

## The Echelon Spectroscope, its Secondary Action, and the Structure of the Green Mercury Line

This content has been downloaded from IOPscience. Please scroll down to see the full text.

1907 Proc. Phys. Soc. London 21 822

(<http://iopscience.iop.org/1478-7814/21/1/356>)

View [the table of contents for this issue](#), or go to the [journal homepage](#) for more

Download details:

IP Address: 146.201.208.22

This content was downloaded on 04/10/2015 at 05:23

Please note that [terms and conditions apply](#).

LVI. *The Echelon Spectroscope, its Secondary Action, and the Structure of the Green Mercury Line.* By HERBERT STANSFIELD, D.Sc., Demonstrator in Physics, Manchester University\*.

Thesis approved for the Degree of Doctor of Science in the University of London †.

[Plates XXX.-XXXII.]

Part I.—THE ECHELON SPECTROSCOPE.

*The Echelon and its Mounting.*

THE echelon spectroscope described in this paper was constructed by Messrs. Adam Hilger for Professor Schuster, a modification, which has proved to be very valuable, being made in the usual design. A front elevation, plan, and two end elevations of the echelon are reproduced on Plate XXX., the plan being drawn with the cover removed.

There are 33 glass plates. The smallest plate is 13 mm. wide, and they increase in width by steps of 1 mm. until the last plate is reached, which is 12 mm. wider than the last but one, the aperture being reduced to 1 mm. by the screen S. The effective aperture of the first plate is similarly reduced in width to 1 mm. by the block B. All the plates are 40 mm. high, and the common thickness is 9.48 mm. The plates are pressed together by the two nickel steel rods marked T, which are intended to have the same coefficient of expansion as the glass. The plates are in very close contact, in most cases the greater part of an interface being taken up by a patch of definite but irregular outline that appears "black" by reflected light. The remaining part of the interface generally shows white of the first order, but here and there the film of air may be thick enough to show the yellow or even the red of the first order.

\* Read June 11, 1909.

† The following alterations have been made:—Equations 2, 3, 4, 9, 9A, and the section on the spectrum given by a hot lamp have been added; also the faint lines previously described as doubtful are shown to have their origin in the echelon.

The common thickness of the glass plates is 9.48 mm.; their refractive index, deduced by a Hartmann formula from the values given by the makers of the glass, is 1.5802 for the green line 5461; and the dispersive power,  $\frac{d\mu}{d\lambda}$ , is -918 per cm. for this wave-length.

*Optical Effects due to clamping the Plates.*

The echelon produces a slight cylindrical convergence in a beam of light, altering the focus of the observing telescope by 1 mm. (the focal length of the object-glass being 53 cms.). This effect is probably due to the clamping, as Twyman\* states that when the clamping pressure is applied a change of focus is produced. He attributes the effect to a uniform increase in the compression of the plates from the largest to the smallest; but there is direct evidence (see p. 830) that the echelon plates become slightly prismatic, and this effect, increasing towards the smaller end, would also produce convergence.

The focus is altered by rotating the echelon about a vertical axis. The convergence is increased, as would be expected, by turning the echelon so as to reduce the width of the emergent beam (see fig. 3 A), and diminished when it is turned so as to make the beam broader (fig. 3 B). Changes in the focus are also produced by covering some of the step-faces at either end of the echelon.

*Use of Auxiliary Prism.*

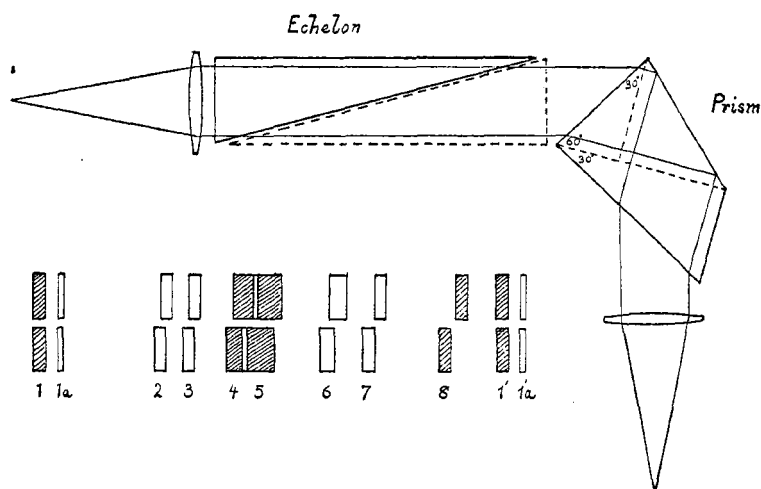
Auxiliary spectroscopes have generally been employed to pick out the particular line in the spectrum to be examined by the echelon, the slit of the echelon spectroscope being illuminated with the selected line; but a modification in the design of this instrument was made for Professor Schuster so that the prism from the auxiliary spectroscope could be mounted next to the echelon, as shown in fig. 1, the prism being made larger than usual in order to take the full width and height of the echelon beam. This arrangement, which appears to have been originally contemplated by Professor Michelson, has been found to have important advantages.

\* Twyman, Proceedings of the Optical Convention, 1905, p. 53.

The dispersion produced by the prism, which is 2 per cent. of that given by the echelon, is subtracted from the echelon dispersion when the echelon is in the usual position shown by full lines, and is added to it when the echelon is in the reversed position shown by the dotted lines.

The change of 4 per cent. in the dispersion obtained in this way produces the alteration in the spectrum shown in fig. 1. The distance apart of successive orders of the same wave-length is not altered; but when the dispersion is reduced by the prism, all the lines belonging to the same order approach one another and draw away from the neighbouring orders. It is evident from fig. 1 that all but two of the lines

Fig. 1.



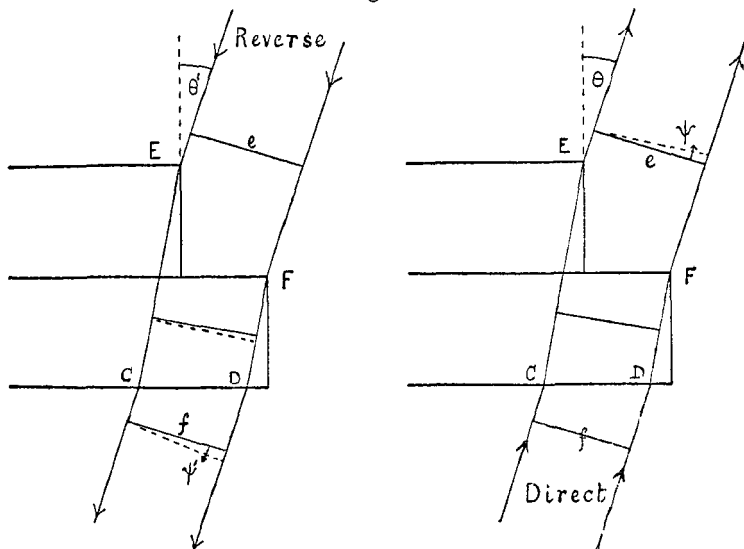
represented belong to the same order, and that these two lines, marked 1' and 1'a, belong to the next higher order. With the prism on an auxiliary spectroscopie, no evidence of this kind is obtained; and it has simply been assumed in some cases that the companion lines belonged to the bright central line nearest to them.

## PRIMARY ACTION OF THE ECHELON IN THE DIRECT AND REVERSED POSITIONS.

*Equations for the Principal Maxima.*

The theory of the echelon given by Michelson \* for the case of light falling normally on the larger end, has been extended by Galitzin †, who has investigated, both by calculation and experiment, the motion and changes in the dispersion of the spectra when the echelon is rotated through small angles within the limits of two or three degrees on either side of the normal position. As the theory of the echelon in the reversed position has not, as far as I know, hitherto been considered, a comparison will be made below with the theory of the ordinary action.

Fig. 2.



In fig. 2 the reversed and direct cases are represented diagrammatically. The rays and wave-fronts drawn with full

\* A. A. Michelson, *Astrophysical Journal*, viii. pp. 36-47 (1898). *American Journal* [4] v. pp. 215-217 (1898). *Journ. de Phys.* [3] viii. pp. 305-314 (1899).

† Fürst B. Galitzin, *Bulletin de l'Académie Impériale des Sciences de St. Pétersbourg*, 5 série, t. xxiii. pp. 67-118 (1905).

lines are those regularly refracted, the dotted wave-fronts are drawn perpendicular to the directions in which a maximum of order  $n$  is formed. The angle between this direction and the normally refracted rays is called  $\psi$  in the direct case and  $\psi'$  in the reversed case. The distance apart in the air of regularly refracted rays passing through corresponding points E and F of neighbouring step-faces is called  $f$  at the end of the echelon and  $e$  on the step side. The condition for a principal maximum is that the sum of the distances of E from the incident wave-front on one side and the dotted wave-front on the other, shall be  $n$  wave-lengths greater than the sum of the distances of the corresponding point F from the same wave-fronts.

When the angles  $\psi$  and  $\psi'$  are sufficiently small, we may employ Fermat's principle and measure the optical paths along the regularly refracted rays. This gives the general equations in the form

$$R - e\psi = n\lambda, \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$R - f\psi' = n\lambda, \quad . \quad . \quad . \quad . \quad . \quad (1A)$$

where  $R$  is the retardation produced in a regularly refracted ray by its passage through a single plate. Here the equations referring to the reversed case are distinguished by a letter A.

The values of  $R$ ,  $e$ , and  $f$  are given below, and are plotted in fig. 4 (p. 831).

$$R = t \left( \mu \sqrt{1 - \frac{\sin^2 \theta}{\mu^2}} - \cos \theta \right), \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$e = s \cos \theta + t \sin \theta, \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$f = s \cos \theta + \frac{t}{\mu} \sin \theta - \frac{\cos \theta}{\sqrt{1 - \frac{\sin^2 \theta}{\mu^2}}}, \quad . \quad . \quad . \quad (4)$$

Here  $s$  is the width of the step-faces,  $t$  the thickness of the plates, and  $\theta$  the angle of incidence of the light on the plates. This angle is in practice generally kept within the narrow limits of  $\pm 2^\circ$ , and it is sufficient to retain only the lowest

powers of  $\theta$ . These expressions then become

$$R = R_0 \left( 1 + \frac{\theta^2}{2\mu} \right), \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$e = s + t\theta, \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

$$f = s + t \frac{\theta}{\mu}, \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

$R_0$  (the value of  $R$  when  $\theta=0$ ) is  $(\mu-1)t$ . The parabolic expression for  $R-R_0$  in equation (5) gives a very close approximation to the value given by (2), the difference, which depends on  $\theta^4$ , being only 1 part in 800 when  $\theta=5^\circ$ . The slight deviations of  $e$  and  $f$  from the linear expressions (6) and (7) may be detected in fig. 4.

Substituting the approximate expressions for  $R$ ,  $e$ , and  $f$  in equations (1) and (1A), we obtain the general equations in the form:

$$R_0 \left( 1 + \frac{\theta^2}{2\mu} \right) - s\psi \left( 1 + \frac{t}{s} \theta \right) = n\lambda, \quad . \quad . \quad . \quad (8)$$

$$R_0 \left( 1 + \frac{\theta'^2}{2\mu} \right) - s\psi' \left( 1 + \frac{t}{s} \frac{\theta'}{\mu} \right) = n\lambda. \quad . \quad . \quad . \quad (8A)$$

Equation (8) is in agreement with Galitzin's calculated values and has the support of his measurements which were made on orders close to the position of greatest brightness so that the angle  $\psi$  was always small.

If the principal maxima several places removed from the direction of the regularly refracted rays are considered, it becomes necessary to take into account the terms depending on  $\psi^2$ , which were neglected in applying Fermat's principle. When the path of the diffracted light is measured along the diffracted rays, the equation for the direct case may be written

$$R - e \sin \psi + (t \cos \theta - s \sin \theta)(1 - \cos \psi) = n\lambda. \quad . \quad (9)$$

If we now suppose the light to retrace its path, the angle of incidence,  $\theta'$ , on the step-faces is equal to  $\theta - \psi$  (see fig. 2), and the angle  $\theta' + \psi'$  at which the diffracted rays leave the end plate is equal to  $\theta$ . Hence the equation for the reversed

case may be found by substituting  $\theta' + \psi'$  for  $\theta$  and  $\psi'$  for  $\psi$  in equation (9) after substituting for  $R$  its value from equation (2). The equation thus obtained may be written in the form:

$$t(\sqrt{\mu^2 - \sin^2(\theta' + \psi')}) - \cos \theta' - s\{\cos \theta' \sin \psi' - \sin \theta'(1 - \cos \psi')\} = n\lambda. \quad (9A)$$

If the terms whose order in  $\theta$  and  $\psi$  is higher than the second are neglected, equations (9) and (9 A) reduce to

$$R_0\left(1 + \frac{\theta^2}{2\mu}\right) - s\psi\left(1 + \frac{t}{s}\theta - \frac{1}{2}\frac{t}{s}\psi\right) = n\lambda. \quad (10)$$

$$R_0\left(1 + \frac{\theta'^2}{2\mu}\right) - s\psi'\left(1 + \frac{t}{s}\frac{\theta}{\mu} + \frac{1}{2}\frac{t}{s}\frac{\psi'}{\mu}\right) = n\lambda. \quad (10A)$$

These equations only differ from (8) and (8 A) by the addition of small terms depending on  $\psi^2$  and  $\psi'^2$ , whose values are given in Table I. below.

*Position of the Orders in the Field of View  
and Dispersion.*

According to the first approximation formulæ (1) and (1A)

$$\psi = \frac{R - n\lambda}{e} \quad \text{and} \quad \psi' = \frac{R - n\lambda}{f}.$$

$R$ ,  $e$ , and  $f$  depend on  $\theta$  but not on  $\psi$ ; so the orders are equally spaced and the dispersions, given by

$$\frac{d\psi}{d\lambda} = -\frac{n - t\frac{d\mu}{d\lambda}}{e} \quad \text{and} \quad \frac{d\psi'}{d\lambda} = -\frac{n - t\frac{d\mu}{d\lambda}}{f},$$

do not vary along the spectrum.

The second approximation formulæ (10) and (10 A) give

$$\psi = \frac{R - n\lambda}{s\left(1 + \frac{t}{s}\theta - \frac{1}{2}\frac{t}{s}\psi\right)} \quad \text{and} \quad \psi' = \frac{R - n\lambda}{s\left(1 + \frac{t}{s}\frac{\theta}{\mu} + \frac{1}{2}\frac{t}{s}\frac{\psi'}{\mu}\right)}$$

If the echelon is rotated a little, so that one order, say the  $n$ th, is in the position of maximum brightness where



$\psi$  or  $\psi' = 0$ ,

$$\psi = \frac{m-n}{1 + \frac{t}{s} \theta - \frac{1}{2} \frac{t}{s} \psi} \left( \frac{\lambda}{s} \right) \text{ and } \psi' = \frac{m-n}{1 + \frac{t}{s} \frac{\theta}{\mu} + \frac{1}{2} \frac{t}{s} \frac{\psi'}{\mu'}} \left( \frac{\lambda}{s} \right)$$

Table I. gives the values of the small terms in the denominators depending on  $\psi$  and  $\psi'$  for the first five orders on either side of the central order.

TABLE I.

$m \sim n.$	$\frac{1}{2} \frac{t}{s} \psi.$	$\frac{1}{2} \frac{t}{s} \frac{\psi'}{\mu}.$
1 .....	·003	·002
2 .....	·005	·003
3 .....	·008	·005
4 .....	·010	·006
5 .....	·013	·008

*The Wave-Length Intervals between the Orders.*

The repetition of a line in a number of orders provides an echelon spectrum with a wave-length scale of nearly equal divisions, and the wave-length value of these divisions,  $\Delta\lambda$ , is a constant of the echelon which can be calculated for any wave-length from the thickness of the plates and the refractive indices given for the glass. The expressions for  $\Delta\lambda$  for the direct and reversed cases may be obtained from the general equations. Neglecting the terms depending on  $\theta^2$ , which do not affect the values by more than one part in ten thousand, they both reduce to

$$\Delta\lambda = \frac{\lambda}{n - t \frac{d\mu}{d\lambda}}.$$

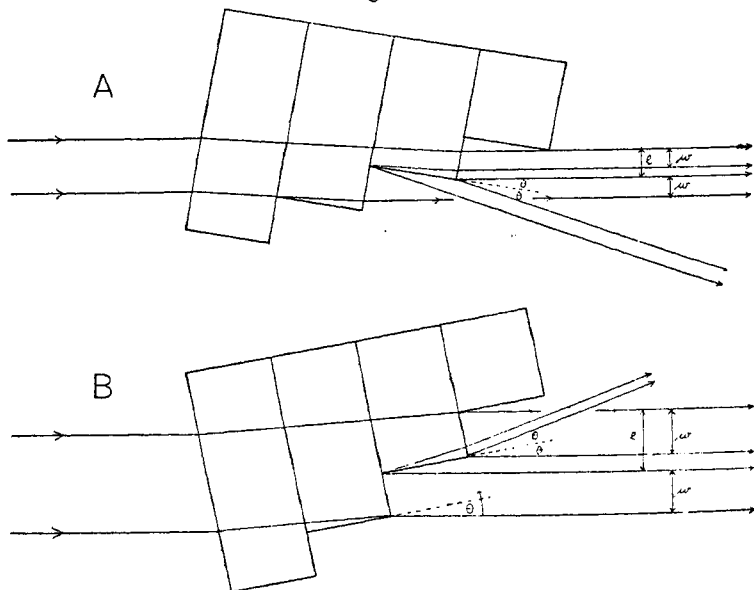
The changes that are made in  $n$  by rotating the echelon are so small compared with its whole value (10,070 for this echelon and the green mercury light) that  $\Delta\lambda$  is sensibly constant for all positions. On the other hand,  $\Delta\lambda$  may be increased or decreased by 2 per cent. by means of the auxiliary prism, as described on p. 824.

NOTES ON CHANGES PRODUCED BY ROTATING THE ECHELON  
ABOUT A VERTICAL AXIS.

*Changes in brightness, and the position of greatest brightness —*  
As an order is moved across the field of view by rotating the echelon, it crosses the lateral maxima of the distribution of light due to the individual step apertures, disappearing as it crosses the diffraction minima, and having its greatest brightness as it crosses the central diffraction maximum. This position of greatest brightness corresponds to the direction of the regularly refracted rays, and is the origin from which the angle  $\psi$  is measured. It does not quite coincide with the position of the image of the slit when the echelon is removed, the position of greatest brightness being displaced towards the side on which the step-faces of the echelon lie. This displacement indicates that the echelon-plates are slightly prismatic.

*Changes in dispersion and retardation.—*Some of the effects produced by rotating the echelon about a vertical

Fig. 3.

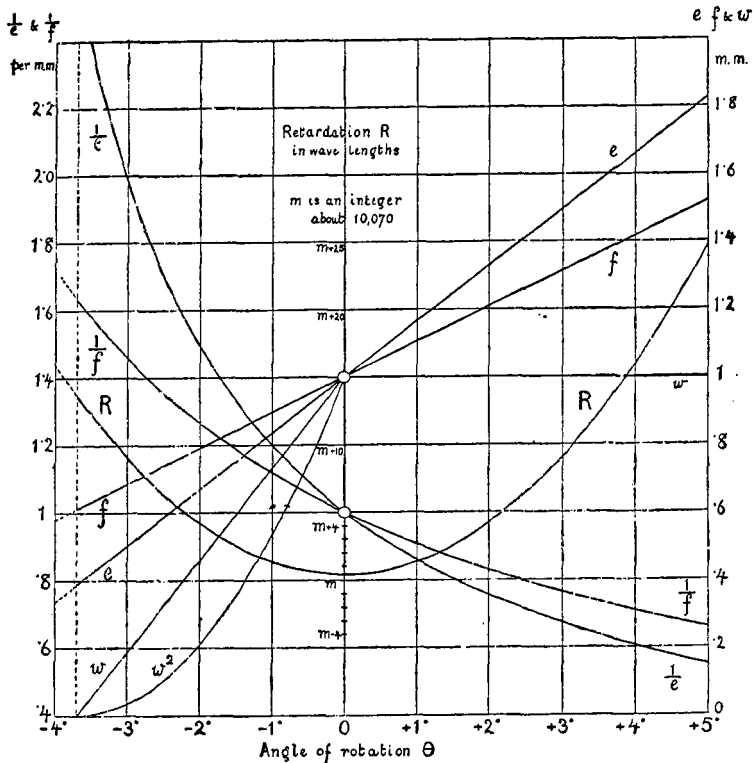


axis are represented in fig. 3. The echelon is shown at A

rotated so as to make  $\theta$  negative; in this case  $e$  is smaller and therefore (see p. 828) the dispersion is larger than in the normal position. B shows the echelon rotated in the opposite direction so that  $e$  is increased and the dispersion decreased.

The way in which  $R$ ,  $e$ , and  $f$  change with  $\theta$  is represented in fig. 4. The scale for  $R$  is given in wave-lengths,  $m$  being the number of wave-lengths in the retardation of the order

Fig. 4.



which is nearest to the position of greatest brightness on the lower retardation side when  $\theta=0$ . The positions of the row of marks on the  $\theta=0$  line represent, by their distances from the apex of the curve, the distances of the orders near the  $m$ th from the position of greatest brightness when  $\theta=0$ , and the abscissæ of the curve passing through the marks which

lie within it, give the angle through which the echelon must be turned in either direction in order to bring orders higher than the  $m$ th up to the position of greatest brightness.

The reciprocals of  $e$  and  $f$  are also plotted to show how the dispersion is changed by the rotation in the direct and reversed cases. These changes in dispersion are simply changes in magnification, as they are not accompanied by any sensible change in the wave-length intervals between the orders.

The way in which the effective width,  $w$ , of the individual step apertures is reduced when the echelon is rotated so as to make  $\theta$  negative, is represented in figs. 3 and 4;  $w^2$  is also plotted in fig. 4, to indicate the falling off in the intensity of the light on this account.

*Width of the diffraction-bands.*—It will be seen from fig. 4 that both  $e$  and  $f$  are greater than  $w$ , except when  $\theta=0$ .

Hence, the angular interval,  $\frac{\lambda}{w}$ , between the equally spaced diffraction minima is in general greater than  $\frac{\lambda}{e}$  or  $\frac{\lambda}{f}$ , the angular intervals, in the direct and reversed cases, between successive orders.

*Light reflected from the ends of the plates.*—The step ends of the echelon plates, which are finely ground, scatter some of the light falling upon them and also give regularly reflected beams, represented in fig. 3. The reflected beams produce echelon spectra; but as the ends are not polished, the spectra are poorly defined and the spreading of the light brings into greater prominence the broad diffraction-bands corresponding to the narrow sources represented by the illuminated end-faces of the plates. When the echelon and prism are used together, as in fig. 1, the reflected spectrum of the green mercury-line may be made to overlap the direct spectrum of the violet line by giving  $\theta$  a small negative value. I think it would be an advantage to have the ends of the plates polished or blackened.

*The position of minimum deviation.*—As the echelon is rotated from one side of the normal position to the other, the orders first move across the field of view in the direction of decreasing deviation (measured by  $\psi$ ) and then turn round and go back again. If the value of  $R$  happens to be a whole

number of wave-lengths for normal incidence, say  $m\lambda$ , then the  $m$ th order will come to its position of minimum deviation in the position of greatest brightness when  $\theta=0$ , but higher and lower orders will not then be quite in their positions of minimum deviation. Consider, for example, a lower order:  $\psi$  (or in the reversed case  $\psi'$ ) will be positive, and its value when  $\theta=0$  can be reduced a little by increasing  $\theta$  (in the positive direction), as at first the value of  $R$  is almost stationary, while  $e$  is increasing, and therefore the dispersion is decreasing.

By differentiating the general equations (1) and (1 A), it will be found that the conditions for the turning-points in the direct and reversed cases are,

$$\frac{dR}{d\theta} = \psi \frac{de}{d\theta} \quad \text{and} \quad \frac{dR}{d\theta} = \psi' \frac{df}{d\theta}.$$

Substituting the values of the differential coefficients and calling the minimum values of the deviation in the two cases  $\psi_M$  and  $\psi'_M$  and the corresponding angles of incidence  $\theta_M$  and  $\theta'_M$ ,

$$\theta_M = \frac{\mu}{\mu-1} \psi_M \quad \text{and} \quad \theta'_M = \frac{1}{\mu-1} \psi'_M.$$

Hence as the echelon is rotated, so as to increase  $\theta$ , the orders higher than the  $m$ th come to their minimum deviation positions before the echelon is normal to the light, and the lower orders have their minimum deviation after the normal position is passed. The central orders are very nearly in their positions of minimum deviation when  $\theta=0$ . If  $\psi_0$  and  $\psi'_0$  are written for the values of  $\psi$  and  $\psi'$  in this case, it may be shown that

$$\frac{\psi_0 - \psi_M}{\psi_M} = \frac{1}{2} \frac{t}{s} \frac{\mu}{\mu-1} \psi_M \quad \text{and} \quad \frac{\psi'_0 - \psi'_M}{\psi'_M} = \frac{1}{2} \frac{t}{s} \frac{1}{\mu-1} \psi'_M,$$

and the values calculated from these formulæ show that, when this echelon is in the direct position,  $\psi_0 - \psi_M$  is .7 per cent. of  $\psi_M$  for an order one place from the position of greatest brightness, 1.4 per cent. of  $\psi_M$  for an order two places away, and so on. The corresponding values for the reversed position are .4 per cent. and .9 per cent.

*Effects produced by Temperature Changes.*

The position of the various orders in the field of view when the echelon is in a definite position, such as the normal position, depends on the temperature of the echelon and on the refractive index of the surrounding air. Records of the temperature of the echelon and micrometer readings of the position of the orders in the field of view, taken from day to day, show that a rise of temperature of  $8.6^{\circ}$  C. moves the spectrum through a distance equal to the interval between neighbouring orders, indicating an increase of one wave-length in the retardation,  $R$ , for this rise of temperature.

*Curving of the Spectrum Lines.*

The echelon spectrum lines, like those of a prism spectrum are curved with the concave side toward the violet end. The curving is due to the variation in the angle of incidence, and therefore in the retardation,  $R$ , of light from different points along the length of the slit. The theory has been given by Laue\*.

Consider the simplest case, in which the slit is vertical and the plates are normal to the axis of the collimator: then light from points above and below the centre of the slit will have a vertical plane of incidence on the plates. Writing  $i$  instead of  $\theta$  for the angle of incidence in equation (3),

$$R - R_0 = \frac{1}{2} t \frac{\mu - 1}{\mu} i^2;$$

but from equation (1),

$$R - R_0 = e(\psi - \psi_0);$$

and in this case  $e = s$ , so that

$$\psi - \psi_0 = \frac{1}{2} \frac{t}{s} \left( \frac{\mu - 1}{\mu} \right) i^2. \quad . \quad . \quad . \quad (8)$$

Hence, for small values of  $i$ , the spectrum lines are parabolas. The curvature is very small. If, for example, the extreme values of  $i$  are ten times the angular separation of the orders,

\* *Physikalische Zeitschrift*, vi. pp. 283-285 (1905).

$\Delta\psi$ , then the top and bottom of the curved image of the slit, representing one order of the spectrum, will be 2 per cent. of  $\Delta\psi$  to the violet side of the centre of the image.

*Effects produced by Rotating the Echelon about a Horizontal Axis parallel to the Plates.*

The echelon can readily be tilted in this way by placing a block under the foot at the small end. The block which I employ tilts the echelon through an angle of nearly  $3^\circ$ , and the photographs Nos. 1 to 3 in Pl. XXXI. of the green mercury line were taken with this angle of tilt. The horizontal axis of the parabolic lines, which passes through the normal to the plates, has been raised much above the field of view of these photographs, so that the lines where they cross the field of view are considerably inclined to the vertical. The reproductions in Pl. XXXI. have the centre of the curves below them, as they have been turned round in order to put the shorter wave-lengths on the left.

Part II.—SECONDARY ACTION OF THE ECHELON.

*Secondary Bands in the Primary Spectrum.*

The inclined spectrum lines of photograph No. 1, Pl. XXXI. are broken up and have a rosy or screw-like appearance because they are crossed by a secondary system of bands. This screw-like structure of echelon spectrum lines was observed by Gebrocke\*. He does not explain how the new bands are produced, but shows that the appearance can be imitated by tilting an echelon with only two apertures, so that the echelon bands slope across the central vertical diffraction band.

When the echelon is in the ordinary position the secondary bands are parallel to the spectrum lines, and so their effects, though very important, are not so easily recognized as they are in this photograph taken with the echelon tilted.

*Character of the Secondary Bands.*

The secondary bands are superposed on the echelon lines and resemble them in appearance; they are also affected in the

\* *Annalen der Physik*, xviii. p. 1074 (1905).

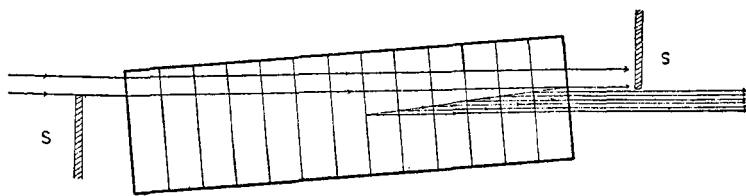
same way, but to a greater extent, by adjustments of the echelon. When the echelon is rotated, for example, the secondary bands move faster than the spectrum lines in the same direction and so move across them. When the echelon plates are vertical, the secondary bands, like the echelon lines, are vertical in the centre of the field; they are also curved in the same direction but more strongly. Their width is about the same as that of the finer spectrum lines, and so, in the ordinary position of the echelon, they are not easily recognized; but when the echelon is tilted they become more inclined than the spectrum lines in the same direction and can then be seen as in photograph No. 1, Plate XXXI., intersecting all the spectrum lines.

The behaviour of the secondary bands suggested the idea that they might be spectrum lines of a higher order, such as might be produced by the reflexion of light in the echelon.

*Fabry and Perot Spectrum produced by the Secondary Action.*

The secondary light, which has been twice reflected in the echelon, is by no means negligible. The echelon was tilted as shown in fig. 5, screens SS being arranged to cut out the

Fig. 5.



primary light, and it was found that the echelon was bright with secondary light coming out below the second screen, the brightness extending down to the bottom of some of the step apertures.

Suppose that each interface reflects a very small proportion of the light falling upon it and leave out of account for a moment the step structure of the echelon. There will be a secondary beam produced by the combination of the  $n$  faint secondary beams which have gone back through one plate,  $n$



being the number of plates coming into action; let the retardation of this beam be  $\Delta$ . There is another secondary beam whose retardation is  $2\Delta$ , made up of  $n-1$  faint beams which have each gone back through two plates, and so on. Hence the secondary action of the pile of plates in the echelon is similar to that of a Fabry and Perot spectroscope, and, under suitable conditions of illumination, the secondary light by this action would be thrown into a ring spectrum of a high order. The retardation is  $2\mu t$  for normal incidence, so it is  $\frac{2\mu}{\mu-1}$  times as great, in this case about five and a half times as great, as that of the primary spectra.

The secondary light also undergoes the ordinary echelon treatment as it leaves by the step-faces, and it is therefore confined to the primary spectrum lines.

The photograph No. 2, Pl. XXXI., which shows short portions of the rings of the Fabry and Perot spectrum crossing the echelon spectrum, was obtained by stopping the primary light in the way described above, and making the echelon lines broad by widening the slit. The spectrum lines produced by the primary action of the echelon upon the secondary light are inclined because the echelon is tilted as shown in fig. 5, and the centre of the ring system is above the field of view for the same reason. The echelon has also been rotated (in the positive direction of  $\theta$ ), so the centre of the ring system is displaced laterally as well as vertically.

The Fabry and Perot spectrum lines are not dependent for their definition, like the echelon spectrum lines, on the narrowness of the slit; their want of clearness in this photograph is partly due to the overlapping of lines belonging to different orders, which is produced by the orders overlapping four deep.

#### *The Secondary Point Spectrum.*

If the slit, instead of being opened wide to show portions of the rings, is made narrow enough to give good definition to the primary echelon action, the secondary light which has undergone both actions is confined to dots indicating the position of the points of intersection of the spectrum lines in

one system with corresponding lines, that is lines representing the same wave-length, in the other system.

The horizontal dispersion of the dots, given by the ordinary echelon action, now prevents overlapping, as the wave-length interval between the orders in this system is a little greater than the length of the spectrum, and the secondary light gives in this point spectrum, therefore, the advantages of a very high order without the usual overlapping.

Gehrcke and Baeyer \* obtained similar spectra which they call interference points, by crossing plane parallel plates, and have pointed out the advantages of combining two independent high dispersions.

A photograph of the secondary point spectrum, obtained with an exposure of one hour, is reproduced in Plate XXXI., No. 3. The echelon had the usual tilt, about  $3^\circ$ , and the echelon table was rotated about  $2^\circ$  in the positive direction of  $\theta$  from the normal position. The dispersion in the Fabry and Perot system is, in this case, twelve times that in the echelon system, so the wave-length intervals can be best determined by the position of the dots in the former system, while the dots can be recognized and their wave-lengths can be roughly fixed by their position in the latter system.

There was no difficulty in recognizing the dots marked 1, 2, 3, 4, 5, 6, and 7 in the photograph, which represent well-known lines in the green line spectrum.

In order to determine the wave-length intervals, it is necessary to calculate  $\Delta\lambda$ , the wave-length interval between neighbouring orders of the Fabry and Perot spectrum. It was found from the formula

$$\Delta\lambda = \frac{\lambda}{p - 2t \cos r \frac{d\mu}{d\lambda}},$$

where  $p$ , the order of the spectra, is given by

$$p = \frac{2\mu t \cos r}{\lambda},$$

$r$  being the angle of refraction of the light in the plates, and  $t$  their thickness. The value found in this way for the centre

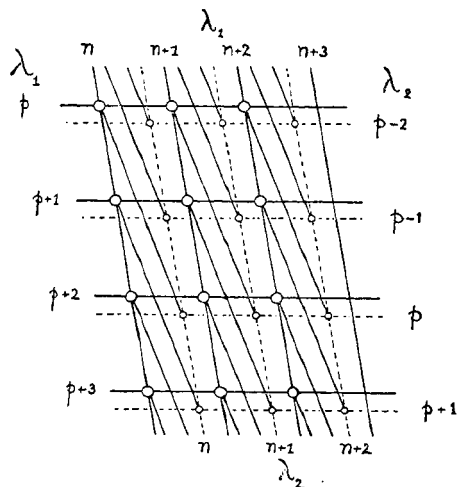
\* E. Gehrcke and O. v. Baeyer, *Annalen der Physik*, xx. p. 269 (1906).

of the photograph is  $96.5 \text{ m.}\text{\AA}$ . The wave-length intervals between the components deduced from the measurements of the photograph and this value of  $\Delta\lambda$  are given in Table III., p. 393; most of them agree closely with the values which have been found by other methods.

*Character of the Point Spectra produced by the  
Secondary Light.*

Fig. 6 is a diagram representing the type of spectrum produced by the secondary light when the echelon is in the direct position and tilted about a horizontal axis, that is, the

Fig. 6.



type represented in the photograph of the secondary point spectrum No. 3, Pl. XXXI.; but in order to make the diagram clearer, the ratio of the dispersion in the Fabry and Perot system to the dispersion in the primary echelon system has been made much smaller than it is in the photograph.

In this case all the rays of light from a given point of the slit are parallel to one another during their passage through the plates, as the lateral diffraction does not take place until the light leaves the echelon by the step-faces. The spectra, or lines of constant retardation, in the Fabry and Perot system are therefore drawn in the diagram as horizontal lines.

When the photograph was taken the echelon had been rotated (from the normal position about a vertical axis) as well as tilted, so the plane parallel to the axis of the collimator and the diffracting apertures would not be quite vertical, but the deviation from the vertical is a small angle of the second order (equal to the product of the small angles of tilt and rotation), and it will be seen that lines joining two dots representing the same wave-length and of the same order in the Fabry and Perot system, would be sensibly horizontal, although if the slit had been opened wide so as to show portions of the Fabry and Perot rings, they would have been inclined about  $45^\circ$ , as in photograph No. 2.

When the echelon is reversed, the lateral diffraction takes place as the light enters the echelon and the lines of equal retardation in the Fabry and Perot system will be represented by circles whose centre is the point in the image plane corresponding to the direction of the normal to the plates.

The four long inclined lines in the diagram (fig. 6) represent spectra in the primary echelon system; they are inclined to the vertical because the echelon is tilted. The orders of a wave-length  $\lambda_1$  are represented in the two systems by full lines, the dotted lines representing similarly the spectra of a second wave-length  $\lambda_2$  greater than  $\lambda_1$ . The order of the spectra in the two systems is indicated at the ends of the lines. The points where the full lines of the two systems intersect give a series of points (marked by the larger circles in the diagram) which represent  $\lambda_1$  in the joint system, each point being defined by two orders. The top left-hand point in fig. 6 may be described for example as the  $np$  order of wave-length  $\lambda_1$ . In the same way the intersections of the dotted lines give the positions where  $\lambda_2$  is represented in the joint system, and the shorter inclined lines joining the  $\lambda_1$  and  $\lambda_2$  points of the same denomination represent the appearance of the joint spectra corresponding to a spectrum continuous between these limits.

It will be seen that there is no chance of spectra of different denominations overlapping in the joint system as long as there is no overlapping in one of the two systems which are combined.

The spectra in the two systems may be regarded as forming

an oblique system of coordinates ; the echelon system giving horizontal dispersion may be called the X system and the Fabry and Perot system the Y system. Then the slope of the spectra in the joint system,  $\frac{dy}{dx}$ , is the ratio of  $\frac{\partial \lambda}{\partial x}$ , the dispersion in the X system, keeping  $y$  constant, to  $\frac{\partial \lambda}{\partial y}$ , the dispersion in the Y system, keeping  $x$  constant.

One important feature of these point spectra is that they give a system of lines whose definition depends in general on the defining power of the two systems which are combined, but does not depend on the smallness of the range of wave-length in the light examined, so long as that range does not exceed a certain relatively large limit.

If the definition is poor in one of the two systems, a monochromatic radiation would be represented by dots elongated in the direction of the spectrum lines of the other system, and this would in general spoil the sharpness of the lines representing in the joint system a spectrum continuous between narrow limits ; but if these lines are nearly parallel to the spectrum lines of one system, the want of definition in the other will not spoil the definition of the lines, as each dot representing a single wave-length will be drawn out in a direction nearly parallel to the length of the joint spectra. This special case is realized when the echelon is in the direct or reversed normal positions.

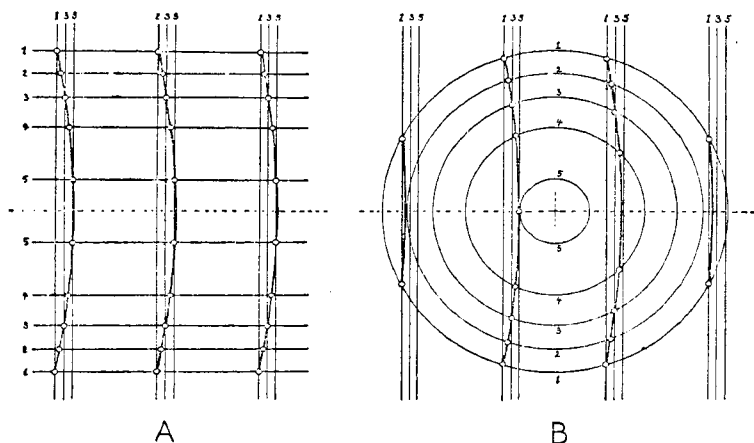
The diagrams A and B in fig. 7 represent the conditions in these cases. They are similar to fig. 6, but spectrum lines have been drawn in the Fabry and Perot system for a series of five wave-lengths whose values increase by equal increment from  $\lambda_1$  to  $\lambda_5$ . In the primary echelon system the lines representing alternate wave-lengths have been omitted. B represents the case in which the echelon is in the reversed position, the radii of the circles, representing the spectra in the Fabry and Perot system, being chosen so that their squares increase by equal increments. The horizontal lines in diagram A represent the same spectra when the echelon is turned round into the direct normal position.

For convenience in drawing the diagrams, the dispersion in the Fabry and Perot system has been represented as only

about ten times as great as that in the primary echelon system. It should be several hundred times as great even for those parts of the diagram farthest from the centre.

It will be seen that in both diagrams the secondary point spectra, drawn through the points of intersection of lines

Fig. 7.



representing the same wave-length in the two systems which are combined, are curved so as to be concave towards the side of shorter wave-length in the primary echelon system; the curvature has been much exaggerated by taking the ratio of the dispersions so much smaller than it actually is.

#### *Origin of the Secondary Bands.*

There is no doubt that the secondary bands which are observed when the echelon is employed in the usual way, that is, with the secondary light superposed on the primary, are very closely connected with the point spectra produced by the secondary light. All the characteristics of the secondary bands described on page 836 may be explained by supposing that they are, like the point spectra of the secondary light, the loci of the points of intersection of lines representing the same wave-length in the primary echelon and Fabry and Perot spectra.

Another characteristic which may be explained in the

same way, with the help of fig. 7, is that when the echelon is in the direct normal position the secondary bands affect each order of the spectrum in the same way, while, when the echelon is reversed, there are marked differences between the different orders.

The secondary bands have other characteristics which may find their explanation in interference taking place between the secondary light and the primary. The dark secondary bands appear to cut through the bright primary spectra. This is shown to some extent in photograph No. 1, Pl. XXXI., but it is much more marked when the secondary light is made relatively stronger by covering half the echelon apertures so as to stop all the light which has passed through less than half the plates in the echelon. The brightness of the primary lines is no doubt increased above and below the dark bands, making them appear darker by contrast, but I think there is little doubt that there has been an actual reduction in the brightness of the primary light in the dark secondary bands.

Another feature of the secondary bands, which may be seen in photograph No. 1, Pl. XXXI., where they cross the line 1, is that they always appear in pairs, fainter and stronger bands alternating with one another. This may be connected with the difference in phase between the primary and the secondary light produced by the two reflexions of the latter. Apart from these phase changes at reflexion, the primary and secondary light would be in phase at the points of intersection of their maxima, as the retardations in both systems are whole numbers of wave-lengths in the direction in which the maxima are formed. There are two cases to be considered for the secondary light, according as it has undergone both reflexions at interfaces or one reflexion at an interface and the other at an external surface. In the former case, as the air-films are very thin, it is possible that a change of phase of  $\frac{\pi}{2}$  may be introduced at each reflexion which would give the best phase conditions for interference between the maxima. The latter case may account for the second set of bands shifted relatively to the first.

PART III.—STRUCTURE OF THE GREEN MERCURY LINE.  
(5461.)

*Description of Spectrum given by an Arons Lamp, and  
Comparison with the Results obtained by other Observers.*

This spectrum consists of a bright principal line, which is a close double, with six companion lines, three on either side. A photograph of the primary echelon spectrum is reproduced in Plate XXXII. together with a diagram of the spectrum. The photograph shows, in addition to the genuine components, a number of faint lines which have their origin in the echelon. The genuine components are numbered from 1 to 8, and the false lines are marked 1 *a*, 1 *b*, 3 *a*, &c. The lines 1 *a*, 1 *b*, 1 *c*, &c., mark the positions of the secondary diffraction maxima on the longer wave-length side of the principal diffraction maximum 1, and the lines 3 *a*, 5 *a*, 6 *a*, 7 *a*, and 8 *a* represent the first secondary maxima on the longer wave-length side of the lines 3, 5, 6, 7, and 8. When the aperture of the echelon is reduced by covering the first ten step-faces at the smaller end, the faint lines move away from their parents into the new positions of the secondary maxima corresponding to the reduced number of apertures.

The numbers in Table II. and Table III. give the distances of the various components from the component of shortest wave-length. I adopted this method of measuring the positions because the bright central line appeared to be the most variable component, while the component of shortest wave-length is a good reference line. The results of the other observers, given in the Tables, are not convenient for comparison in their original form, because Gehrecke and Baeyer\*, Janicki†, and Galitzin and Wilip‡, give the distances from the centre of the principal line, while Fabry and Perot§, Baeyer|| (in a later paper), and Nagaoka¶, divide

\* F. Gehrecke and O. Von Baeyer, *Annalen der Physik*, vol. xx. p. 269 (1906). † L. Janicki, *Annalen der Physik*, vol. xix. p. 36 (1906).

‡ Fürst B. Galitzin und J. Wilip, *Mémoires de l'Académie Impériale des Sciences de St. Pétersbourg*, sér. 8, vol. xxii. no. 1 (1906).

§ Astrophysical Journal, xv. p. 218 (1902).

|| O. v. Baeyer, *Verhandlungen der Deutschen Physikalischen Gesellschaft*, ix. no. 4, p. 84 (1907). ¶ Nagaoka, 'Nature,' vol. lxxvii. p. 582 (1908).



the principal line into two components and measure the distances from the brighter one.

Another difficulty in the comparison is that the values obtained by Gehrcke and Baeyer, and later by Baeyer, by the use of crossed plane-parallel plates, are systematically higher than the values obtained with echelon spectroscopes, which agree fairly well amongst themselves. I have reduced all Baeyer's intervals 5 per cent., and the earlier values of Gehrcke and Baeyer (the means of three sets of measurements given in their paper) by 3 per cent.; and it will be seen that, apart from these differences in the constants, the two independent methods are in close agreement.

TABLE II.—Measurements of the Green Mercury-line Spectrum.

The distances of the component lines from the component of shortest wave-length are given in milli-Ångström units.

Fabry & Perot.	Crossed Plates.		Echelon Spectroscopes.		Reference Numbers.
	Gehrcke & Baeyer.	Baeyer.	Janicki.	Galitzin & Wilip.	
·01 Å. units	reduced 3 p.c.	reduced 5 p.c.			
0	0	0	0	0	1
15	126	136	133	137	2
17	169	169	166	168	3
...	...	189	...	189	...
<u>22</u>	} <u>234</u> {	214	} <u>232</u> {	<u>236</u>	4
23		<u>238</u>			5 } Central Band.
31	318	320	320	321	6
36	364	363	365	365	7
...	...	449	...	...	8

The brightest component in each case is underlined.

Fabry and Perot's values\* given in the Table are those published by Zeeman† in 1902. They help to confirm

\* M. Perot informs me (in a letter dated Oct. 13th, 1908) that they have not published any particulars of this line since then.

† Astrophysical Journal, xv. p. 218 (1902).

TABLE III.—Measurements of the Green Mercury-line Spectrum (*continued*).

Nagaoka.	Author.			Reference Numbers.
	Primary Spectrum.	Secondary Point Spectrum.	Width.	
0	0	0	17	1
31	...	...	...	...
72	...	...	...	...
105	...	...	...	...
137	135	141	13	2
163	164	167	13	3
189	...	...	...	...
223	<u>232</u> { (217) (243)	217	16	4
<u>247</u>		<u>243</u>	24	5
280	...	...	...	...
315	319	322	17	6
356	363	365	12	7
390	...	...	...	...
448	448	...	14	8
477	...	...	...	...

the accuracy of the constants employed in the echelon calculations.

The results obtained by several other observers are given by Janicki\*.

The series of faint lines given by Nagaoka resembles the series of false lines in my primary echelon photographs. I published my measurements for comparison with his before discovering that the faint lines in my photographs were not genuine †.

#### *Width of the Components.*

The mean values of the widths of the photographic images of the components are given in Table III. If the echelon acted perfectly, the width of a principal light maximum

\* *Loc. cit.* p. 61.

† 'Nature,' vol. lxxviii. p. 8 (1908).

constituting one order of the primary echelon spectrum of a monochromatic radiation, with a narrow slit, would be  $\frac{2}{33}$  of the interval between the orders, which corresponds in this case to a width of 30 m.Å. The narrowest lines in the photographs which still have a measurable width, are about 10 m.Å. wide. The widths of the brighter lines vary considerably with their exposure, while the width of the companion line 8, which is too faint to be overexposed, is the same on each of the three plates on which its width could be measured.

*Measurement of Secondary Spectrum.*

The results obtained by measuring the secondary point spectrum are also entered in Table III. The photograph measured was exposed for an hour, and is the one reproduced in Plate XXXI., No. 3. It will be seen that the agreement of these results, obtained by an independent method, with the ordinary echelon values, is fairly close, except for the component 2. The methods agree very closely as to the position of the central line. They both give the centre of the double line at 232 and the dividing dark line at 228. The two components of the central line were not measured separately in the primary spectrum photographs, but the position of the dark dividing line was sometimes recorded. The positions of the centres of the components given in brackets in the second column of Table III., are deduced from the position and width of the whole line and the position of the dark dividing line (neglecting its width).

*Spectrum given by a Bastian Lamp.*

A striking variation in the spectrum of the green line was observed with a Bastian mercury arc-lamp. The glass tube through which the discharge passes is bent nearly into the form of an S in a horizontal plane, so that when one part of the discharge is parallel to the slit plate, another part may be normal to it. When the image of the "end-on" portion of the discharge is put on the slit, the change in the spectrum is so great that it is difficult at first to recognize the components. On measuring a photograph of the "end-on" spectrum, however, it was found that the companion

lines keep their relative positions, although some become broader and brighter, but the dark space between 4 and 5, the two components of the principal line, is greatly increased (from 6 to 26 m.Å.) and, being no longer brighter or broader than the rest, they look like ordinary companion lines.

The "side-on" spectrum of the Bastian lamp resembles more nearly that given by the Arons lamp.

The separation of the components of the principal line in the "end-on" radiation has been investigated by Galitzin and Wilip\*, who observed the phenomenon first in the case of a Geissler tube arranged so that the axis of the discharge was normal to the slit plate.

*Spectrum of the Green Mercury line given by a hot  
Mercury lamp.*

Janicki† describes the broadening of the components of the green and yellow mercury lines which takes place when a mercury lamp is allowed to become sufficiently hot, and a peculiar system of five equidistant bands which he observed when the original components of the lines were lost in a continuous spectrum. Galitzin and Wilip‡, who give measurements of the bands, suggest that they may be due to a reversal of the lines, or perhaps to some peculiar property of the echelon spectroscope.

The theory of the secondary echelon spectra (see page 841) indicates that the secondary bands would be well defined in a short continuous spectrum, and it seemed probable that they were the bands which Janicki and Galitzin & Wilip observed.

I have tested this point with a quartz-lamp fitted with an air-manometer, similar to that described by Galitzin and Wilip§.

When an arc is started with the lamp cold, the central line (4 and 5 together) and all the companion lines are at

\* Fürst B. Galitzin und J. Wilip, "Ueber die Eigenschaften einiger Emissionslinien des Quecksilberdampfes," *Mémoires de l'Académie Impériale des Sciences de St. Pétersbourg*, sér. 8, vol. xxii. no. 1 (1907).

† *Loc. cit.* pp. 49-55.

‡ *Loc. cit.* pp. 34 & 76.

§ *Loc. cit.* p. 4.

first plainly visible, but the pressure in the lamp increases rapidly and the broadening and coalescing of the lines soon takes place. The position occupied by the bright central line in the various orders is now marked by a dark line, probably due to absorption. As the companion lines become merged in the general brightness the fine secondary bands become clearly visible in all the bright parts of the field, the fact that they are secondary bands being shown by their motion relative to the primary spectrum when the echelon is given a slight rotation. The secondary bands show clearly on the green line when the pressure in the lamp is about one atmosphere.

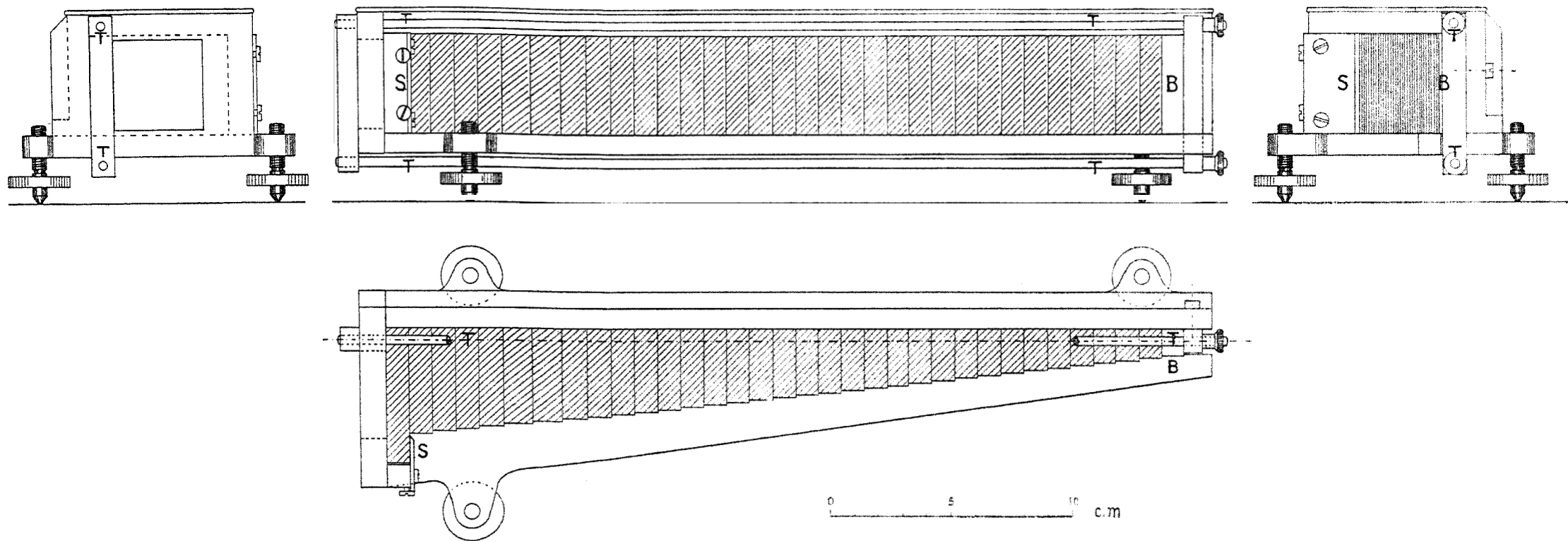
This research was commenced at the request of Professor Schuster, and I have much pleasure in acknowledging the help I have received from the interest he has taken in its progress.

I wish to thank Mr. E. Marsden for assistance in my first experiments on the secondary effects, and Mr. W. A. Harwood, B.Sc., for measuring several of the photographs. My thanks are also due both to Professor Schuster and to Professor Rutherford, for placing the resources of the Physical Laboratories at my disposal.

#### DISCUSSION.

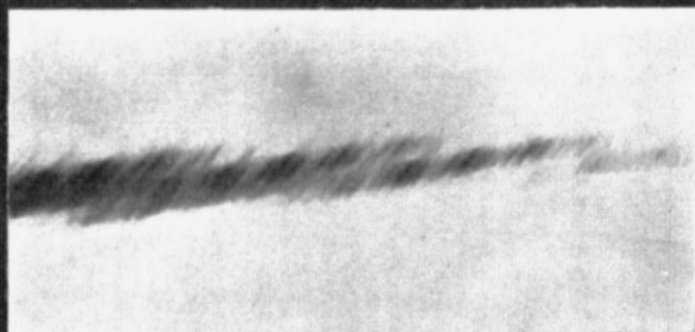
Dr. LEES referred to the importance of the secondary action and asked the Author if it was now possible to say definitely whether a line observed in an echelon spectrum is genuine or is produced by the instrument.

The AUTHOR said that Gebrcke and Baeyer had hoped to supply the means of settling doubts of this kind when they eliminated the ghosts from their green line spectrum by their method of "interference-points." Since then, however, two faint lines had been added to the list of components. With the possible exception of one faint line agreement had now been arrived at between two independent methods.

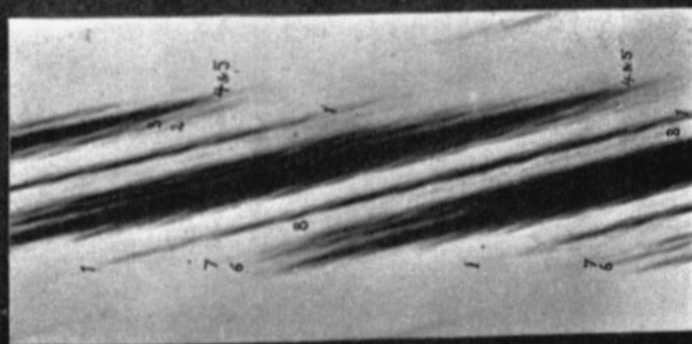




3.



2.



1.

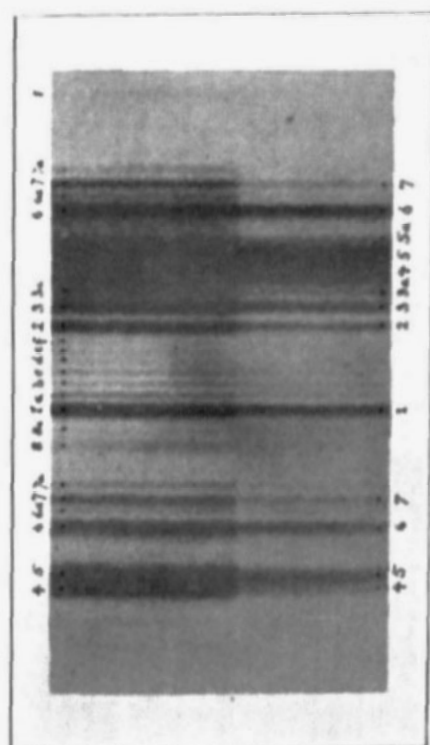


Diagram of Spectrum

