Addendum 4 — Correction to the α_i Range and Its Physical Meaning

In Section 1 of the main paper, the components of the internal change-vector α were mistakenly restricted to:

$$0 \leq \alpha_i < 1.$$

This was an oversight. Each component α_i represents the fractional orientation of internal change assigned to the imaginary axis paired with a given spatial direction. Because this orientation can point in either direction along that imaginary axis, and because it may span the full range from purely real to purely imaginary, the correct domain is:

$$-1 \leq \alpha_i \leq 1.$$

Why α_i can be positive or negative

α_i is a directional component of the internal change-vector's magnitude. It contributes to the overall magnitude, but each component itself carries sign, because it can point in either direction along its imaginary axis. Its sign represents the orientation of the internal change-direction in that imaginary plane. This sign freedom is essential for later sections of the paper that discuss:

- curvature orientation,
- geometric asymmetries,
- particle/antiparticle inversion,
- and the handedness of internal rotation.

Restricting α_i to nonnegative values eliminates these directional distinctions and was inconsistent with the geometry used throughout the rest of the paper.

Why α_i can reach ±1 (the photon-like limit)

The magnitude $|\ \alpha_i\ |$measures how strongly the internal change-vector is tilted into the imaginary component along spatial axis i.

- When $|\ \alpha_i\ | = 0$:
  internal change lies entirely along the real spatial direction.
- When $0 < |\ \alpha_i\ | < 1$:
  the internal change-direction has a mixed real–imaginary orientation (massive states with internal evolution).
- When $|\ \alpha_i\ | = 1$:
  the internal change-direction lies fully in the imaginary axis, with no real component along that spatial direction.
  This produces a perfect orthogonality between internal change and real displacement.

This extreme case corresponds precisely to the massless / photon-like limit described later in the paper:

- no internal evolution along the direction of propagation,
- no rest frame,
- pure propagation through the geometry,
- and no coupling between real displacement and internal change.

This boundary is not a singularity but simply a natural geometric endpoint – the full imaginary orientation of internal change.

Summary

- α_i measures the imaginary-axis orientation of internal change assigned to spatial axis i.
- The correct range is:

$$-1 \leq \alpha_i \leq 1.$$

- The sign gives orientation; the magnitude gives degree.
- $|\ \alpha_i\ | = 1$ naturally represents the photon boundary – a state with no internal evolution along the direction of motion.

This correction aligns the initial definition of α with the geometry and interpretations already used throughout the main paper, without altering any subsequent equations or conclusions.