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Recent Writings on Index-Numbers

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of £100 a year is worth, say, £1,000 a year?' as he finds in the case of an example designed to 'show in the clearest way that the measurement of total utility by price is purely illusory.'<sup>1</sup>

The gist of the difficulty, as I understand, is not the disproportion of the magnitudes 100 and 1,000 (which presents no difficulty), but the inexactness and indefiniteness of the result. First then it is to be observed that Professor Nicholson has exaggerated that inexactness by attempting to measure in money the complete utility derived from the entire income. This spurious specimen of Consumers' Rent being disallowed, the legitimate analogy of Professor Nicholson's example would be as follows:—To  $x$  spent upon extras and luxuries corresponds a total utility equal to ten times  $x$ . 'Of what avail' is such a proposition? Well, certainly not for any purpose which requires the exactness of a commercial account, or even of a physical constant. Our result could in general claim only such accuracy as pertains to the rougher statistical calculations—such as the determination of the variation in the value of money, or in the amount of the national capital—calculations which Professor Nicholson has countenanced by making important contributions to them.

Thus understood, the proposition may avail to verify and illustrate many curious propositions in the theory of *taxation*—employing the term *tax* in the generalised sense of any change which may cause an increase in the supply price for each several amount of the commodity;<sup>2</sup> and to refute many unfounded assumptions on that subject.<sup>3</sup>

A more immediately practical application has been indicated already.<sup>4</sup> Suppose that a tax must be imposed, or a railway rate fixed by Government, and that the amount of Consumers' Rent lost by the tax, or not obtained by the rate, were estimated to be according to one proposed plan  $x$ , and, according to another,  $\frac{1}{2}x$ ; it would be advisable to adopt the latter rather than the former plan. This is a rough, but may be a useful, conclusion.

F. Y. EDGEWORTH

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#### RECENT WRITINGS ON INDEX-NUMBERS.

ONE of the problems which has exercised economists for some years, the determination of variations in the value of the monetary standard, bears some not wholly accidental resemblances to one of the problems which has exercised philosophers in all ages, the determination of the standard of moral action. With respect to both

<sup>1</sup> P. 58.

<sup>2</sup> Marshall, *Principles of Economics*, Book V., ch. xii., p. 4. See the remarks made above p. 152. See also in Auspitz and Lieben's *Theorie der Preises* the propositions relating to taxes and bounties on exports and imports. I venture also to refer to propositions which I have stated in the article on *International Trade* in another part of this number of the Journal, almost all deduced with the aid of the principle of Consumers' Rent.

<sup>3</sup> For the negative use of the principle to dispel the vain appearance of knowledge, see Jevons's *Theory*, ch. iv., section on *Gain by Exchange*.

<sup>4</sup> Above, p. 151.

problems there are wise men who despair of determinateness ; there are enthusiasts of whom each is confident that he has obtained *the* solution. With respect to both problems the discrepancy of principles is greater than the difference in practice ; within certain limits almost any formula, accompanied with common sense, will lead to good results.

These reflections are illustrated by the treatment which the monetary problem has received in Dr. Lindsay's recent volume.<sup>1</sup> The writer proposes an Index-number which may be described as a weighted (arithmetic) mean, based on national consumption. This proposal will command general assent ; the proposed method is indeed that which was recommended by the Committee of the British Association appointed to consider this subject. It is a pity that Dr. Lindsay in reviewing his predecessors did not look out for those with whom he could agree, instead of censuring those from whom he differs. His criticisms deserve notice only as typical of the sort of intolerance which—in monetary, as in ethical theory—is apt to characterise common sense.

A certain narrowness appears in the very first paragraph of Dr. Lindsay's criticism where he places in a preeminent category three simple methods, the Arithmetic, Geometric and Harmonic means. For there are two other species which are 'not unbefitting highest place,' and which are relevant to the problem in hand in some of its aspects : namely the Median and the Greatest Ordinate.<sup>2</sup> Dr. Lindsay criticises Jevons's method with a severity which I cannot regard as justified. For the Geometric mean would be appropriate on the tenable assumptions (1) that the *quasitum* is a real thing,<sup>3</sup> or at least a unique type (such as the average stature of a nation) ; (2) that the 'errors,' or deviations, from that true mean which the data present obey a certain law of dispersion which there is some reason for expecting prices to fulfil.<sup>4</sup>

Among mathematical toys [*mathematische Spielerei*], the childish things which Dr. Lindsay puts away, next comes the original statistical method proposed by Prof. Poynting.<sup>5</sup> The authority with which Dr. Lindsay pronounces condemnation may be estimated from the circumstance that he describes Prof. Poynting's method as a variety of the Harmonic Mean (p. 14), because in the profound remarks on the theory

<sup>1</sup> *Die Preisbewegung der Edelmetalle seit 1850, verglichen mit der andern Metalle* . . . Jena : G. Fischer. 1893. (Chap. III. *et passim*.)

<sup>2</sup> See the masterly chapters on averages in Dr. Venn's *Logic of Chance*. (Third edition.)

<sup>3</sup> See below pp. 164-5.

<sup>4</sup> The *à priori* reason may be gathered from Mr. Galton's paper *Law of the Geometric Mean* in the *Proceedings of the Royal Society*, 1879 ; an *à posteriori* verification is presented in section viii. p. 31 *et seq.* of the first memorandum contributed by the present writer to the British Association Committee for ascertaining and measuring variations in the monetary standards (Report of the British Association for 1887). I desire to refer to this memorandum and its successors in 1888 and 1889 in proof of several assertions made in the present paper. The references are to the pages of the memoranda (not of the Reports in which they are embodied).

<sup>5</sup> *Journal of the Statistical Society*, for 1884.

of averages on which Prof. Poynting based his method he had employed harmonic analysis—just as if an historian were to describe the Jacobins as a variety of the Jacobites. Prof. Poynting's potent method being, as I understand, directed to somewhat different purposes from those of an ordinary index-number, it is no disparagement to the method that its results do not tally with those of Soetbeer. It is difficult to see how a comparison could be instituted.<sup>1</sup>

Proceeding to German index-numbers Dr. Lindsay notices the want of *rationale* in Drobisch's method of comparing the value at different epochs of an average hundredweight of goods. It should be observed that the method is not so deficient in practice as in theory. This is exemplified by an English variety, the average ton, independently constructed and successfully employed by Sir Rawson Rawson.<sup>2</sup>

I agree with Dr. Lindsay in not employing Dr. Julius Lehr's cumbrous formula; but not for the reason assigned, that the alteration in the quantities consumed plays as important a part in the formula as the alteration of the prices.<sup>3</sup>

It is pleasant to turn from Dr. Lindsay's theory to his practice, and to be able to admit that he has made good use of a sound method in determining the decline in price of a group of metals. The number of metals not being very great and their prices presumably much affected by a common cause—decrease in the cost of production—it is not surprising that there is a considerable difference in the results obtained by the simple Arithmetic mean and Dr. Lindsay's index-number weighted according to the quantities consumed. It has even happened in one or two instances that for a particular quinquennium one method shows a rise, the other method a decline in the average price of the group of metals.<sup>4</sup>

The decline in the price of metals in the principal countries of the world since the period 1851–55 is found by Dr. Lindsay to be from 25 to 30 per cent. Of this 25 or 30 he estimates that from 10 to 15 may be accounted for by the diminution in the cost of production. The remaining 15 or 10 per cent. he is disposed to attribute to the appreciation of the monetary standard.

His remarks on the means of rectifying instability of the standard, seem characterised by his usual practical good sense. In a weighty passage (p. 204) he points out how unworkable a tabular standard would be: exposed to the corrupt influence of politics. He looks rather to international action. A commission representing the leading nations should first try to correct the evil by the subsidiary use of

<sup>1</sup> See Dr. Lindsay, p. 15.

<sup>2</sup> See Third Memorandum (Report of the British Association for 1889), p. 20.

<sup>3</sup> Dr. Lehr's index-number does not take account of the decrease of final utility attributable to the increase of national wealth (First Memorandum, 1886 p. 11); it belongs to a class of index-number, in which the quantities consumed are used as 'weights,' and do *not* play as important a part in determining the result as the prices (Second Memorandum, p. 14 *et ante*).

<sup>4</sup> See the splendid diagram at p. 188, and the author's comments at p. 192.

silver. Should this fail, they might resort to bimetallism of the ordinary type, or of that which Prof. Marshall has proposed.

He concludes with a useful analysis of Prof. Suess's authoritative treatise on the future of gold—not a very brilliant future, if the distinguished geologist is right in inferring that the yield of gold will become inadequate to supply the currency.

The strife between rival methods may be somewhat abated by considering the second report of the United States Finance Committee upon the course of prices and wages; the results of which are summed up by Prof. Taussig in a masterly paper read before the International Statistical Institute at Chicago.<sup>1</sup> There is hardly any difference between the index-numbers for the course of prices since 1860, as determined by a simple average, or by one weighted according to the importance of each article to the consumer—an importance which was measured by the proportions in which different articles entered into the average 'budget' or expenditure of families of small means. Prof. Taussig says: 'If these two methods of simple arithmetical average on the one hand and average weighted by family budget importance on the other hand yielded greatly different results, we might be perplexed which to use as significant of the general course of prices.'

It must not be supposed that this sort of perplexity is always equally well avoided. There has lately been agitated a question of principle upon the answer to which depends a material difference in practice. Should the standard of deferred payments—the amount payable at future epochs to a creditor—be the product of a constant quantity of effort and sacrifice, the same "value" in Ricardo's terminology,<sup>2</sup> or a constant quantity of commodity, the same amount of "riches"?

Prof. Simon Newcomb<sup>3</sup> goes so far as to say—

'The fundamental idea on which the tabular standard ["twenty years ago supposed to afford a satisfactory solution to the problem"] was based, was that human labour itself furnished the best possible standard.'

In a similar spirit Mr. Leonard Courtney, in his candid article on Bimetallism, writes:—<sup>4</sup>

'We may aim not at a redelivery of article by article, but at a repayment of labour by labour or of sacrifice by sacrifice. . . I do not stop to investigate the ethical foundation of this principle, which might lead us far afield; but I believe the standard so described does represent what would commonly be accepted as the *desideratum*.'

<sup>1</sup> Published in the *Yale Review* for November 1893.

<sup>2</sup> *Political Economy*, ch. xx.

<sup>3</sup> In his article in the September number of the *Journal of Political Economy* Chicago, Vol. I., p. 505.

<sup>4</sup> In the *Nineteenth Century* for April 1893.

This standard derives some support from the argument employed by Dr. L. S. Merriam in a recent article<sup>1</sup> that 'the restoration of equal value, or equal amounts of final utility'—a principle underlying approved standards—'means also the restoration of equal amounts of final disutility.'<sup>2</sup>

Mr. E. A. Ross objects<sup>3</sup> that, as the goods restored would not all be employed at the margin of expenditure, the increase in the quantity of goods payable by the debtor should not be measured by the decrease in their *final* utility. This objection is valid against the exact correspondence between the labour standard and the utility standard which Dr. Merriam had suggested in virtue of the condition that final utility balances final disutility. But Prof. Ross does not disprove a rough correspondence between the utility standard as corrected by reference to total rather than final utility, and the disutility standard in the only form in which it is practically proposed to employ it—viz., assuming the total labour per head at the periods compared to be constant, and taking the ratio between the total quantity of goods produced per head now, and the corresponding total at a former epoch, as the measure of the increase in the quantity of goods produced by a unit of labour.<sup>4</sup> The depreciation of goods, if I may be allowed the expression, thus determined by the disutility standard may well coincide with—there is no reason why it should exceed—the depreciation determined by the (total) utility standard.

This possibility becomes fortified by the consideration that, as Mr. Ross well puts it, 'the total well being we derive from goods depends' 'not only on the positive satisfaction experienced in use or consumption,' but also 'on the social satisfactions that flow to us in consequence, the latter largely determined by the relation of our consumption to that of our neighbours.'<sup>5</sup> In a progressive state of society the second circumstance as well as the first tends to depreciate goods with respect to utility, and *pro tanto* increases the probability that the appreciation of money as measured by the corrected utility standard will not be materially greater than as measured by the proposed labour standard.

One objection against the Labour standard recently made by Professor Foxwell does not seem to me decisive: namely that it is impossible to define "a unit of labour."<sup>6</sup> A similar objection might be

<sup>1</sup> 'The Standard of Deferred Payments.' *Amer. Ac. Pol. Sci.*, January, 1893. It is sad to learn that the promising author of this just and ingenious argument has been the victim of a boating accident.

<sup>2</sup> On the idea of the final utility of wealth decreasing with the progress of society see the first of the British Association Memoranda above referred to, p. 14.

<sup>3</sup> *Amer. Ac. Pol. Sci.*, Nov. 1893.

<sup>4</sup> This argument may be illustrated by the use of diagrams such as Jevons has employed in his *Theory* to denote the total and final utility of consumption and disutility of production.

<sup>5</sup> *Loc. cit.*, p. 104.

<sup>6</sup> In speaking before Section F of the British Association 1893; as reported in the December number of the *Journal of the Statistical Society*, p. 645. *Cp.* Report of Annual Meeting of the Bi-metallic League, 1894, p. 56.

made to the most generally received index-number based upon consumption ; which seems to involve implicitly what Dr. Julius Lehr with his *genusseinheit* has the courage to state explicitly—the measurement of utility.

If the objection is directed, not so much against the difficulty of conceiving, as that of carrying out the labour standard ; it may be replied that statistics of wages, which may be regarded as giving the average increase in the amount of money procured by a day's work,<sup>1</sup> are not altogether wanting. For example Professor Taussig in the paper already referred to exhibits the rise of wages, as well as the fall of prices, during recent years. He remarks :—

‘ The average, or index-numbers, are in one sense more accurate and significant as to wages than they are as to prices.

‘ The inevitable fictitious quality of a general index number thus calls for less constant allowance in using these results of the statistics of wages than in using the figures for prices.’

An index-number based on such statistics is accurate enough for the conclusion to which it is applied : *quieta non movere*—that for the purpose of assuring to creditors the produce of a constant quantity of labour an alteration of the standard of deferred payment is not called for.

But this purpose may not be accepted as just and expedient by currency reformers whose end is to minimize the drag on the producer caused by continually shrinking prices.

For the construction of an index-number which should indicate that danger retail prices are less appropriate than wholesale prices. Accordingly when Mr. Cannan, criticising Bimetallism,<sup>2</sup> doubts the fact of appreciation as not evidenced by retail prices he is not persuasive. But the same consideration, with reference to the purpose of endowment—keeping a teacher or preacher on the same level of comfort and respectability—would be pertinent.

It is with the index numbers as with conduct ; in order to form a just judgment, we must always look to the underlying idea and purpose.

As another example of misunderstanding occasioned by diversities of purpose, I may refer to that variety of index-number which purports to determine a real quantity, a cause or characteristic, such as ‘ scarcity of gold,’ in some more objective sense than a mere fall of prices on an average. The *quæsitum* in this case may be likened to a physical quantity which is to be ascertained from a set of measurements. The method accordingly presents certain peculiarities derived from the theory of errors-of-observation.<sup>3</sup>

<sup>1</sup> The other element of effort-and-sacrifice, abstinence, is less easily taken account of. On an average, statistics relating to numerous different occupations, the errors due to the neglect of this element might disappear through compensation.

<sup>2</sup> *Economic Review*, October 1893.

<sup>3</sup> See First Memorandum, p. 29 ; Third Memorandum, p. 25.

I have been unfortunate in not making this view clear to Professor Laughlin. Some years ago<sup>1</sup> he had seemed to deny that there had occurred a general fall in prices in a sense which could prove the existence of 'scarcity of gold.' After accounting for the fall of prices in several species of commodity, he goes on :—

'The preceding discussion however does not account for a general fall in prices. If the fall of prices had been general, it might suggest a single cause affecting all commodities, such as the scarcity of the medium, by which goods are exchanged, in fact, it seems to be quite necessary to a theory which explains the fall in prices by the scarcity of gold that the fall should be universal.'<sup>2</sup>

Referring to this passage and the similar views of other writers I maintained :

'To assert with Mr. Laughlin and others that, in order to prove a general fall, you must prove a fall in every article, is wholly to ignore the character of the investigation.'

'The phenomenon under examination is of the nature of what Mill called a "residual phenomenon," like the difference in the mean height of the barometer between two hours of the day, the so-called "diurnal variation." On an average of many days there is found to be a fall, but it is not necessary nor true that every day's experience should present that phenomenon. The theory of probabilities is satisfied with a majority of days.' . . .

'It seems to be taken for granted that, when we can show a reason why each price should have varied in the direction actually observed, we are thereby debarred from inferring a general displacement due in the phrase of Mill, to "causes that operate on all goods whatever." But this assumption is quite erroneous. The meteorologist may be able to assign the reason why between morning and noon each particular day there has been a rise or fall of the barometer. But the mathematician is not thereby precluded from extricating by the theory of probabilities a mean variation between those hours.'<sup>3</sup>

Referring last year to this criticism Professor Laughlin complains that I have 'wholly misunderstood' his argument.<sup>4</sup>

I am very sorry to have unconsciously misrepresented the argument which I was disputing. I can only console myself by reflecting that no amount of care on my part could have averted the mistake, since even after Professor Laughlin's explanation I am unable to discern any appreciable difference between the position which he takes up and that which was the object of my attack. He explains :

'I at least never contended that "in order to prove a general fall you must prove a fall in every article." Accepting the fact of a decline in prices, my contention was solely that the *cause* of the decline could not be scarcity of gold ; since, if there was a single cause for the fall then this cause should show itself in all,<sup>5</sup> or nearly all, the commodities quoted.'<sup>6</sup>

<sup>1</sup> In his paper on 'Gold and prices since 1873,' in the *Quarterly Journal of Economics* for April, 1887.

<sup>2</sup> *Loc. cit.* p. 340.

<sup>3</sup> *Quarterly Journal*, Vol. III., (1889) p. 107.

<sup>4</sup> *Journal of Political Economy*, (Chicago), Vol. II., p. 279.

<sup>5</sup> His former words (above quoted) are 'it is quite necessary . . . that the fall should be universal'; excusing I think my expression 'a fall in every article.' But I am quite willing to substitute 'all or nearly all' for 'every.'

<sup>6</sup> *Loc. cit.*



Now my contention was and is that, though there be a common cause it need *not* 'show itself in all or nearly all' the commodities quoted.'

To take a new example, for which the data happen to be ready to hand; suppose that the average height of a regiment of 1,000 Italian recruits selected indiscriminately from all the provinces was returned as half an inch in excess of the average height of the whole army; one might infer with certainty that the difference was due to a real cause (as distinguished from chance); and that cause might well be 'single,' such as the circumstance that the men in the regiment were (contrary to the general practice) measured with their shoes on. But it does not follow that this cause should show itself—by excess above the average of the kingdom—in a large majority. The proportion of men above the general average might be about 57 per cent., 570 out of the 1000.<sup>2</sup> Is that 'all, or nearly all'?

F. Y. EDGEWORTH

#### HINDRANCES TO THE ACCEPTANCE OF BIMETALLISM.

The following letter will explain the origin of this communication.

DEAR MR. PALGRAVE,

It would, in my opinion, be a very useful thing to form a collection of the objections to bimetallism which would naturally strike men of business; not so much those who have been engaged in the controversy, as those who are concerned in the ordinary affairs of life in which money bears so great a part.

This can only be effectively done by an impartial person who is well acquainted with the main features of the controversy.

You are eminently that person, for you have studied the matter for many years and have, as yet, not declared yourself on either side.

H. H. GIBBS.

(I.) How can an exact ratio be constantly preserved between two different things? One ounce of pure silver is worth as much as another ounce of pure silver, and the same with gold. But how can an ounce of one metal be made constantly equal in value to a fixed number of ounces of another metal?

<sup>1</sup> See penultimate note.

<sup>2</sup> The *modulus* of errors, or deviation for Italian recruits, as found by Perozzo from some hundreds of thousands of observations (see *Journal of the Statistical Society*, Jubilee volume, p. 195) being about 3·7; the modulus for the difference between the average of the regiment and the general average is about  $\sqrt{\frac{3\cdot7}{1000}}$  say ·06. Whence for the limit of difference possibly due to accident we have, say ·15; and for the amount of difference certainly attributable to cause ·5 - ·15 = ·35. Now ·35 is about a *tenth* of the modulus. And (by well-known tables) the proportion of a group (ranging under a probability curve) intercepted between the centre and ·1 of modulus is about ·05. Whence the proportion of the regiment showing excess over the general average is at least 55 per cent. By parity of reasoning—as the real difference cannot exceed ·5 + ·15—the proportion is at most 60 per cent.