

## PSYCHOLOGICAL LITERATURE.

### CALCULUS OF PROBABILITY.

*Le calcul des probabilités et la méthode des majorités.* EMILE BOREL. Année Psychologique, 1908, XIV., 125-158.

The procedure of regarding the views expressed by the majority of a group as practically true for this group, is called the method of majorities. It is best illustrated by an example given by the author. The photographs of 40 children, of whom 17 were normal and 23 backward, were shown to 20 subjects, who had to state whether the picture represented a normal or a backward child. The percentages of correct judgments varied from 67 to 92 per cent. and their average was 78 per cent. From these data alone one may conclude that the pictures — or at least some of them — gave an indication as to the intelligence of the children; but this conclusion can be made considerably more definite by the application of the calculus of probabilities. The author starts by asking what results one would have to expect, if the judgments were mere guesses. The difference between the number of correct judgments which were given on 27 photographs (or approximately two thirds of all the cases) and the number of correct judgments which one would obtain by random guessing, is so great that it cannot be attributed to chance, whereas the judgments on the remaining 13 pictures are such that they may be a coincidence due to random guesswork. From this one must conclude that in two thirds of all the cases a certain influence was at work which tended to produce correct judgments, and it is only reasonable to suppose that this was the inspection of the pictures which gave some indication as to the intelligence of the person represented. Since no such influence can be shown for the remaining third of the pictures one must conclude that traces indicating the intelligence were absent in these pictures.

A further application of this method is made in a study of the relation of the form of the hand to intelligence. Pictures of the hands of children were shown to different observers, who had to decide whether the hand belonged to a male or a female, and whether the child was bright or dull. The agreement of the judgments is not as well marked as in the first example, but here also it is possible to

show an influence which determines the judgment. Binet made the hypothesis that the child is judged stupid if the thumb is short and ugly, whereas it is judged intelligent if the thumb is long and slender. This hypothesis is confirmed by further experiments, in which the observers had to base their judgment on this criterion exclusively. The calculus of probabilities thus enables one to show the presence of a cause, but it does not give any indication as to what this cause may be. A hypothesis as to the nature of this cause must be the result of experience and must be verified by further experiments, and here again the calculus of probabilities enables one to form a judgment as to the value of the hypothesis. Mathematical deductions are only the connecting link between two experiments, but this part of guiding the investigation is, though modest, of the highest utility.

The bearing of Professor Borel's deduction on the questionnaire method is obvious, and it may be hoped that his principles will give to this method the precision and elegance which have hitherto been lacking in some of the investigations using the questionnaire. The author's views about the rôle which mathematical reasoning may play in psychological experiments are sound and in perfect agreement with the spirit of statistical investigations. The demonstration that certain events are or are not such as they probably would be if they were chance events warrants the conclusion as to the presence or absence of a knowable cause, but does not give an indication of what this cause may be. This is the way of reasoning applied in all such cases ever since Laplace's demonstration that the orbits of the planets are not the work of chance. Borel's analysis of the method of majorities has the merit of reducing a method which was rather doubtful and led to loose results, to sound and well-established principles.

The author tries to apply his ideas to the method of right and wrong cases and uses for illustration the example given in Titchener's *Manual* ('Student's Manual,' p. 107), combining the results for the two time orders. He imagines a set of results similar to these, but which was not obtained from one individual but from a group of 100 individuals, each one of which gave a judgment on the comparison of the standard with the seven comparison stimuli, and tries to determine the collective sensitivity of the group. After having determined the point of subjective equality as that intensity of the comparison stimulus on which 'greater' and 'smaller' judgments are given in equal number, the author defines the upper and lower threshold as those intensities of the comparison stimulus for which one can conclude with a high probability that the standard is smaller or greater than the comparison stimu-

lus. These values are found by interpolation. The difference between Titchener's results and the results obtained by the author is considerable. The author sees that the cause of this discrepancy is a difference of the definition of the threshold, and explains it by his intention to determine the collective, not the individual sensitivity as Titchener does.

Borel's definition of the threshold is closely related to the one given by Scripture, which is even a little more definite in so far as Scripture asks for a certain high probability (the so-called statistical certainty) with which certain judgments may be expected. Leaving aside the objection that the notion of collective sensitivity is not very well defined, we certainly could not expect to determine it by other means than individual sensitivity. Here the author misses an important point apparently on account of his not being thoroughly conversant with the problems of psychophysical measurement. The fundamental definition of the threshold is the one supplied by the method of just perceptible differences, and no measurement of the accuracy of sense perception is acceptable unless the relation of the proposed measurement to the result of this method is shown. The difficulty of the method of constant stimuli lies, therefore, in part in the method of just perceptible differences. There are very numerous possibilities of defining the threshold so far as the method of constant stimuli is concerned, but since it was demonstrated that the method of just perceptible differences determines the interval inside of which the judgments 'greater' and 'smaller' have probabilities smaller than one half, no measurement of the accuracy of sense perception can be accepted unless it uses this quantity or quantities related to it.

In the course of his argument the author calls Titchener's calculations 'rather long and very correct.' These words, of course, refer not only to the numerical calculations but also to the algorithm of Müller's method of constant stimuli. The deductions in Müller's method are perfectly correct except the formula for the determination of the weight of the observation equations, which, however, is not given in Titchener's *Manual*. This formula is incomplete and the numbers in the table of Müller's Coefficients of Weights (Titchener, *loc. cit.*, p. 101) are false, as the eminent mathematician certainly would have seen had he looked into this matter.

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