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He, likewise, controverts the view that the seat of the electromotive force in gas batteries is the place of contact of metal, liquid, and gas ; but he comes to the conclusion which in the present communication I have declared is not universally valid—that the entire current of the gas battery owes its rise to the dissolved gases. At the same time he does not admit that the gradual falling-off of the current of a closed gas battery is to be attributed to polarization coming in, but seeks its cause solely in the diminution of the volume of gas dissolved in the liquid. As, however, he does not measure the electromotive forces by momentary closings of the circuit, as Gaugain and I have done, but calculates them from the current-intensity observed during a continued closing, and from the resistance, it is not possible from his measurements to distinguish the primary from the secondary actions. That a mixture of this sort has not been avoided is shown also by the proposition at which Mr. Morley arrives:—that the electromotive force of the gas battery is not constant, but increases with the resistance.

Munich, May 1878.

II. *On the Mechanical Theory of Crookes's Force.*

By GEORGE FRANCIS FITZGERALD, *M.A., F.T.C.D.**

WHEN two surfaces at different temperatures are in presence of one another with a gas between them, there exists a force tending to separate them. The assumption of this force explains a very great number of phenomena, including the motion of the arms in Mr. Crookes's radiometers, and the so-called spheroidal state of liquids. That this force was due to some sort of unequal stress in the gas between the two surfaces, was pointed out by Mr. Stoney in the *Philosophical Magazine*, March and April 1876, where he attempted to show that such a state of stress would arise. An attempt to explain the motion of the arms of a radiometer had been made previously by Professor O. Reynolds ; but his conclusion, that it was principally due to evaporation and condensation, is manifestly inadequate to explain a continuous action, such as that in a radiometer ; and the method by which he tried to show that a surface, when communicating heat to gas, is subject to an increased pressure, is open to the overwhelming objection that this increased pressure would be almost instantaneously transmitted to all parts of the enclosed gas, and so

* From the *Scientific Transactions of the Royal Dublin Society* for October 1878. Communicated by the Author.

could not possibly be the source of such a force as would explain the motion of the arms of a radiometer.

In amplification of a letter I wrote to 'Nature' on the 17th of December 1877, and which was published on the 4th of January, 1878, I now intend to prove that such a state of stress as Mr. Stoney's theory requires would exist under the assumed conditions. My letter contains a proposed application of Clausius' investigation for finding the conducting-power of a gas, as published in the *Philosophical Magazine*, vol. xxiii. 4th series. Mr. Stoney, in a paper read before the Royal Dublin Society on Monday, the 18th of February, 1878, [*Phil. Mag.* Dec. 1878, p. 401] has obtained results somewhat like those obtained by my method by applying a method similar to one he originally employed.

I may first observe that the only way in which a state of other than uniform stress can exist in a gas is by the distribution of the mean velocities, and number of molecules, being different in different directions, or, as Mr. Stoney has called it, by the gas being polarized. That the distribution is not uniform when heat is being conducted through a gas has been pointed out long ago by both Clausius and Maxwell; and what is required is, to show that the distribution will then be such as to develop a force like Crookes's.

Following the method adopted by Clausius in his paper already referred to, I assume that the mean velocity of a molecule is a function of its direction of motion, and that the number of molecules in the unit volume moving in a given direction is also a function of that direction. If, then, we define the direction by means of μ , the cosine of the angle it makes with a given direction, ϕ the angle the plane of these two directions makes with a fixed plane through the given direction, we may evidently assume

$$v = v_0 f(\mu\phi), \quad n = n_0 F(\mu\phi),$$

where v and n are the mean velocities and number of molecules moving in this direction, and v_0 and n_0 are certain given values of v and n when f and F are unity. Now we may evidently in addition take $n_0 = \frac{N}{4\pi} N d\mu d\phi$, where N is the total number of molecules per unit of volume; so that we have, generally,

$$n = \frac{N}{4\pi} F(\mu, \phi) d\mu d\phi.$$

The quantities I intend to calculate are—the number of molecules carried through the unit area in any direction, the total

momentum carried through the same, and the quantity of energy carried through it. The number of molecules going in one direction through the unit area must evidently be equal to that of those going in the opposite direction, if there are no gaseous currents going on; and even if present, their existence is evidently beside the question in hand. Hence, if we sum the number of molecules passing the unit area, taking those that go in opposite direction through it with opposite signs, the sum must vanish. I shall calculate the numbers in three cases of unit areas:—1st, perpendicular to the line from which π is measured, or X; 2nd, parallel to the plane from which ϕ is measured (*i. e.* perpendicular to Y); and, 3rd, for the case of a unit area perpendicular to these two (*i. e.* perpendicular to Z). The number of molecules going in the direction (μ, ϕ) that pass through the first of these per unit of time is evidently $=nv\mu$; and it is likewise evident that the number going in the opposite direction will have an opposite sign; so that we have the sum of all such zero. Similarly, for the other two planes the numbers are

$$nv\sqrt{1-\mu^2} \cdot \sin \phi \text{ and } nv\sqrt{1-\mu^2} \cdot \cos \phi;$$

so that we get

$$0 = \Sigma nv\mu = \Sigma nv\sqrt{1-\mu^2} \sin \phi = \Sigma nv\sqrt{1-\mu^2} \cos \phi.$$

The momentum carried through the first of these unit areas per unit of time by molecules moving in the direction (μ, ϕ) is $= Mnv^2\mu^2$, if M be the mass of each molecule; and as it does not change sign with μ , we see that the sum of all such will represent the normal pressure per unit area at the given place. We can similarly get the normal pressures on the other two unit areas; and calling them P_{xx} , P_{yy} , and P_{zz} , we obtain

$$\begin{aligned} P_{xx} &= M\Sigma nv^2\mu^2, \\ P_{yy} &= M\Sigma nv^2(1-\mu^2) \sin^2 \phi, \\ P_{zz} &= M\Sigma nv^2(1-\mu^2) \cos^2 \phi. \end{aligned}$$

Proceeding similarly, we can get the tangential pressures on these areas; and we easily see that they are

$$\begin{aligned} P_{yz} &= P_{zy} = M\Sigma nv^2(1-\mu^2) \sin \phi \cos \phi, \\ P_{zx} &= P_{xz} = M\Sigma nv^2\mu\sqrt{1-\mu^2} \cos \phi, \\ P_{xy} &= P_{yx} = M\Sigma nv^2\mu\sqrt{1-\mu^2} \sin \phi. \end{aligned}$$

If now we proceed to calculate the energy carried across these areas per unit of time, we get $knv^3\mu$ as that carried across the first area by molecules moving in the direction (μ, ϕ) when k is the coefficient by which the energy of translation must

be multiplied in order to obtain the total energy. Calling the quantities of energy Q_x, Q_y, Q_z , we thus get

$$\begin{aligned} Q_x &= Mk \Sigma n v^3 \mu, \\ Q_y &= Mk \Sigma n v^3 \sqrt{1-\mu^2} \sin \phi, \\ Q_z &= Mk \Sigma n v^3 \sqrt{1-\mu^2} \cos \phi. \end{aligned}$$

In order to be able to perform these summations, it is necessary to know the mean values of nv , nv^2 , and nv^3 in terms of μ and ϕ ; and I shall, in the first place, merely assume that they can be expanded in a series of spherical harmonics, thus:

$$\begin{aligned} \overline{nv} &= \frac{N v_0}{4\pi} (A_0 + A_1 + A_2 + \dots) d\mu d\phi, \\ \overline{nv^2} &= \frac{N v_0^2}{4\pi} (B_0 + B_1 + B_2 + \dots) d\mu d\phi, \\ \overline{nv^3} &= \frac{N v_0^3}{4\pi} (C_0 + C_1 + C_2 + \dots) d\mu d\phi. \end{aligned}$$

The effect of this is to obtain our former results under the following simplified forms. Our first series of equations gives $A_1=0$; and as A_1 must be of the form

$$A_1 = a_1 \mu + a_2 \sqrt{1-\mu^2} \sin \phi + a_3 \sqrt{1-\mu^2} \cos \phi,$$

we get

$$a_1 = a_2 = a_3 = 0.$$

The second system of equations gives

$$\begin{aligned} P_{xx} &= \frac{MN v_0^2}{4\pi} \iint (B_0 + B_2) \mu^2 d\mu d\phi, \\ P_{yy} &= \frac{MN v_0^2}{4\pi} \iint (B_0 + B_2) (1-\mu^2) \sin^2 \phi d\mu d\phi, \\ P_{zz} &= \frac{MN v_0^2}{4\pi} \iint (B_0 + B_2) (1-\mu^2) \cos^2 \phi d\mu d\phi, \\ P_{yz} &= P_{zy} = \frac{MN v_0^2}{4\pi} \iint B_2 (1-\mu^2) \sin \phi \cos \phi d\mu d\phi, \\ P_{zx} &= P_{xz} = \frac{MN v_0^2}{4\pi} \iint B_2 \mu \sqrt{1-\mu^2} \cos \phi d\mu d\phi, \\ P_{xy} &= P_{yx} = \frac{MN v_0^2}{4\pi} \iint B_2 \mu \sqrt{1-\mu^2} \sin \phi d\mu d\phi. \end{aligned}$$

If now we assume

$$B_2 = b_1(\mu^2 - \frac{1}{3}) + b_2(1 - \mu^2) \cos 2\phi + b_3(1 - \mu^2) \sin \phi \cos \phi \\ + b_4\mu\sqrt{1 - \mu^2} \cos \phi + b_5\mu\sqrt{1 - \mu^2} \sin \phi,$$

as it must be of this form, we get, on putting our other quantities into the forms of spherical harmonics,

$$P_{xx} = \frac{MNv_0^2}{3} \left(B_0 + \frac{4}{15} \cdot b_1 \right), \\ P_{yy} = \frac{1}{3} MNv_0^2 \left(B_0 - \frac{2}{15} b_1 - \frac{2}{5} b_2 \right), \\ P_{zz} = \frac{1}{3} MNv_0^2 \left(B_0 - \frac{2}{15} b_1 + \frac{2}{5} b_2 \right), \\ P_{yz} = \frac{1}{15} MNv_0^2 b_3 = P_{zy}, \\ P_{zx} = \frac{1}{15} MNv_0^2 b_4 = P_{xz}, \\ P_{xy} = \frac{1}{15} MNv_0^2 b_5 = P_{yx}.$$

Similarly, for the quantities of energy transferred we get

$$Q_z = \frac{MNv_0^3}{4\pi} k \iint C_1 \mu d\mu d\phi, \\ Q_y = \frac{MNv_0^3}{4\pi} k \iint C_1 \sqrt{1 - \mu^2} \sin \phi d\mu d\phi, \\ Q_x = \frac{MNv_0^3}{4\pi} k \iint C \sqrt{1 - \mu^2} \cos \phi d\mu d\phi;$$

so that if we assume, as we evidently may,

$$C_1 = c_1\mu + c_2\sqrt{1 - \mu^2} \sin \phi + c_3\sqrt{1 - \mu^2} \cos \phi,$$

we get

$$Q_x = \frac{k}{3} MNv_0^3 c_1, \\ Q_y = \frac{k}{3} MNv_0^3 c_2, \\ Q_z = \frac{k}{3} MNv_0^3 c_3.$$

Even in this most general form we can see that there will in general be a difference of pressure in different directions;

for it is evident that the pressures in the three directions cannot be equal unless b_1 and b_2 both vanish, which will not in general be the case. Without a knowledge of the nature of the distribution of the velocities and numbers of molecules moving in the different directions, it would be impossible to calculate the values of b_1 , b_2 , b_3 , b_4 , and b_5 ; but I think we can see that they will in part at least vary as the square of the quantity of heat passing. This can be seen from the following considerations. No matter what the distribution of the velocities and numbers of molecules moving in the different directions may be, it is plain that terms occurring in the coefficients of $\sqrt{1-\mu^2} \sin \phi \sqrt{1-\mu^2} \cos \phi$ (*i. e.* in the spherical harmonics of the first order in u and v) will occur in the terms of the same order in nv , nv^2 and nv^3 , and that linearly; while these same terms will occur squared in the spherical harmonics of the second order in nv , nv^2 and nv^3 . Hence we see that terms occurring linearly in the spherical harmonics of the first order in nv^3 will occur as squares in the spherical harmonics of the second order in nv^2 ; so that b_1 , b_2 will contain c_1 , c_2 , and c_3 in the second degree, *i. e.* will contain terms varying as the squares of the quantities of heat passing. It is also to be observed that terms occurring in the spherical harmonics of the second order can never come into those of the first, except as products with terms belonging to spherical harmonics of the third order; so that a hypothetical distribution which gave correct values for the quantities of heat passing might very well be quite inadequate as a means of calculating the difference of pressure in different directions. This remark is of importance when we come to consider the results of Clausius' hypothesis, and was suggested to me by Mr. Stoney in conversation.

As an example of what I am insisting upon, we may take two opposite extreme cases:—first, the case of B_2 vanishing, and, secondly, the case of C_1 doing so. In the first case there would be a distribution of velocities and numbers such that, though heat would be conducted across the layer, nevertheless there would be no resultant inequality of stress; while in the second case, though no heat would be conducted, yet there would be inequality of stresses. It seems, however, certain that neither of these extreme cases can exist as a permanent distribution in gases. Before calculating the values of these quantities upon particular hypothetical distributions, it may be well to see what they are in the simple case of two parallel planes, each at a uniform temperature.

In this case it is evident from symmetry that, if we take X normal to the planes, we must have all our equations indepen-

dent of ϕ , as the effect is evidently symmetrical with regard to X. Then we get

$$b_2 = b_3 = b_4 = b_5 = 0 = c_2 = c_3,$$

and there are no tangential forces, while all the heat is transferred in the direction X, and our pressures become

$$P_{xx} = \frac{1}{3} MN v_0^2 \left(B_0 + \frac{4}{15} b_1 \right),$$

$$P_{yy} = P_{zz} = \frac{1}{3} MN v_0^2 \left(B_0 - \frac{2}{15} b_1 \right),$$

while the heat transferred is

$$Q_x = \frac{k}{3} MN v_0^3 c_1.$$

The excess of pressure in X over that in the normal directions is

$$P_{xx} - P_{yy} = \frac{2}{15} MN v_0^2 b_1 = K;$$

and this has been called Crookes's force.

That it depends wholly upon b_1 can be seen by the following simple method, mentioned to me by Mr. Stoney.

Our expressions for P_{xx} and P_{yy} are

$$P_{xx} = M \Sigma n v^2 \mu^2,$$

$$P_{yy} = M \Sigma n v^2 (1 - \mu^2) \sin^2 \phi;$$

so that, calling

$$n = \frac{N}{4\pi} I d\mu d\phi,$$

when I depends upon the distribution of numbers only, we can write the pressures

$$P_{xx} = \frac{MN}{4\pi} \iint I v^2 \mu^2 d\mu d\phi,$$

$$P_{yy} = \frac{MN}{4\pi} \iint I v^2 (1 - \mu^2) \sin^2 \phi d\mu d\phi.$$

We can integrate them with respect to ϕ ; for we know that $I v^2$ is independent of ϕ in the case we are considering;

$$\therefore P_{xx} = \frac{1}{2} MN \int I v^2 \mu^2 d\mu,$$

$$P_{yy} = \frac{1}{4} MN \int I v^2 (1 - \mu^2) d\mu;$$

$$\therefore P_{xx} - P_{yy} = \kappa = \frac{3}{4} MN \int I v^2 (\mu^2 - \frac{1}{3}) d\mu;$$

so that if Iv^2 be expanded in spherical harmonics, K depends only upon the spherical harmonic of the second order. Similarly, if Iv^3 be similarly expanded, it is easy to see that

$$Q_x = \frac{1}{2} MNk \int Iv^3 \mu d\mu$$

can only depend upon the spherical harmonic of the first order in Iv^3 .

If now we turn to particular hypotheses as to the character of the distribution of velocities and numbers, the first that claims our attention is Clausius's. He starts from the assumption that the distribution of velocities among the molecules that have just encountered one another in any given layer may be perfectly represented by supposing a small constant velocity in the direction of the transference of heat to be superposed upon a uniform distribution. This is the same as supposing that these velocities in various directions may be represented by the radii drawn to the surface of a sphere from a point slightly displaced from its centre. It is worthy of remark, in connexion with what I mentioned before with reference to the way the quantities in the various spherical harmonics are related to one another, that, supposing the sphere to be an ellipsoid of even greater ellipticity would not have affected his results; for it is easy to show that the ellipticity of an ellipsoid of revolution only enters into the spherical harmonics of the second and higher orders; so that it would not enter into the equation giving the quantity of heat, except when multiplied by terms of at least the order of the quantity of heat. Thus, even though the square of the ellipticity were of the order of the displacement from the centre of the point from which the radii representing the velocities are drawn, nevertheless that would at most only have introduced terms depending upon the product of these two, which would not have materially affected his results. Hence we see that Clausius' success in calculating the quantity of heat conducted is no proof that his hypothesis is by any means a sufficient representation of the actual distribution for the purpose of calculating the resultant stresses; and that it is not is proved by calculating what the Crookes's force would be upon his hypothesis. If this be done with the help of the quantities he gives in his note (see *Phil. Mag.* [IV.] vol. xxiii. p. 526), we get

$$K = \frac{1.8}{k^2} \cdot \frac{\rho_0 T_0}{P_0} \cdot \frac{Q^2}{PT};$$

and the pressures deduced from this formula are very much smaller than those observed; so that it seems certain that the

hypothetical distribution Clausius assumed is not at all adequate to represent the actual one. The pressures obtained by this formula are so insignificant that it is not worth while giving the details of the method by which it is deduced. That Clausius' hypothesis is by no means adequate, can also be seen by the consideration that it is only after the Clausian laws for the conduction of heat have ceased to apply, owing to the rarefaction of the gas, that Crookes's force becomes remarkable, as well as by considering what the distribution tends towards, as has been done by Mr. Stoney, in his paper published in the December Number of this Magazine. He shows that the distribution lies between one which could be represented by two unopposing streams of molecules, moving one towards the heater and the other towards the cooler and unpolarized gas. With such a distribution the laws of conduction of heat would, of course, differ somewhat from those deduced from Clausius' distribution.

I shall now calculate the result upon an arbitrarily assumed distribution, which, however, probably represents the actual one more nearly than Clausius's. I shall assume that the distribution of velocities can be represented by the formula

$$v = v_0(1 + \alpha \cos \theta + \beta \sin \theta \sin \phi + \gamma \sin \theta \cos \phi \\ + a \cos \theta + b \sin^2 \theta \sin^2 \phi + c \sin^2 \theta \cdot \cos^2 \phi + 2f \cdot \sin^2 \theta \sin \phi \cos \phi \\ + 2g \sin \theta \cdot \cos \theta \cdot \cos \phi + 2h \sin \theta \cdot \cos \theta \cdot \sin \phi,$$

where

$$\cos \theta = \mu.$$

This is equivalent to saying that it is represented very nearly by the radii drawn to the surface of a slightly elliptical ellipsoid from a point near its centre. I shall assume that $\alpha, \beta, \gamma, a, b, c, f, g, h$ are all quantities whose squares and products may be neglected. For the number of molecules moving in the given direction θ, ϕ , I shall assume that it varies inversely as the velocity of the molecules moving in that direction, so that $nv = Nv_0$. This evidently satisfies the condition $A_1 = 0$. By these assumptions we obtain approximately $nv^2 = Nv_0 \cdot v$ and $nv^3 = Nv_0 \cdot v^2$, and hence

$$nv^2 = Nv_0^2 \left\{ \begin{aligned} &[1 + \alpha\mu + \beta\sqrt{1-\mu^2}\sin\phi + \gamma\sqrt{1-\mu^2}\cdot\cos\phi \\ &+ \alpha\mu^2 + b(1-\mu^2)\sin^2\phi + c(1-\mu^2)\cdot\cos^2\phi \\ &+ 2f\sqrt{1-\mu^2}\sin\phi\cos\phi + 2g\mu\sqrt{1-\mu^2}\cdot\cos\phi \\ &+ 2h\mu\sqrt{1-\mu^2}\cdot\sin\phi], \end{aligned} \right\}$$

or, turning it into the form of a series of spherical harmonics,

$$nv^2 = Nv_0^2 \left\{ \begin{aligned} &1 + \frac{1}{3}(a+b+c) + \left(a - \frac{1}{2}\overline{b+c}\right)(\mu^2 - \frac{1}{3}) \\ &\quad + \frac{1}{2}(c-b)(1-\mu^2)\cos 2\phi \\ &+ 2f\sqrt{1-\mu^2} \cdot \sin \phi \cos \phi + 2g\mu\sqrt{1-\mu^2} \cdot \cos \phi \\ &\quad + 2f\mu\sqrt{1-\mu^2} \cdot \sin \phi \\ &+ \alpha\mu + \beta\sqrt{1-\mu^2} \cdot \sin \phi + \gamma\sqrt{1-\mu^2} \cdot \cos \phi, \end{aligned} \right\}$$

from which we see that

$$b_1 = a - \frac{1}{2}(b+c), \quad b_2 = \frac{1}{2}(c-b),$$

$$b_3 = 2f, \quad b_4 = 2g, \quad b_5 = 2h.$$

We may evidently include the $\frac{1}{3}(a+b+c)$ in the mean value of Nv_0^2 , and take $B_0 = 1$; so that, calling $MN = \rho$ the density of the gas, our pressures become

$$P_{xx} = \frac{1}{3}\rho v_0^2 \left[1 + \frac{4}{15} \left(a - \frac{1}{2}(b+c) \right) \right],$$

$$P_{yy} = \frac{1}{3}\rho v_0^2 \left[1 + \frac{4}{15} \left(b - \frac{1}{2}(c+a) \right) \right],$$

$$P_{zz} = \frac{1}{3}\rho v_0^2 \left[1 + \frac{4}{15} \left(c - \frac{1}{2}(a+b) \right) \right],$$

$$P_{yz} = \frac{2}{15}\rho v_0^2 \cdot f = P_{zy},$$

$$P_{zx} = \frac{2}{15}\rho v_0^2 \cdot g = P_{xz},$$

$$P_{xy} = \frac{2}{15}\rho v_0^2 \cdot h = P_{yx}.$$

Similarly, from $nv^3 = Nv_0 \cdot v^2$ we can get

$$c = 2\alpha, \quad c^2 = 2\beta, \quad c^3 = 2\gamma,$$

and hence

$$Q_x = \frac{2}{3}\overline{kp v_0^3} \cdot \alpha, \quad Q_y = \frac{2}{3} \cdot \overline{kp v_0^3} \cdot \beta, \quad Q_z = \frac{2}{3}\overline{kp v_0^3} \cdot \gamma.$$

The normal pressures may also be put into the form

$$P_{xx} = \frac{1}{3} \rho v_0^2 \cdot \left\{ 1 + \frac{1}{15} (a+b+c) + \frac{1}{5} (a-b-c) \right\},$$

$$P_{yy} = \frac{1}{3} \rho v_0^2 \cdot \left\{ 1 + \frac{1}{15} (a+b+c) + \frac{1}{5} (b-c-a) \right\},$$

$$P_{zz} = \frac{1}{3} \rho v_0^2 \cdot \left\{ 1 + \frac{1}{15} (a+b+c) + \frac{1}{5} (c-a-b) \right\};$$

so that the state of stress is a uniform pressure, and superposed upon it a system of pressures represented by the equations

$$p_{xx} = \frac{1}{3} \rho v_0^2 \cdot \frac{1}{5} (a-b-c),$$

$$p_{yy} = \frac{1}{3} \rho v_0^2 \cdot \frac{1}{5} (b-c-a),$$

$$p_{zz} = \frac{1}{3} \rho v_0^2 \cdot \frac{1}{5} (c-a-b),$$

$$p_{yz} = \frac{1}{3} \rho v_0^2 \cdot \frac{2}{5} f = p_{zy},$$

$$p_{zx} = \frac{1}{3} \rho v_0^2 \cdot \frac{2}{5} g = p_{xz},$$

$$p_{xy} = \frac{1}{3} \rho v_0^2 \cdot \frac{2}{5} h = p_{yx}.$$

Now it is remarkable that, if

$$ax^2 + by^2 + cz + 2yz + 2gzx + 2hxy = (lx + my + nz)^2,$$

we should have expressions for these additional unequal pressures the same as Professor Clerk Maxwell gives (see his 'Electricity and Magnetism,' vol. i. p. 129, and vol. ii. p. 256) as expressing that state of stress in the æther which produces electrical phenomena. In order to make them identical, all that is necessary is to put

$$X = l \sqrt{\frac{8\pi}{15} \rho v_0^2},$$

$$Y = m \sqrt{\frac{8\pi}{15} \rho v_0^2},$$

$$Z = n \sqrt{\frac{8\pi}{15} \rho v_0^2};$$

so that the resultant unequal pressures in the gas may be

represented by a pressure $p = \frac{R^2}{8\pi}$ when $R^2 = X^2 + Y^2 + Z^2$ in the direction given by

$$\mu : \sqrt{1-\mu^2} \sin \phi : \sqrt{1-\mu^2} \cos \phi :: X : Y : Z :: l : m : n,$$

and an equal diminished pressure in every direction at right angles to this line. Double this pressure will be the Crookes's force, which is consequently in this case

$$K = \frac{1}{3} \rho v_0^2 \cdot \frac{1}{5} (l^2 + m^2 + n^2);$$

and it is in the direction whose direction-cosines are proportional to $l : m : n$; so that, if we put

$$l = \nu \mu, \quad m = \nu \sqrt{1-\mu^2} \sin \phi, \quad n = \nu \sqrt{1-\mu^2} \cos \phi,$$

$$R = \frac{1}{15} \rho v_0^2 \cdot \nu^2.$$

The direction-cosines of the line of transference of heat are evidently $\alpha : \beta : \gamma$, and so far there is no reason why these two lines should coincide, although of course in most cases they probably differ but little in direction.

The only other distribution I shall consider is one suggested by Mr. Stoney's investigation (Scientific Transactions of the Royal Dublin Society, p. 39) of the nature of the distribution of the velocities in the gas between two large parallel surfaces at uniform unequal temperatures. He has shown that it tends towards a distribution which would be represented by two streams of unpolarized gas moving in opposite directions across the layer. Now the actual distribution is never exactly this, and possibly, as he has mentioned, departs in various degrees from it as you pass across the layer. If, however, we assume the distribution to be the same all the way across, and that consequently the mean temperature of each stream is that due to the surface it is leaving, we can calculate the resultant pressures.

If v_1 and v_2 be the mean velocities of the molecules in each stream respectively relatively to the centres of mass of the molecules, and if u_1 and u_2 be the velocities of the streams (*i. e.* of these centres of mass), and ρ_1 and ρ_2 their densities, the pressure upon a fixed plane normal to the direction of the streams is

$$P = \frac{1}{3} \rho_1 v_1^2 + \frac{1}{3} \rho_2 v_2^2 + \rho_1 u_1^2 + \rho_2 u_2^2,$$

while the pressure sideways is

$$p = \frac{1}{3} \rho_1 v_1^2 + \frac{1}{3} \rho_2 v_2^2;$$

so that the Crookes's pressure in this case is

$$K = P - p = \rho_1 u_1^2 + \rho_2 u_2^2.$$

In order that there be no accumulation of gas at either surface, we must evidently have

$$\rho_1 u_1 = \rho_2 u_2.$$

If V_1^2 and V_2^2 be the total mean squares of the velocities of agitation, $V_1^2 = v_1^2 + u_1^2$, $V_2^2 = v_2^2 + u_2^2$, and the quantity of heat transferred is

$$Q = k(\rho_1 V_1^2 u_1 - \rho_2 V_2^2 u_2),$$

k being, as before, the coefficient by which the *vis viva* of translation has to be multiplied in order to get the total energy of the gas.

From these we easily obtain

$$K = \rho_1 u_1 (u_1 + u_2),$$

$$Q = k \rho_1 u_1 (V_1^2 - V_2^2);$$

$$\therefore Q = kK \cdot \frac{V_1^2 - V_2^2}{u_1 + u_2}.$$

We have besides $\rho_1 + \rho_2 = \rho$, where ρ is the density of the gas. Hence there are six equations between the six unknowns,

$$\rho_1, \rho_2, v_1, v_2, u_1, u_2;$$

and in order to eliminate them and obtain an equation between K and Q , it is necessary to make one further assumption. I assume, then, that $u_1 = \lambda v_1$ and $u_2 = \lambda v_2$, so that $V_1^2 = (\lambda^2 + 1)u_1^2$ and $V_2^2 = (\lambda^2 + 1)u_2^2$. I assume this because, if the streams did not interfere with one another at all, we should have

$$u_1^2 = \frac{1}{6} V_1^2;$$

so that, if $\lambda^2 + 1 = \alpha^2$, we should have

$$\alpha^2 = 6 \text{ and } \alpha = 2.5 \text{ } q. \text{ } p.$$

Our equations then become

$$V_1 - V_2 = \alpha^2 (u_1^2 - u_2^2);$$

$$\therefore Q = kK\alpha^2(u_1 - u_2).$$

From these we can eliminate u_1, u_2, ρ_1, ρ_2 ; and putting

$$V_1^2 - V_2^2 = X^2,$$

we get

$$Q^4 + 4 \frac{\alpha^2 k^2}{\rho} K^2 Q^2 - \alpha^2 k^4 X^4 \cdot K^4 = 0,$$

which is a quadratic for Q^2 or a biquadratic for K .

Solving for Q , we get

$$Q = \frac{kK\sqrt{\alpha}}{\sqrt{\rho}} \{ \sqrt{X^4 \rho^2 + 4\alpha^2 K^2} - 2\alpha K \}^{\frac{1}{2}},$$

as evidently the other solutions are inadmissible.

From this we may get an approximate value for K in terms of Q ; for, unless α be very large, or the density or difference of temperature very small, $X^2 \rho$ is much greater than $2\alpha K$. For instance, if V_1 and V_2 correspond to a difference of 10° C.,

$$V_1 = 48500 \sqrt{\frac{T_1}{273}},$$

$$V_2 = 48500 \sqrt{\frac{T_2}{273}},$$

and consequently

$$X^2 = \frac{(48500)^2}{27 \cdot 3};$$

$$\therefore X = 9700,$$

while $\rho = \frac{1}{800}$ for air at atmospheric pressure ;

$$\therefore X^2 \rho = 107600.$$

And K would be large if it were 100; so that even if α were 50, $2\alpha K$ would still be less than $\frac{1}{10}$ of $X^2 \rho$; and so we may take approximately

$$Q = k\sqrt{\alpha} \cdot KX ;$$

$$\therefore K = \frac{Q}{k\sqrt{\alpha} \cdot X}.$$

From this we can calculate K ; for $k = 1.6$ in most gases, and, if $\alpha = 2.5$, $\sqrt{\alpha} = 1.5$, and $X = 9700$, as above ;

$$\therefore k\sqrt{\alpha}X = 22310 = 2 \times 10^4 q. p.$$

Now, at a distance of a fourth-metre in air at atmospheric pressure, and with a difference of temperature of 10° C.,

$$Q = 10^6 q. p. ;$$

so that in this case

$$K = 50 q. p.,$$

which is within the limits of the quantities obtained in the case of the spheroidal drops on liquids.

That by this formula K varies nearly as Q , and not as Q^2 , is not to be wondered at, because in the first place the formula only professes to represent an approximation to the true state of affairs, and in the second place it is only at distances and pressures at which the ordinary laws of conduction of heat cease to apply that it professes even approximately to represent it.

The whole of these investigations are unsatisfactory to this extent—that I have been unable, from a consideration of the molecular encounters themselves, to discover what is the actual distribution of velocities even in the simple case of two parallel surfaces. This is hardly to be wondered at; for the problem is extremely complicated, and evidently depends upon the undecided point in molecular physics, namely the proportion of the molecules encountering in a given direction that are thrown off in the various other directions. We might very well assume, with Maxwell, that they are uniformly distributed in every direction after the encounter; but even this does not simplify the question sufficiently to bring it within my present powers of solution.

III. *The Electric Light.* By W. H. PREECE, *Memb. Inst. C.E., V.P. Soc. T.E., Electrician General Post-Office, &c.**

1. **T**HE theory of the electric light cannot be brought absolutely within the domain of quantitative mathematics, for the reason that we do not yet know the exact relation that exists between the production of heat and the emission of light with a given current; but we know sufficient to predicate that what is true for the production of heat is equally true for the production of light beyond certain limits.

The work done in a battery, or any source of current-electricity, is expended outside the battery in a closed circuit in the form of heat. When this heat acquires a certain temperature per unit mass, we have light. If the heat be confined to a mass of metal wire like platinum, we have light by *incandescence*; if it be expended in the transference of minute particles of incandescent matter like carbon across an air-space, we have the *electric arc*. The exact relations between current, heat, temperature, mass, and light have yet to be determined by experiment.

2. The arc is thus a form of energy developed in one point

* Communicated by the Author.