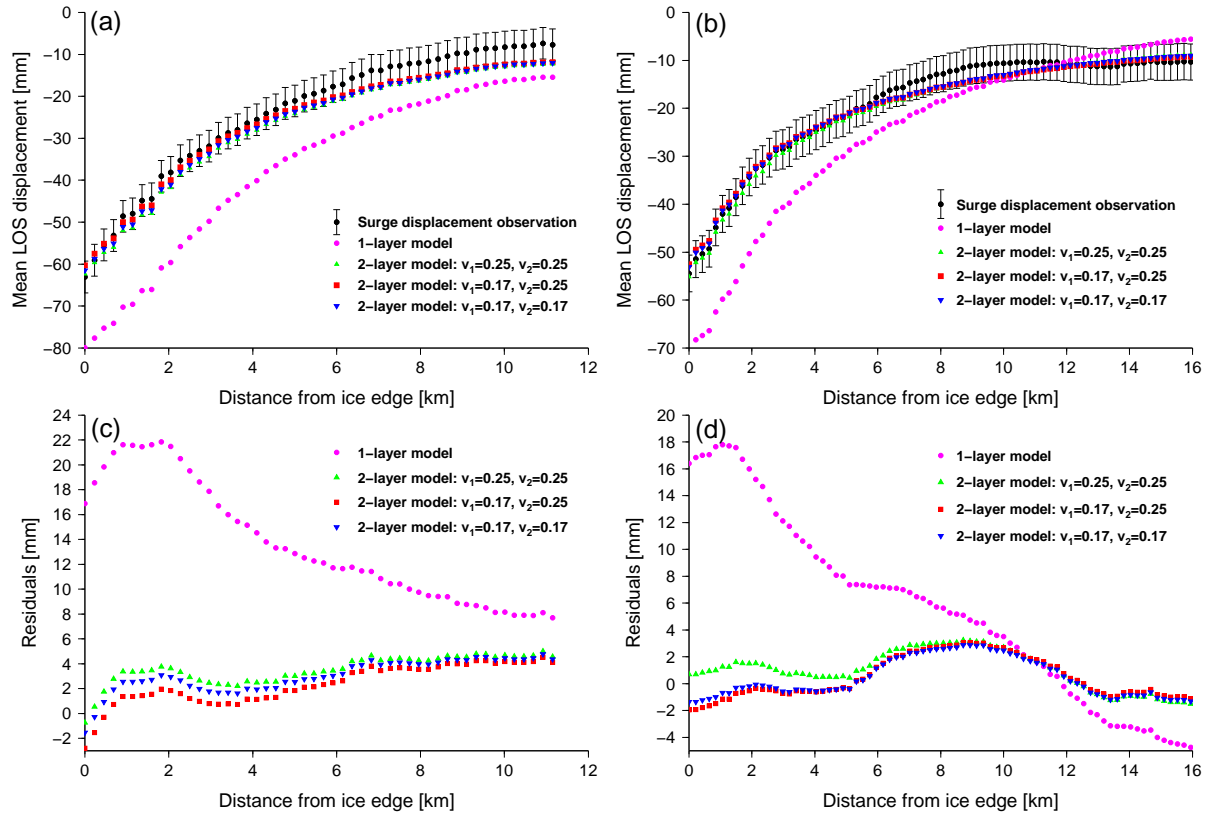


Figure 11. Plots showing the deformation along two profiles (location on Fig. 8). Results from profile A are displayed on the left side panels and results from profile B are shown in the right side panels. (a) and (b) Comparison between the surge displacement field (corresponding to \mathbf{d}_{surge}) and the best-fit models (where we added the ramp and offset estimated by the Bayesian approach), in black and coloured symbols respectively. (c) and (d) Residual displacement along each profile for each of the best-fit models. In all four panels, purple circles indicate the results obtained with the best-fit one-layer model, the green triangles are used for the best-fit two-layer model with $v_1=v_2=0.25$, the red squares correspond to the best-fit two-layer model with $v_1=0.17$ and $v_2=0.25$ (preferred model), and the blue inverted triangles show the best-fit two-layer model with $v_1=v_2=0.17$.