values $\begin{vmatrix} x_1-y, & x_2-y, & x_3, & 0 \\ 0\ , & 0\ , & 0, & 0 \\ 1\ , & 1\ , & 1, & 1 \end{vmatrix}$, $\begin{vmatrix} x_1-y, & x_2-y, & x_3, & 0 \\ 1\ , & 1\ , & 1, & 1 \\ 1\ , & 1\ , & 1, & 1 \end{vmatrix}$,

and adding the results: we thus find

(56) $2\theta_{0,1}(x_1-y)\,\theta_{0,1}(x_2-y)\,\theta_{0,1}(x_3)\,\theta_{0,1}(0)$

$=\theta_{0,1}(x_1)\,\theta_{0,1}(x_2)\,\theta_{0,1}(x_3+y)\,\theta_{0,1}(y)+\theta_{1,1}(x_1)\,\theta_{1,1}(x_2)\,\theta_{1,1}(x_3+y)\,\theta_{1,1}(y).$

Changing y into $-y$, dividing the first result by the second, and substituting the resulting expression of $\dfrac{\theta_{0,1}(x_1-y)\,\theta_{0,1}(x_2-y)}{\theta_{0,1}(x_1+y)\,\theta_{0,1}(x_2+y)}$ in the equation (52), we arrive at the formula (55).

Jacobi has given, in the *Fundamenta Nova*, two other expressions of this addition theorem, both of which are easily inferred from (52) by using appropriate particularisations of the formula (3).

On an Involution System of Circular Cubics, and description of the curve by points, when the double focus is on the curve.

By T. COTTERILL, M.A.

It follows from a theorem given by Chasles, that, if a system of circular cubics can pass through seven points, the tangents at each point sweep through equal angles; and consequently, if the system can break up in at least three ways into a circle and line, the circle described by the double focus or intersection of the tangents at the circular points and the conic of intersection of the tangents at any two of the other points are determined. The six intersections of four lines in a plane, and the focus of the inscribed parabola, afford an instance of such a system.

A particular case of this is given from the three points of a triangle (determined by a point on a circle and its intersections with a line), and the four centres of the circles touching its sides.

Any point on a circular cubic through seven such points determines another point on the curve, its confocal to the triangle, the line joining them being parallel to the real asymptote, so that a circular cubic is the locus of the real foci lying on parallel axes of conics inscribed to a triangle. The intersection of the tangents to the cubic at these confocal points, is the confocal of the centre of the conic. In other words, the locus of the intersection of tangents to a circular cubic at the points in which it is cut by a line parallel to its real asymptote, is the confocal of the polar conic of the real point at infinity.

The tangents at the four centres are parallel to the real asymptote. The tangents at the angles of the triangle and the real asymptote meet at the remaining intersection of the cubic and circle circumscribing the triangle, the asymptote being easily drawn, as it is the tangent in a given direction to the three-cusped hypocycloid touching the six lines through the centres, one pair of which is always real.

The double focus is on the same circle, diametrically opposite the last mentioned point.

The single foci are known to be in fours on the polar circles of the triangles formed from the centres, and found from the principles laid down in Dr. Salmon's "Higher Geometry" by means of double lines of Involution.

The cubics break up in six ways into a circle and line, one such system being orthogonal to another. The common foci (which coalesce in fours) of an orthogonal pair are the nodes of each, considered as a cubic.

When the asymptote is perpendicular to a side of the triangle, and the double focus at its opposite angle and therefore on the curve, the genesis of the curve is so easy, and its properties, both metric and graphic, become so simple, as to entitle it to hold the rank amongst cubics somewhat similar to that of the circle amongst conics.

The locus of the intersections of circles through two fixed points with their diameters through another fixed point, is a circular cubic having the last mentioned point for its double focus.

Hence it is also the locus of the points of contact of tangents through the fixed point to the orthogonal system of circles.

The envelope of the line joining two corresponding points on a 3rd order cubic, considered as a Hessian, is called by Prof. Cremona a *Cayleyan*, in honour of the discoverer of the remarkable relations of the two curves.

In the circular cubic in question, the distance between one set of pairs of corresponding points (the circular points at infinity being one pair) is bisected by the line of centres of the circles, and corresponding tangents to the Cayleyan are the orthogonal lines through any point on the Hessian, cutting the axis parallel to the line of centres through the double focus, in points equidistant from it.

The tangent at a point of the Hessian, and point of contact of a tangent to the Cayleyan, are found from the property given by Prof. Cayley; viz., that corresponding points are harmonicals to corresponding tangents.

According as the axis of the coaxial circles is real, imaginary, or evanescent, the Hessian is complex, simplex, or orthogonally crunodal, and the Cayleyan is complex, simplex, or a parabola, and the limiting point of the evanescent axis.

The Hessian is the locus of the points of contact of pairs of conics with fixed foci, the tangent and normal at the point of contact being corresponding tangents to the Cayleyan.

It is also the locus of the foci of conics touching four lines, the Cayleyan being the envelope of their axes.

October 15, 1866.

Special Meeting held at University College (Professor De Morgan in the Chair).

Professor Hirst read a letter from Sir John Lubbock with reference to a present, which he intended to make to the Society, of a considerable portion of the Mathematical Library of the late Sir John Lubbock.

It was resolved, That the warmest thanks of the Society be returned to Sir John Lubbock for his very generous and valuable gift.

A new scheme of rules, which had been drawn up by Professors Hirst and Sylvester at the request of the Council, was then read by Professor Hirst. These rules were passed with certain alterations.