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THE ALTERNATING-CURRENT RAILWAY MOTOR

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For electric railroading a motor is required which maintains a high value of efficiency over a wide range of speed. That is, the torque per ampere input at constant impressed voltage must increase with decrease of speed, the speed increase with decrease of load.

In electric motors, torque is produced by the action of a magnetic field upon currents flowing in an armature movable with regard to the field. If then the field is constantly excited—shunt motor on constant potential—the torque is approximately proportional to the current, the speed approximately constant at all loads. If the field is excited by the main current of the motor—series motor on constant potential—the field strength and thereby the torque per ampere varies approximately proportional to the current, and thereby the load, the whole torque approximately proportional to the square of the current and the speed inversely proportional to the current, leaving saturation out of consideration. That is, the motor has the characteristic specified above for a railway motor.

Since the direction of rotation of the direct-current motor is independent of the direction of the impressed e.m.f., with laminated field the direct-current motor can be operated with alternating currents. By the use of alternating currents it becomes possible to transfer current from circuit to circuit by induction, and instead of passing the main-line current through the armature of the alternating-current motor, the armature circuit can be closed upon itself and the current induced therein as transformer secondary by a stationary primary coil in the main circuit surrounding the armature.

Condition of operation of the direct-current motor type on alternating current is, however, that the current in field and armature reverses simultaneously. This is by necessity the case in the series motor. In the shunt motor, however, the armature current as energy current should be in phase with the impressed e.m.f., while the field current as magnetizing current lags nearly 90°. To bring it back into phase, W. Stanley tried condensers in series in the field circuit, but failed, due to the impossibility of neutralizing the self-induction of the field which varies with the commutation and the frequency, by the negative self-induction of the condensers, which varies with the frequency in the opposite direction. The solution of the problem has been found by the use of polyphase systems, by utilizing for the field excitation an e.m.f. in quadrature with the armature currents acted upon by the field magnetism. As I have shown elsewhere, the polyphase induction motor can be considered as a development of the direct-current shunt motor for alternating-current circuits, and indeed has all the shunt motor characteristics regarding speed, torque, etc. As railway motor the induction motor has therefore not been exploited, although it has been strongly recommended in those very few cases where it appeared good engineering. Experimental work with polyphase induction motor railways has been carried on continuously since 1893.

While in the early days of alternating-current motor development, all other engineers were industriously developing the constant-speed stationary motor type, with shunt motor characteristic, only Rudolph Eickemeyer of Yonkers was far-sighted enough to realize the vast future of alternating-current railroading, and to appreciate the absolute necessity of the series motor characteristic for railway work, and undertake the development of the single-phase alternating-current series motor. I had the good fortune at that time to be associated with Mr. Eickemeyer.

As was pointed out by G. Kapp, I believe in 1888, the power-factor of the alternating-current series motor is inherently low, since the same magnetic flux which induces, proportional to the frequency of rotation, the e.m.f. of useful work in the armature conductors, induces in the field coils an e.m.f. of self-induction, proportional to the frequency of alternation. Hence, giving the armature the same number of turns as the field (which is more than permissible in good practice, since good practice requires weak armature and strong field), even at synchronous speed the e.m.f. of rotation of the armature would still only be equal to the e.m.f. of self-induction of the field; and the power-factor, allowing for an additional self-induction of the armature, would be below 70 per cent. This probably deterred the other engineers from considering the alternating-current series motor.

Eickemeyer solved the difficulty by designing the armature with a number of turns several times greater than the field (24 to 7 in the first motor built) and neutralizing the armature self-induction and reaction by a stationary secondary circuit surrounding the armature at right angles electrically to the field circuit (the "cross-coil," as he called it), and either short-circuited upon itself or energized by the main current in opposite direction to the current in the armature.

In January, 1891, I tested the first motor of this type, a bipolar motor with the following constants:

Field: Two coils of 14 turns No. 10 B & S wire, connected in parallel.

Armature: 24 coils of four turns each of No. 12 B & S wire.

Secondary circuit: Two coils of 18 turns each of No. 10 B & S wire connected in parallel.

At 100 cycles and 150 volts impressed e.m.f., this motor gave at three-fourths synchronous speed:

Current: 45 amperes.

$I^2 R$: 400 watts.

Hysteresis and eddy currents: 900 watts.

Total output, including friction: 4000 watts.

Hence:

Efficiency, 75.5%

Power-factor, 79%

The starting current of this motor at 150 volts was 70 amperes.

As bipolar motor with the very high frequency then used, the speed, 4500 revolutions at three-fourths synchronism, was undesirably high, so we immediately proceeded to build an 8-pole motor In this, solid copper rings were used as secondary circuits surrounding the armature and neutralizing its self-induction, with an effective copper-section more than four times that of the armature conductors. The ratio of armature series turns to field series turns was about 4. This motor was tested in 1892. The record of tests is given in Fig. 1, the observed values being marked on the curves. For comparison on this sheet is also given the direct-current voltage required to operate this motor at the same speed and current.

As seen, when approaching synchronous speed, the power-factor is nearly 90 per cent. The commutation was fair at 85 cycles, the highest frequency at which our factory engine was able to drive the alternator, and perfect at 33 cycles.

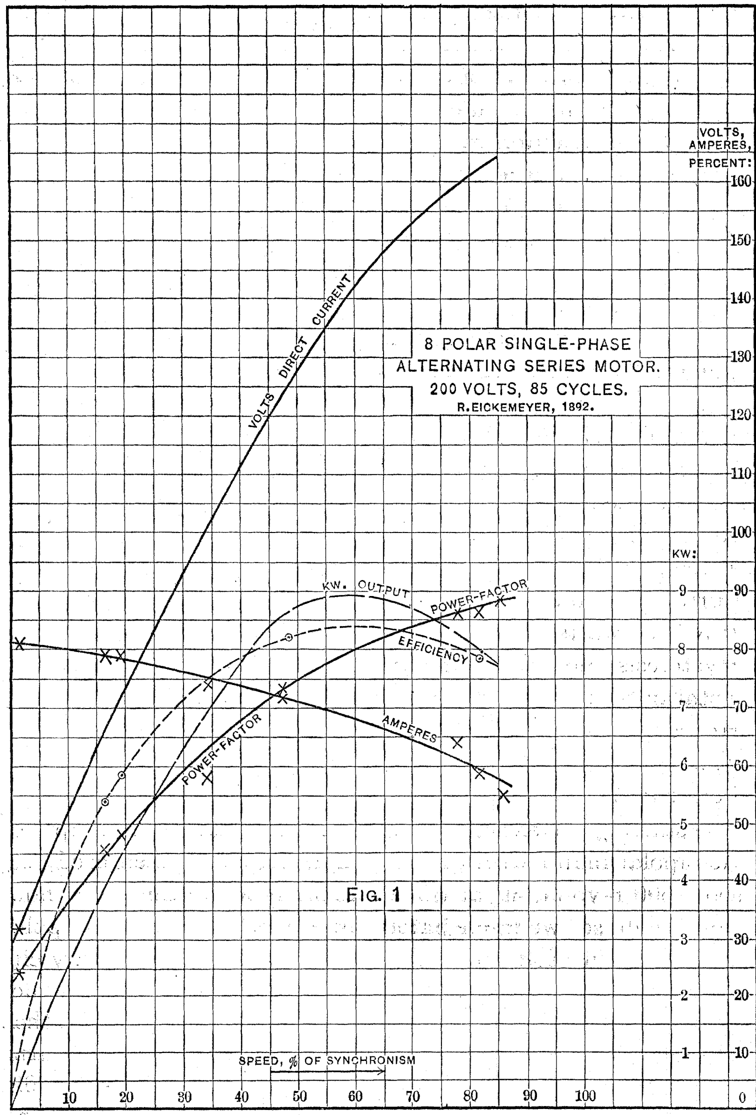


FIG. 1

A number of railway motors of this type were designed. The great difficulty, however, was that during these early days 125 to 133 cycles was the standard frequency in this country, 60 cycles hardly considered, and 25 cycles not yet proposed.

The efficiency of this alternating-current series motor is slightly lower than that of the same motor on direct-current circuit, due:

1. To the hysteresis loss in the field.
2. The hysteresis loss in the armature core, which is of full frequency up to synchronism and of still higher frequency, the frequency of rotation, beyond synchronism.
3. The I^2R loss in the short-circuited secondary conductors surrounding the armature.

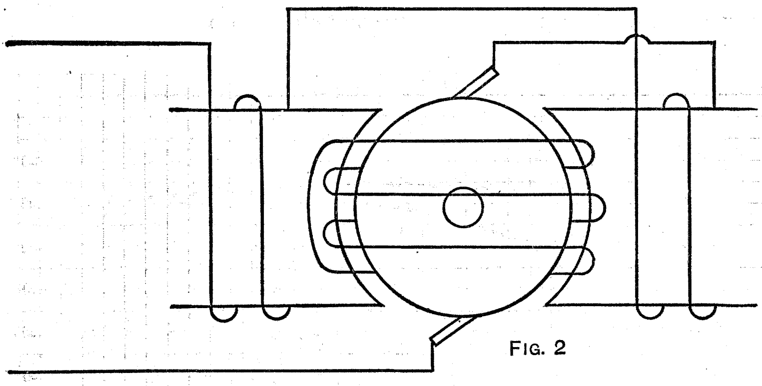


FIG. 2

As seen, to make the alternating-current series motor practicable, the transformer feature must be introduced, by having its armature as primary circuit closely surrounded by a short-circuited secondary circuit, as shown diagrammatically in Fig. 2.

Instead of closing the stationary circuit upon itself as second-

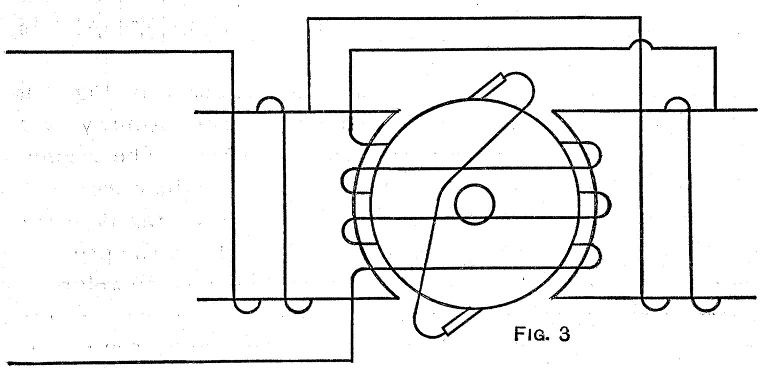


FIG. 3

ary circuit and feeding the main current into the rotating armature as primary circuit, mechanically the same results would obviously be obtained by using the stationary circuit as primary energized by the main current, and closing the armature upon itself as secondary by short circuiting the brushes and thereby keeping the main current and the line potential away from the armature, as shown diagrammatically in Fig. 3. This introduces the great advantage of reversing the sign of the uncompensated part of the armature self-induction, so that it is subtractive, which results in an essential improvement of the power-factor, especially at low speed.

This is shown in Fig. 4, where with the speed as abscissas, in per cent of synchronism, are plotted the power-factor of the Eickemeyer compensated series motor of Fig. 1, of ratio armature to field = 4, and the power-factor of one of the first railway repulsion motors, of ratio armature to field = 3.5.

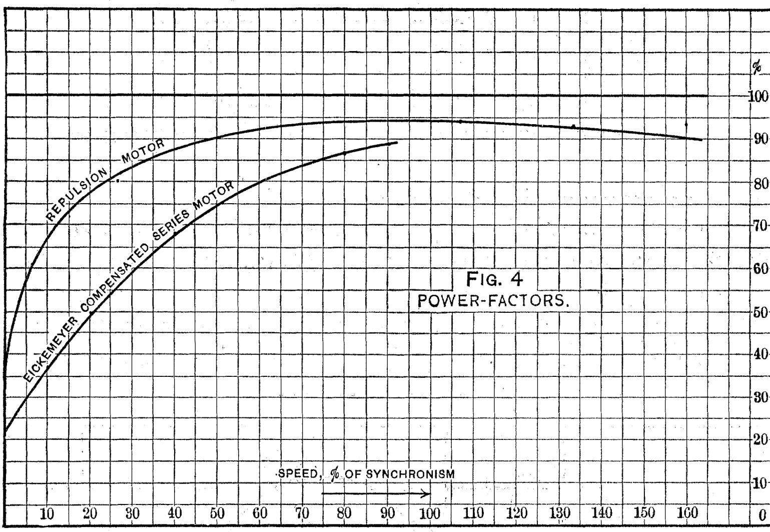


FIG. 4
POWER-FACTORS.

The compensation of the armature self-induction in Fig. 3 is based on the feature of the transformer that primary and secondary current are in opposition to each other. The secondary current of the transformer, however, lags slightly less than 180° behind the primary current; that is, considering it in the reverse direction, is a leading current with regard to the primary current. The current in the armature in Fig. 3 is, therefore, a leading current with respect to the line current, and so not only does not add an additional lag but reduces the lag caused by the self-induction of the field-exciting coil.

This motor then consists of a short-circuited armature surrounded by two coils at right angles with each other and connected in series, as illustrated in Fig. 5; the one, A_2, parallel with the effective armature circuit, acting as primary of a transformer to induce the secondary armature current; the other, A_1, the field-exciting coil. The ratio of turns of these coils, n_2, to n_1, is the ratio of effective armature series turns to field turns, as discussed before. Obviously, these two coils can be replaced by one coil at an angle with the position of brushes as shown in Fig. 6, and the cotangent of the angle of the axis of this coil with the position of the brushes is above ratio; that is, the smaller this angle the greater is the ratio of armature to field turns; that is, the better the power-factor of the motor.

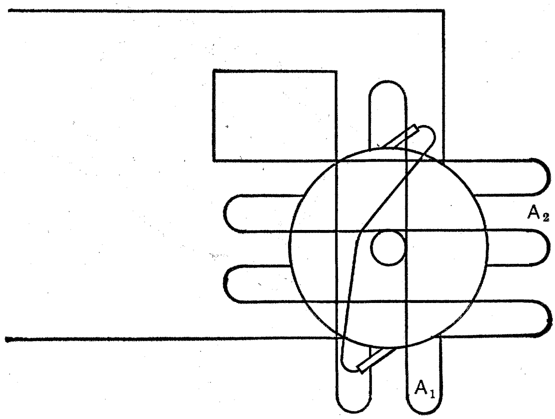
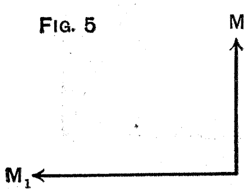


FIG. 5



This motor, Fig. 6, is Professor Elihu Thomson's repulsion motor.

In the armature an e.m.f. is induced by the alternation of the magnetic field, M_2, of coil A_2, proportional to M_2 and to the impressed frequency and in quadrature with M_2; and an e.m.f. is induced by the rotation through the magnetic flux M_1 of coil A_1, proportional to M_1 and to the frequency of rotation and in phase with M_1. These two e.m.fs. must be equal and opposite, since the armature is short circuited (neglecting the resistance and self-inductive reactance of the armature), and at synchronism, M_1 and M_2 are, therefore, equal and in quadrature with each other; that is, in the armature of the motor, Fig. 5, and therefore of the repulsion motor, Fig. 6, at synchronism a uniform rotating field exists and the hysteresis loss in the armature core is therefore zero at synchronism and at other speeds proportional to the difference between speed and synchronism; that is, to the slip, just as in the polyphase induction motor; while in the motor, Fig. 2, the hysteresis loss in the armature core is proportional to the impressed frequency or the frequency of rotation, whichever is the higher frequency. The hysteresis loss of the repulsion motor is therefore lower than that of the same motor as compensated series motor.

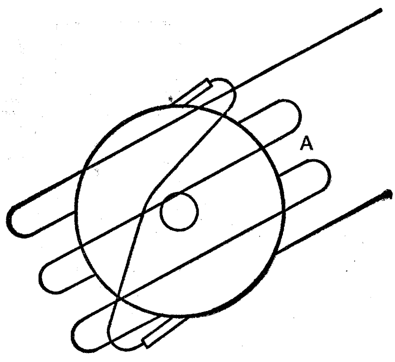
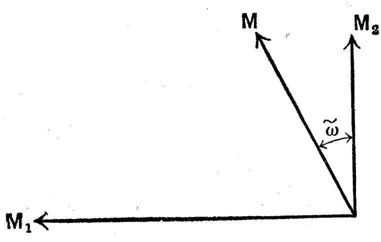


FIG. 6



In general, if in the repulsion motor in Fig. 6 the magnetic flux in line with the brushes, which does not induce e.m.f. by rotation but only by alternation, is denoted by M_2; the magnetic flux in quadrature with the brushes which induces e.m.f. in the armature by its rotation but not by the alternation of the flux, by M_1; and the magnetic flux in the axis of the primary coil A, which is much nearer to M_2 than to M_1, since good power-factor requires a small angle ω, by M, the two e.m.fs. induced in the armature, by the rotation through the flux M_1, are:

$$E_1 = 2 \pi N_0 \; M_1 n,$$

By the alternation of the flux M_2.

$$E_2 = -j \, 2 \pi N M_2 n$$

Where n = number of armature turns,
N = impressed frequency;
N_0 = frequency of rotation.

Since, approximately, $E_2 = E_1$, it is,

$$j N_0 M_1 = N M_2$$

or

$$j M_1 : M_2 = N : N_0 \tag{1}$$

That is: M_1 and M_2 are in quadrature in phase and the ratio of their intensity is inversely proportional to the ratio of speed to synchronism.

That is, in the repulsion motor an elliptically rotating field exists which becomes circular; in other words, a uniformly rotating field, at synchronism. Below synchronism the component M_1, which induces e.m.f. by the rotation of the armature is greater than M_2, the more the lower the speed. The flux M interlinked with the primary coil is, however, nearer to M_2 and therefore below synchronism, especially at low speeds, the magnetic flux which induces e.m.f. by the rotation of the armature and so represents the useful work, is greater than the magnetic flux which interlinks with the primary coil and so gives the lag of the primary current. This accounts for the high power-factor of the repulsion motor at low speeds.

Neglecting resistance and self-inductive reactance, it is,

$$M = \text{constant},$$

corresponding to the impressed voltage.

From equation (1) it follows:

$$M_2 = j M_1 (N_0/N)$$

Hence,

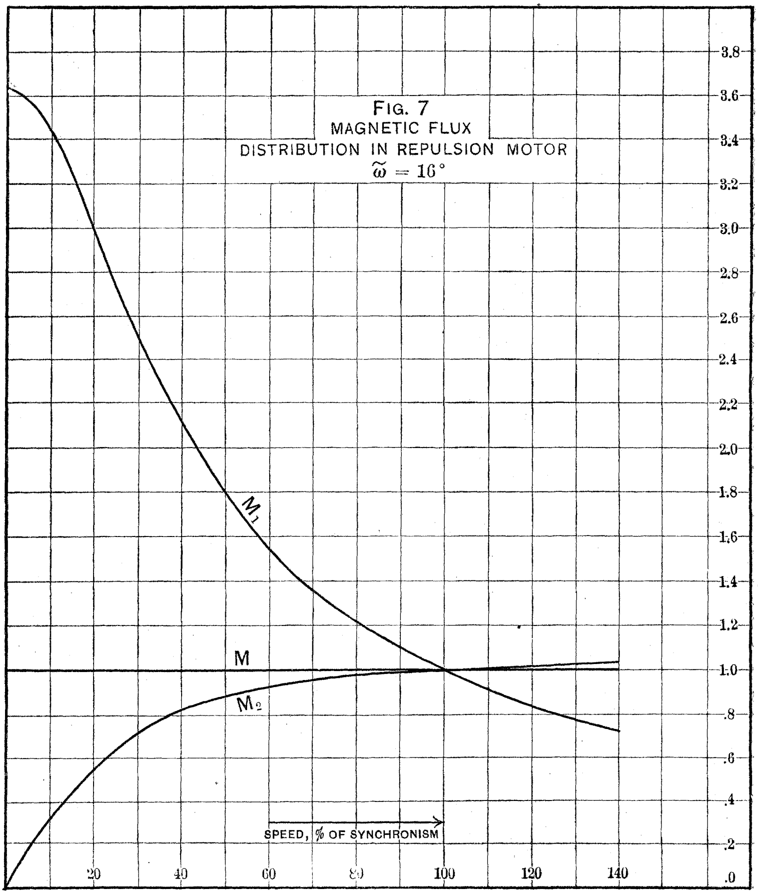
$$\begin{aligned} M &= M_2 \cos \omega + M_1 \sin \omega \\ &= M_1 [\sin \omega + j (N_0/N) \cos \omega] \end{aligned}$$

$$\left. \begin{aligned} M_1 &= \frac{M}{\sin \omega + j (N_0/N) \cos \omega} \\ M_2 &= \frac{M}{\cos \omega - j (N/N_0) \sin \omega} \end{aligned} \right| \tag{2}$$

or, absolute:

$$M_1 = \frac{M}{\sqrt{\sin^2 \omega + (N_0/N)^2 \cos^2 \omega}}$$

$$M_2 = \frac{M}{\sqrt{\cos^2 \omega + (N/N_0)^2 \sin^2 \omega}} \qquad (3)$$



At synchronism: $N_0 = N$, it is:

$$M_1 = M_2 = M$$

At standstill: $N_0 = 0$, it is:

$$M_1 = \frac{M}{\sin \omega}$$

$$M_2 = 0$$

In Fig. 7 are shown the two quadrature components M_1 and M_2 of the magnetic field of the repulsion motor, for $\omega = 16°$ and $M = 1.0$.

In the theoretical investigation and practical calculation of the repulsion motor, just as in the polyphase and single-phase induction motor, the transformer feature is made the starting point and the motor considered as a transformer, the secondary of which is standing at a constant angle ω with regard to the primary, so that motion results from the repulsive thrust existing between primary and secondary.

Let in vector denotation:

$\dot{E}_0$ = impressed e.m.f., used as zero vector e_0;

$\dot{I}$ = primary current;

$\dot{I}_1$ = secondary current reduced to the primary by the ratio of turns (the same as customary in induction motors);

Z = primary exciting impedance that is, primary induced e.m.f., divided by primary current at open secondary circuit;

Z_0 = primary self-inductive impedance;

Z_1 = secondary self-inductive impedance reduced to the primary circuit;

ω = angle between primary and secondary axis; that is, angle of shift of brushes from the position of complete transformer action;

$a = (N_0/N)$ = ratio of speed to frequency.

The total m.m.f. acting upon the primary coil is then: Primary current $\dot{I}$ plus component of secondary current acting in the direction of the primary, or $\dot{I}_1 \cos \omega$.

Hence,

$$\dot{I} + \dot{I}_1 \cos \omega$$

This, then, is the primary exciting current; and the primary induced e.m.f. is therefore $(\dot{I} + \dot{I}_1 \cos \omega) Z$.

The primary impedance voltage is $\dot{I} Z_0$.

Hence,

$$e_0 = Z(\dot{I} + \dot{I}_1 \cos \omega) + \dot{I} Z_0.$$

In the secondary circuit an e.m.f. is induced by its rotation through the magnetic flux $\dot{M}_1$ in quadrature with the secondary axis, which is in phase with this magnetic flux. This magnetic flux $\dot{M}_1$ is due to the component of primary current acting in this direction: $\dot{I} \sin \omega$, since the secondary current exerts no m.m.f. at right angles to the axis of the secondary coil. The e.m.f. induced thereby is in phase with $\dot{I}$.

This e.m.f., induced by flux $\dot{M}_1$ is therefore,

$$\dot{E}_1 = a\,x\,\dot{I} \sin \omega \qquad (4)$$

In the secondary, an e.m.f. is induced also by the alternation of the magnetic flux M_2 in the axis of the secondary coil.

This magnetic flux is produced by the m.m.f. of the secondary current $\dot{I}_1$ and the component of primary current acting in this direction: $\dot{I}$ cos ω, hence, due to

$$\dot{I}_1 + \dot{I} \cos \omega$$

and the e.m.f. induced hereby is

$$\dot{E}_2 = -Z\,(\dot{I}_1 + \dot{I} \cos \omega)$$

The impedance e.m.f. of the secondary circuit is $Z_1 \dot{I}_1$, and since the secondary is short circuited, it is

$$Z\,(\dot{I}_1 + \dot{I} \cos \omega) + Z_1 \dot{I}_1 - a\,x\,\dot{I} \sin \omega = 0$$

These are the two fundamental equations of the repulsion motor:

Primary circuit:

$$e_0 = Z\,(\dot{I} + \dot{I}_1 \cos \omega) + \dot{I}\,Z_0$$

Secondary circuit: (5)

$$0 = -a\,x\,\dot{I} \sin \omega + Z\,(\dot{I}_1 + \dot{I} \cos \omega) + Z_1 \dot{I}_1$$

Where we assumed the impressed e.m.f. e_0 as zero vector

From the second equation it follows:

$$\dot{I}_1 = \frac{-\dot{I}\,(Z \cos \omega - a\,x \sin \omega)}{Z + Z_1}$$

Substituting this in the first equation, gives:

Primary current;

$$I = \frac{e_0}{Z} \frac{Z + Z_1}{a\,x \sin \omega \cos \omega + Z \sin^2 \omega + Z_0 + Z_1 + \frac{Z_0 Z_1}{Z}} = e_0\,(a_1 + j\,a_2) \quad (6)$$

Secondary current;

$$\dot{I}_1 = \frac{e_0}{Z} \frac{Z \cos \omega - a\,x \sin \omega}{a\,x \sin \omega \cos \omega + Z \sin^2 \omega + Z_0 + Z_1 + \frac{Z_0\,Z_1}{Z}} = e_0\,(b_1 + j\,b_2)$$

Hence,

Power-factor: $\cos \phi = \dfrac{a_1}{\sqrt{a_1^2 + a_2^2}}$

Volt-ampere input; $Q_0 = e_0\,I = e_0^2 \sqrt{a_1^2 + a_2^2}$

Power input; $P_0 = /e_0, \dot{I}/^1 = e_0^2\,a_1$

Power output; $P = /\dot{E}_1 \dot{I}_1/^1 = /a\,x\,\dot{I} \sin \omega, \dot{I}_1/^1$

$$= a\,x \sin \omega /\dot{I}, \dot{I}_1/^1$$

$$= a\,x\,e_0^2 \sin \omega\,(a_1\,b_1 + a_2\,b_2)$$

Torque;
$$T = (P/a) = x \sin \omega / \dot{I}, \dot{I}_1/^1$$
$$= x\, e_0^2 \sin \omega \,(a_1\, b_1 + a_2\, b_2)$$

Efficiency;
$$\gamma = \frac{P}{P_0} = \frac{a\, x \sin \omega \;(a_1\, b_1 + a_2\, b_2)}{a_1}$$

etc.

Substituting

$Z_0 + Z_1 = Z^1 = r^1 - j\, x^1 =$ total internal self-inductive impedance. Neglecting minor terms, it is, approximately:

Primary current;

$$\dot{I} = \frac{e_0}{(a\, x \sin \omega \cos \omega + r^1) - j\,(x \sin^2 \omega + x^1)} \tag{7}$$

Secondary current;

$$\dot{I}_1 = -\frac{e_0\,(\cos \omega - j\, a \sin \omega)}{(a x \sin \omega \cos \omega + r^1) - j\,(x \sin^2 \omega \;\; x^1)} \tag{8}$$

Angle of lag; $\tan \phi = \dfrac{x \sin^2 \omega + x^1}{a\, x \sin \omega \cos \omega + r^1}$ (9)

Torque; $T = \dfrac{e_0^2\, x \sin \omega \cos \omega}{(a\, x \sin \omega \cos \omega + r^1)^2 + (x \sin^2 \omega + x^1)^2}$ (10)

The secondary current leads the primary current by angle

$$\tan x_1 = a \tan \omega$$

The secondary current comes in phase with the primary impressed e.m.f. e_0,

$$\text{if} \qquad \frac{a \sin \omega}{\cos \omega} = \frac{x \sin^2 \omega + x^1}{a\, x \sin \omega \cos \omega + r^1}$$

hence,

$$a = -\frac{r^1}{2\, x \sin \omega \cos \omega} + \sqrt{\frac{x^1}{x \sin^2 \omega} + \frac{(r^1)^2}{4\, x^2 \sin^2 \omega \cos^2 \omega}}$$

Above this speed, the secondary current leads the primary impressed e.m.f. e_0, hence magnetizes.

At $a = 1$, that is, at synchronism, the secondary current equals the primary current, $I_1 = I$, but leads by the angle of brush-shift ω.

At infinite speed, $a = \infty$.

$$I = 0$$

That is; the primary current decreases with increasing speed, down to zero at infinite speed. (In the ordinary induction motor the current can never fall below a certain minimum value, the exciting current).

In this case,

$$I_1 = j \frac{e_0}{x \cos \omega}$$

that is, lags 90° behind the primary impressed e.m.f., hence is 90° ahead of the phase of a transformer secondary current, and supplies the magnetizing current of the magnetic circuit, just as a condenser in the secondary circuit of a transformer would do.

At standstill:

$$a = 0;$$

$$I = \frac{e_0}{r^1 - j\,(x \sin^2 \omega + x^1)}$$

$$I_1 = \frac{e_0 \cos \omega}{r^1 - j\,(x \sin^2 \omega + x^1)}$$

$$\tan \phi = \frac{x \sin^2 \omega + x^1}{r^1}$$

$$T = \frac{e_0^2\, x \sin \omega \cos \omega}{(r^1)^2 + (x \sin^2 \omega + x^1)^2}$$

Current and torque are a maximum, the power-factor a minimum at standstill.

Backward rotation:

For negative values of speed a the torque T is still positive, hence opposed to the rotation, and the repulsion motor with reversed brush-angle ω acts as brake.

The primary current,

$$I = \frac{e_0}{(a\, x \sin \omega \cos \omega + r^1) - j\,(x \sin^2 \omega + x^1)}$$

for negative values of speed a becomes wattless, when

$$a\, x \sin \omega \cos \omega + r^1 = 0$$

or

$$a = - \frac{r^1}{x \sin \omega \cos \omega}$$

For higher negative values of a, or if at a speed greater than $a = (r^1/x \sin \omega \cos \omega)$ (a very low speed), the brush-angle ω is reversed, the energy component of the primary current becomes negative; that is, the motor returns power into the line.

Unlike the plain series motor, which never returns power into the line, the repulsion motor when reversed becomes a generator, consumes mechanical power as brake and returns electric power into the line, even at low speeds. Experiment verifies this feature.

Discussion of Theory.

In the preceding outline of the theory of the single-phase alternating-current repulsion motor, the impedances Z, Z_0, Z_1, have been assumed as constant.

While this is approximately the case in the ordinary induction motor, it is not the case with the repulsion motor which works over a wide range of magnetic flux densities.

With increasing load and thereby increasing current and decreasing speed, magnetic saturation is approached and causes a decrease of the impedances Z, Z_0, Z_1, which has to be taken into consideration in predetermining the characteristic curves of such a motor. Furthermore, the different component magnetic fluxes are affected differently by saturation. The flux M interlinked with the primary coil is approximately constant and therefore affected by saturation only indirectly, while the flux M_1 at right angles to the line of polarization of the secondary coil is approximately proportional to the load and so reaches saturation at high loads, and the impedances become thereby different in the different directions of the magnetic structure.

The m.m.fs. in the preceding have been treated as vector quantities, independent of their distribution around the periphery of the armature. This distribution, however, is different with the different m.m.fs. The m.m.f. causing the effective flux M_1 is due to a zone of the primary winding within the angle $\pm\ \omega$ from the axis of the secondary coil, hence nearly a concentrated winding, which gives a flat-topped flux distribution, while the flux M_2 in the direction of the axis of the secondary coil is that of a distributed winding or peaked. For the same m.m.f. the effective flux M_1 is therefore greater than the effective flux M_2. Taking this into consideration, gives the motor somewhat better characteristics than calculated above. Due to the different wave shapes of the fluxes M_1 and M_2, they are affected differently by saturation. The flat-topped flux M_1 reaches saturation at a much higher value, but then over the whole range; while the peaked flux M_2 shows the effect of saturation at a lower value but then gradually, by a rounding off of the peak. In the repulsion motor it is therefore not sufficient merely to consider the resultant m.m.fs. as vectors but their distribution in the air-gap and the effect of saturation must be taken into consideration in the calculation and design of the motor. An exhaustive investigation hereof has been made by my assistant, Mr. M. Milch, and may be communicated at a later date.

The secondary circuit of the motor has been considered as the seat of two e.m.fs. induced respectively by the rotation through flux M_1 and by the alternation of flux M_2. These e.m.fs., however, have no separate existence. At synchronism, for instance, the magnetic field is an approximately uniformly rotating field and therefore no e.m.f. is induced in the armature conductors except that required to overcome the resistance. The secondary frequency varies with the load and thereby the secondary self-inductive reactance which we assumed as constant in the preceding discussion. This is best taken into consideration by a theory developed by my former assistant, Mr. S. Sugiyama, of Japan. The primary impressed alternating m.m.f. of the current I is resolved into two component m.m.fs. of half intensity, revolving synchronously in opposite directions. If now a = the ratio of speed to synchronism, the two oppositely revolving components of I revolve with regard to the secondary system with the speeds $(1 - a)$ and $(1 + a)$ respectively. The same consideration applies to the secondary m.m.f., I_1, and in the secondary system we then have induction at two frequencies, $1 - a$ and $1 + a$, of which the former becomes zero at synchronism. That is, at synchronism the secondary current in the armature conductors is of double frequency, similar as in the ordinary single-phase induction motor. At other speeds it is the superposition of two currents of the frequencies $1 - a$ and $1 + a$, respectively. This theory more closely allies the repulsion motor with the ordinary induction motor. Using the same values of secondary impedance, Z_1, for both components, obviously leads to identically the same equations as given in the preceding.

The complete investigation of the repulsion motor must also take into consideration the current flowing in the armature coil during the moment where the coil is short circuited by the brushes passing from commutator segment to segment. The m.m.f. of this short-circuit current of commutation is at right angles to the axis of secondary polarization of current, I_1, hence has the angle of brush-shift $-(90-\omega)$. A corrective term must therefore be applied, taking this phenomenon into consideration, essentially of the character of a repulsion machine with negative or generator brush-angle of $(90-\omega)$ and very high effective secondary resistance. This term is very small or negligible at speeds up to a point somewhat beyond synchronism but becomes noticeable at speeds considerably above synchronism, due to the decrease of the main current at these speeds. The main effect of this phenomenon is that the power-factor of the motor instead of increasing indefinitely with the speed up to 100% at some very high speed (and then decreasing again slightly, with leading current), reaches a maximum somewhere between 90 and 97% according to the constants of the motor, and then very slightly decreases with increasing speed as shown in the curve of the power-factor of a repulsion motor in Fig. 2.