

PROCEEDINGS
OF THE
LONDON MATHEMATICAL SOCIETY.

VOL. XXV.

THIRTIETH SESSION, 1893-94.

November 9th, 1893.

ANNUAL GENERAL MEETING, held at 22 Albemarle Street, W.

Mr. A. B. KEMPE, F.R.S., President, in the Chair.

SPECIAL MEETING.

The President stated that, in accordance with a notice sent out to all members capable of attending, the meeting had been made a "special" one, for the purpose of considering the following resolution, which would be submitted by the Council, viz., "That the London Mathematical Society be incorporated as a Limited Liability Company, under Section 23 of the Companies' Act, 1867, and that the Council be empowered to take the necessary steps to carry this resolution into effect."

The adoption of the above resolution was moved by Mr. Basset, seconded by Dr. Larmor, and further supported by Mr. S. Roberts and the Chairman. The resolution was carried unanimously, and the meeting then became the

ANNUAL GENERAL MEETING.

The President informed the members of the recent decease of Mr. W. S. B. Woolhouse, F.R.A.S., and gave the following short sketch of his life and work:—

Since our last meeting we have lost by death one who had been a member of our Society for over twenty-five years, viz., Westley

Stoker Barker Woolhouse, who has just passed away at the age of eighty-four. In laying before the Society a very slender sketch of his life and achievements, I must express the obligations I am under to his grandson, Mr. Rea, and our Secretary, Mr. Tucker, for information without which I could not have ventured to make even the brief tribute of respect to his memory which I propose to make. I regret that the time and opportunities at my disposal have not been such as to enable me to do something more to testify to our appreciation of one who was in many respects remarkable.

Mr. Woolhouse was born on May 6th, 1809. As a boy, though fond of practical jokes, he was regarded as particularly dull, his stupidity at figures prompting a mode of education which was at one time much in vogue, viz., blows on the head with a ruler. This mode of stimulating the brain, though not approved of in our own times, seems to have been effective, for he suddenly developed a most pronounced mathematical faculty, which came to light by the discovery of some difficult integrations chalked on the shop shutters in the lower part of the town of North Shields, where he resided. These having been traced to young Woolhouse, the interest of the learned of the town was excited, and his progress became assured.

The communication of problems to the *Newcastle Magazine*, and the *Lady's and Gentleman's Diary*, followed, and at thirteen he obtained a prize from the latter for higher mathematics, over the heads of several mathematicians of repute. At nineteen, without, it is said, ever having seen any treatise on the subject, he published a work on geometry of two dimensions. He also, about the same time, published some interesting investigations in dynamics.

These indications of his powers led to the adoption of a scientific career, and, after beginning as a computer for the *Nautical Almanack*, he rose to the post of Deputy Superintendent. The appendices to the *Almanack* bear witness to the part he played in constructing the formulæ employed in its calculation. Thus, in the *Almanack* for 1835, there are to be found (Appendix, 1-39) "New Tables for computing the occultations of Jupiter's satellites by Jupiter, the transits of the satellites and their shadows over the disc of the planet, and the positions of the satellites with respect to Jupiter at any time," and a paper "On the Computation of an Ephemeris of a Comet from its Elements" (Appendix 40-48). The *Almanack* for 1836 has an Appendix by him (pp. 53-148) "On Eclipses," and that for 1837 one "On the Determination of the Longitude from an observed Solar Eclipse or Occultation" (Appendix, pp. 172-183).

Subsequently he became the actuary of the International Loan Fund. His actuarial work on the tables employed by life assurance offices was of first-rate importance; but perhaps "his most remarkable feat was the solution of a problem in probabilities in connexion with the Ten Hours Bill. The question was how far the factory girls had to run in a day, when attending the 'mules' and trotting backwards and forwards to tie the threads which were constantly breaking. Mr. Woolhouse was engaged by Lord Ashley to go down to Manchester and obtain the necessary data. He performed the journey, obtained the data, solved the problem, wrote his report, and sent it off by the same evening's post. Mr. Woolhouse's calculation showed that the thread girl ran upwards of thirty miles each working day." Some remarks of his on the problem are to be found in the *Lady's and Gentleman's Diary* for 1859, pp. 91-95.

The *Philosophical Magazine* contains several papers from his pen. There was one in 1836, "On the Theory of Gradients on Railways" (*Phil. Mag.*, VIII., 243-6), and another, in the same year, "On the Theory of Vanishing Fractions" (*Phil. Mag.*, VIII., 293-400; IX., 18-26, 209-212). One in 1860, "On the Deposit of Submarine Cables" (*Phil. Mag.*, XIX., 345-361), led to a letter from the late Astronomer Royal, highly complimenting him on having "completely mastered a rather difficult investigation." And a paper in the same magazine in 1861, "On the Rev. T. P. Kirkman's Problem respecting certain Triadic Arrangements of Fifteen Symbols" (*Phil. Mag.*, XXII., 510-515), shows that he must be numbered among those who have attacked the "Girls' School Problem."*

The Royal Society's catalogue of scientific writings refers also to a paper by him in the *Mathematical Miscellany* of 1838 (Vol. I., pp. 336-343), "On the Theory of Exponential and Imaginary Quantities."

To our own *Proceedings* he communicated but one paper, which was "On General Numerical Solution," Vol. II., p. 75, 1868.

In a list of his writings before 1880, which has been furnished me

* In the *Lady's and Gentleman's Diary* for 1862 (pp. 84-88), and again in that for 1863 (pp. 79-90), Woolhouse fully discusses, in very small type, this question, which was first proposed, in the *Diary* for 1860, by the Rev. T. P. Kirkman. Of the previous solutions he says: "They are irregular in construction, and do not in their present state suggest any system of derivation." He proposes, in his remarks, to give "a complete and systematic solution, with a few observations." Further, in the *Diary* for 1865 (pp. 94, 95), he gives a short note on Combinations, in connexion with the *Phil. Mag.* article on Triads.

by Mr. Tucker, I find reference to the following works, besides those of which I have already spoken:—

“On the Application of Algebraic Analysis to Geometry,” Lond., 1831, 8vo;

“On Musical Intervals,” 1835, 8vo;

“Tables of Continental Lineal and Square Measures,” 1836, 8vo;

“On the Mortality in the Indian Army,” 1839, 8vo;

“Elements of the Differential Calculus,” 1852, 8vo;

“Measures, Weights, and Monies of all Nations,” 1856, 12mo;

“Memoirs of the Early Life of W. S. B. Woolhouse,” North Shields, 8vo.

I have reason to believe that the list which I have given from the materials at my disposal is far from exhaustive,* and that other important papers might be referred to. Among his minor works may be mentioned the editing of the Almanacks of the Stationers' Company for half a century, and an edition of Tredgold “On the Steam Engine.” He was a frequent contributor to the mathematical columns of the *Educational Times*.

Some allusion should be made in conclusion to his character, which was one of unblemished simplicity. He was, says his grandson, “a perfect child in business affairs, always too ready to please, and too willing to be led.”

He next gave a brief account of Prof. Klein's mathematical work, in connexion with the fourth award of the De Morgan Medal (made at the June meeting of the Council) to that gentleman. As Prof. Klein was unable to be present, the medal, at his instance, was given in charge to Prof. Greenhill and Dr. Forsyth; these gentlemen made suitable replies to the President's address.†

The Treasurer then read his Report. Its reception was moved by Dr. Forsyth, seconded by Mr. S. Roberts, and supported by Mr. Basset (who expressed the hope that, with a succession of favourable reports and strict economy, the time was not far distant when the Society would have rooms of its own with suitable accommodation for its accumulating library), and carried unanimously.

* The theorem now known as Holditch's Theorem (Williamson, *Int. Calc.*, third edition, p. 206) was first proposed in the *Diary* for 1858 (Prize Quest. 1928). Mr. Woolhouse gives a *general* solution (*cf.* Williamson, *l.c.*) on p. 96, and, in the *Diary* for 1859, he gives (pp. 89–95) some “general theorems in further extension of Quest. 1928.”

† This address is given, *in extenso*, in *Nature*, November 23rd, 1893 (p. 80).

At the request of the Chairman, the Rev. T. R. Terry consented to act as Auditor.

From the Report of the Secretaries, it appeared that the number of the members during the session had been 221, but was now reduced to 216, in consequence of two deaths, two withdrawals, and one removal. The number of compounders is 98.

The Society had to regret the loss, by death, of Mr. Hari Dās Sāstri, M.A., who was elected a member December 11th, 1890; and of Mr. Westley Stoker Barker Woolhouse, F.R.A.S., who was elected a member December 12th, 1867.

The following communications had been made or received:—

- Collaboration in Mathematics (Valedictory Address): Prof. Greenhill.
 Note on the Equation $y^2 = x(x^4 - 1)$: Prof. W. Burnside.
 Some Properties of Homogeneous Isobaric Functions: Prof. E. B. Elliott.
 On certain General Limitations affecting Hyper-Magic Squares: Mr. S. Roberts.
 On a Theorem in Differentiation, and its Application to Spherical Harmonics: Dr. Hobson.
 Notes on Determinants: Mr. J. E. Campbell.
 On the Evaluation of a certain Surface-Integral, and its Application to the Expansion, in Series, of the Potential of Ellipsoids: Dr. Hobson.
 On the Application of the Sylvester-Clifford Graphs to Ordinary Binary Quantics (second part): Mr. A. B. Kempe.
 On the Vibrations of an Elastic Circular Ring: Mr. Love.
 Note on Secondary Tucker-Circles: Mr. J. Griffiths.
 On a Group of Triangles inscribed in a given Triangle ABC , whose Sides are Parallel to Connectors of any Point P with A, B, C : Mr. R. Tucker.
 On the Thirty Cubes that can be constructed with Six differently Coloured Squares: Major P. A. MacMahon.
 Note on the Stability of a Thin Elastic Rod: Mr. Love.
 A Geometrical Note: Mr. R. Tucker.
 The Dioptrics of Gratings: Dr. J. Larmor.
 On a Threefold Symmetry in the Elements of Heine's Series: Prof. L. J. Rogers.
 Note on the Centres of Similitude of a Triangle of Constant Form inscribed in a given Triangle: Mr. J. Griffiths.
 On a Problem of Conformal Representation: Prof. W. Burnside.
 On the Collapse of Boiler Flues: Mr. Love.
 On some Formulæ of Codazzi and Weingarten in relation to the Application of Surfaces to each other: Prof. Cayley.
 On Complex Primes formed with the Fifth Roots of Unity: Prof. Tanner.
 The Singularities of the Optical Wave-Surface, Electric Stability, and Magnetic Rotatory Polarization: Dr. J. Larmor.
 On the Linear Transformations between Two Quadrics: Dr. H. Taber.
 Note on some Properties of Gauche Cubics: Mr. T. R. Lee.
 Pseudo-Elliptic Integrals, and their Dynamical Applications: Prof. Greenhill.

- Complex Integers derived from $\theta^3 - 2 = 0$, and on the Algebraical Integers derived from an Irreducible Cubic Equation : Prof. G. B. Mathews.
- Note on the Centres of Similitude of a Triangle of Constant Form circumscribed to a given Triangle : Mr. J. Griffiths.
- A Note on Triangular Numbers : Mr. R. W. D. Christie.
- The Harmonics of a Ring : Mr. W. D. Niven.
- Toroidal Functions : Mr. Basset.
- On the Expansion of certain Infinite Products (2) : Prof. L. J. Rogers.
- A Theorem for Bicircular Quartic Curves and for Cyclides analogous to Ivory's Theorem for Conics and Conicoids : Mr. A. L. Dixon.
- On Maps and the Problem of the Four Colours : Prince C. de Polignac.
- On Fermat's Proof that Primes of the Form $4n + 1$ can be broken up into the Sum of Two Squares : Mr. S. Roberts.
- On Cauchy's Condensation Test for the Convergency of Series : Dr. M. J. M. Hill.

The same Journals had been subscribed for as in the preceding session. An additional exchange of *Proceedings* had been made with the Mathematical Society of Amsterdam.

As it is some years since the list of exchanges has been printed in the *Proceedings*, it is here given as it stands at the present time.

*Addresses of Societies and Persons, not Members, who receive the
"Proceedings of the London Mathematical Society."*

1. The Royal Society.
2. The Royal Society of Edinburgh.
3. The Royal Irish Academy.
4. The Library of Trinity College, Dublin.
5. The Cambridge Philosophical Society.
6. The Philosophical Society of Manchester.
7. The Institute of Actuaries.
8. The Library of University College, *Gower Street*.
9. The Principal Librarian of the British Museum.
10. The University Library, *Cambridge*.
11. The Bodleian Library, *Oxford*.
12. The Faculty of Advocates, Edinburgh, *Advocates' Library*.
13. The Librarian, Mason Science College, *Birmingham*.
14. The Edinburgh Mathematical Society, Edinburgh.
15. The Editor of *Nature*.
16. The Canadian Institute, *Toronto*.
17. The Smithsonian Institute, *Washington, D.C., U.S.A.*
18. The United States Naval Observatory, *Washington, D.C., U.S.A.*
19. The Connecticut Academy, *New-Haven, Conn., U.S.A.*
20. The Editors of the "American Journal of Mathematics," *Johns Hopkins University, Baltimore, Md., U.S.A.*

21. The Editors of "The Annals of Mathematics," *Leander McCormick Observatory, University of Virginia, U.S.A.*
22. L'Institut National de France, *Paris.*
23. La Société Mathématique, *7 Rue des Grands-Augustins, Paris.*
24. La Société Philomathique, *7 Rue des Grands-Augustins, Paris.*
25. [M. le Général Commandant] l'Ecole Polytechnique, *Paris.*
26. La Société des Sciences physiques et naturelles, *Bordeaux.*
27. Bibliothèque Universitaire de médecine et des sciences alliées, *St. Michel, Toulouse.*
28. L'Académie Royale des Sciences, des Lettres, et des Beaux Arts de Belgique; *Palais des Académies, Bruxelles.*
29. La Société Hollandaise (par l'entremise du Bureau scientifique central Néerlandais), *Haarlem.*
30. M. le Prof. Bierens de Haan, Rédacteur de "Nieuw Archiv," *Leiden.*
31. The Editors of the "Annales de l'Ecole Polytechnique à Delft."
32. The Mathematical Society of Amsterdam.
33. Reale Istituto Lombardo di Scienze e Lettere, *Milan.*
34. Reale Accademia dei Lincei, *Palazzo delle Scienze, Lungara 10, Roma.*
35. Reale Accademia di Scienze, Lettere, ed Arti, *Modena.*
36. Reale "Accademia delle Scienze fisiche e matematiche," *Napoli.*
37. Reale Istituto Veneto di Scienze, Lettere, ed Arti, *Venezia.*
38. Circolo Matematico di Palermo.
39. M. le Prof. F. Gomes Teixeira, *Coimbra.*
40. La Société Mathématique, [Cabinet de Mécanique, Université,] *Odessa.*
41. Akademie der Wissenschaften, *Berlin.*
42. The Editor of the "Journal für die reine und angewandte Mathematik (Crelle)."
43. The Authors of the "Jahrbuch über die Fortschritte der Mathematik." *Berlin.*
44. Die Königliche Gesellschaft der Wissenschaften, *Göttingen.*
45. [Dem Herrn Bibliothekar von der] Universität (an der Physischen und Medicinischen Gesellschaft), *Erlangen.*
46. "Beiblätter zu den Annalen der Physik und Chemie," *Leipzig.*
47. Die Königliche Sächsische Gesellschaft, *Leipzig.*
48. Die Naturforschende Gesellschaft, *Zürich.*
49. The Editors of the "Prace matematyczno fizyczne" of Warsaw.

The meeting next proceeded to the election of the new Council. Mr. Jenkins having read the rules bearing on the election, the President nominated, as Scrutators, Prof. Hudson and the Rev. T. R. Terry. These gentlemen, having examined the balloting lists, declared the following gentlemen duly elected:—

Mr. A. B. Kempe, F.R.S., President; Mr. A. B. Basset, F.R.S., Prof. Elliott, F.R.S., Prof. Greenhill, F.R.S., Vice-Presidents; Dr. J. Larmor, F.R.S., Treasurer; Messrs. M. Jenkins and R. Tucker, Honorary Secretaries. Other Members of the Council: Lieut.-Col. J. R. Campbell, F.G.S., Lieut.-Col. A. J. Cunningham, R.E., Dr. A. R.

Forsyth, F.R.S., Dr. J. W. L. Glaisher, F.R.S., Dr. M. J. M. Hill, Dr. E. W. Hobson, F.R.S., Mr. A. E. H. Love, Major MacMahon, R.A., F.R.S., Mr. J. J. Walker, F.R.S.

The following communications were made :—

A Mechanical Solution of the Problem of Tethering a Horse to the Circumference of a Circular Field, so as to Graze over an n th part of it: Prof. L. J. Rogers. (The solution turned on a property of the cycloid.)

The Stability of certain Vortex Motions: Mr. A. E. H. Love.

Cyclotomic Quartics: Prof. G. B. Mathews.

On the Application of Elliptic Functions to the Curve of Intersection of Two Quadrics: Mr. J. E. Campbell.

Notes on the Theory of Groups of Finite Order: Prof. W. Burnside.

Prof. Hudson showed, and explained, some mechanical constructions by his son, R. W. Hudson, for the Parabola, Hyperbola, Cubical Parabola, and Semi-Cubical Parabola.

Messrs. Hill, Basset, Greenhill, Walker, and the President, took part in the discussions which followed the reading of the papers.

The following presents were received :—

Cabinet Likeness of Mr. G. Heppel, for the Album, from Mr. Heppel.

Mukhopadhyay, A.—“Elementary Treatise on the Geometry of Conics,” 8vo; London, 1893.

“Proceedings of the Royal Society,” Vol. LIV., No. 327.

“Beiblätter zu den Annalen der Physik und Chemie,” Band XVII., Stück 9; Leipzig, 1893.

“Journal of the Institute of Actuaries,” Vol. XXXI., Part 1, No. 171; October, 1893.

“Institute of Actuaries,” List of Members to September, 1893.

“Proceedings of the Royal Irish Academy,” Vol. II., Nos. 4, 5, N. S., May, August, 1893; Dublin.

Issaly, Mons. l'Abbé.—“Optique Géométrique,” 5^e Mémoire, Extrait des Mémoires de la Société des Sciences physiques et naturelles de Bordeaux, T. IV., 4^e Série.

“Bulletin of the New York Mathematical Society,” Vol. III., No. 1; October, 1893.

“Rendiconti del Circolo Matematico di Palermo,” Tomo VII., Fasc. 3, 4, and 5.

“Atti della Reale Accademia dei Lincei,” 1893, Serie 5^a, Rendiconti, Vol. II., Fasc. 7 and 8, 2^e Sem.; Roma, 1893.

“Educational Times,” November, 1893.

“Annals of Mathematics,” Vol. VII., Nos. 5 and 6; October, 1893; University of Virginia.

“Indian Engineering,” Vol. XIV., Nos. 13, 14, 15, 16.