



# LLI. On the proper relative sectional areas for copper and iron lightning-rods

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mass of similar calculations, came into the possession of Dr. Olinthus Gregory, and were purchased by the Royal Society at the sale of his books in 1842." Nothing is known of them, however, at the Royal Society. The Cambridge University Library contains two copies of each of Mr. Goodwyn's publications of 1818 and 1823 \*, but no copy of the specimen of 1816. There is an account of Mr. Goodwyn's works in the British Association Report on Tables (Bradford, 1873, pp. 31-33), where I have erroneously attributed the property enunciated by Mr. Farey to Cauchy. A more complete description of Mr. Goodwyn's works is contained in a paper "On Circulating Decimals, with special reference to Henry Goodwyn's 'Table of Circles' and 'Tabular Series of Decimal Quotients' (London, 1818-1823)," printed in the 'Proceedings of the Cambridge Philosophical Society, vol. iii. (1879) pp. 185-206.

Trinity College, Cambridge.  
February 15th, 1879.

LII. *On the proper Relative Sectional Areas for Copper and Iron Lightning-Rods.* By R. S. BROUGH†.

SO far as mere conductivity is concerned, a comparatively thin wire of either copper or iron would suffice for the loftiest conductor; but such a thin conductor would be dangerous, because it would be fused by a heavy discharge of lightning. Now the problem is to determine what relative sectional areas should be given to copper and iron rods, in order that neither should be more liable to fusion than the other.

The usual answer given is, that an iron rod should have four times the sectional area of a copper rod‡. The result is, I suppose, arrived at in the following way. The conductivity

\* One of the copies of the 'First Centenary' (1818) contains the following letter, "September 16th, 1831. Mrs. Catherine Goodwyn presents to the Library of the University of Cambridge a complete set of the works of her late father, Henry Goodwyn, Esq., of Blackheath, Kent. Royal Hill, Greenwich." Mr. Goodwyn also published a few folding sheets on weights and measures &c., which are bound up at the end of this copy of the 'First Centenary.'

† Communicated by the Author, having been read before the Asiatic Society of Bengal in November 1878.

‡ War-Office Memorandum by General Sir Frederick Chapman, R.E., G.C.B.

of copper is about six times as great as that of iron; but the melting-point of iron is about 50 per cent. higher than that of copper; therefore  $\frac{6.0}{1.5} = 4$  is the ratio for the sectional area of iron to copper.

This simple treatment of the problem, however, is incomplete, because it neglects to take three most important factors into consideration—namely (1) the influence of the rise of temperature in increasing the electrical resistance of the metal, (2) the difference between the “thermal capacity” or “specific heat” of copper and iron, and (3) the fact that, the iron rod being made several times more massive than the copper rod, it will require a proportionately greater quantity of heat to increase its temperature. These omissions introduce an enormous error in the result.

The effect of the passage of a discharge of lightning through the rod will be to raise its temperature.

The temperature ( $T$ ) to which a given length of the rod will be raised will depend on:—

- (1) The quantity of heat developed by the discharge.
- (2) The mass of the rod.
- (3) The specific heat  $\sigma$  of the metal composing the rod.

This may be expressed mathematically as follows:

$$T = \text{const} \frac{H}{\sigma m};$$

where  $m$  is the mass of a unit length of the rod, which we shall assume to be uniform in sectional area throughout its length, and  $H$  is the quantity of heat developed by the discharge.

We may take  $\sigma = 0.1013$  for copper, and  $\sigma = 0.1218$  for iron. These figures were only verified, by Dulong and Petit, up to  $300^\circ \text{C}$ . It is probable, however, that their ratio, with which we are only here concerned, would not greatly alter at higher temperatures. At any rate, comparing the specific heats between  $0^\circ$  and  $100^\circ \text{C}$ . with those between  $0^\circ$  and  $300^\circ \text{C}$ ., we infer that any alteration would be in favour of iron, *i. e.* that the specific heat of iron would increase in a quicker ratio than that of copper.

Adopting the centimetre as the unit of length, the mass of one centimetre of the rod  $= \rho a$ , where  $a$  is the sectional area of the rod in square centimetres, and  $\rho = 8.9$  for copper, and  $\rho = 7.8$  for iron.

Further, assuming the quantity and duration of the discharge to be constants,  $H = \text{const} \times R$ , where  $R$  is the resistance of the unit length of the conductor.

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But  $R = \frac{\lambda}{a}$ , where  $\lambda$  is the specific resistance of the metal per cubic centimetre at its temperature of fusion.

We may take the melting-point of copper as  $1400^{\circ}\text{C}.$ , and that of wrought iron as  $2000^{\circ}\text{C}.$ ,\* and, in order to find  $\lambda$ , assume that Dr. William Siemens's formula, which he verified to  $1000^{\circ}\text{C}.$ , holds good†, viz.

$$\lambda_t = \lambda_0(0.026577 t^{\frac{1}{2}} + 0.0031443 t - 0.29751) \text{ for copper,}$$

$$\lambda_t = \lambda_0(0.072545 t^{\frac{1}{2}} + 0.0038133 t - 1.23971) \text{ for iron.}$$

The temperature  $t$  in these formulæ is to be measured from the absolute zero; so that we have  $t = 1673$  for copper, and  $t = 2273$  for iron.

The value of  $\lambda_0$  per cubic centimetre of copper is  $1.652$  microhm, and per cubic centimetre of iron is  $9.827$  microhms‡.

Thus the value of  $\lambda_t$  per cubic centimetre of copper becomes about  $10$  microhms at  $1673^{\circ}\text{C}.$ , and per cubic centimetre of iron becomes about  $107$  microhms at  $2273^{\circ}\text{C}.$

Hence

$$\left. \begin{aligned} H &= \text{const} \frac{10}{a} \text{ for copper,} \\ \text{and} \\ H &= \text{const} \frac{107}{a} \text{ for iron.} \end{aligned} \right\}$$

Therefore

$$\left. \begin{aligned} T &= \text{const} \frac{10}{0.1013 \times 8.9 \times a^2} \text{ for copper,} \\ \text{and} \\ T &= \text{const} \frac{107}{0.1218 \times 7.8 \times A^2} \text{ for iron.} \end{aligned} \right\}$$

Now, putting  $T =$  temperature of fusion in each case,

$$\left. \begin{aligned} 1400 &= \text{const} \frac{11.09}{a^2} \text{ for copper,} \\ 2000 &= \text{const} \frac{112.63}{A^2} \text{ for iron.} \end{aligned} \right\}$$

Therefore

$$\begin{aligned} \left(\frac{A}{a}\right)^2 &= \frac{1400}{2000} \cdot \frac{112.63}{11.09} \\ &= 7.112, \end{aligned}$$

whence

$$\frac{A}{a} = \frac{8}{3} \text{ nearly ;}$$

\* Rankine's Tables.

† Bakerian Lecture, 1871.

‡ Jenkin's Cantor Lectures, from Mathiessen's experiments.

or, the sectional area of an iron rod should be to the sectional area of a copper rod in the ratio of 8 to 3.

This result is an argument in favour of the use of iron as being the less expensive.

Calcutta, March 3, 1879.

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LIII. *Experiments for determining the Correction to be added to the Length of a Cylindrical Resonant Tube to find the true Wave-length, and the Velocity of Sound in small Tubes.*

*To the Editors of the Philosophical Magazine and Journal.*

GENTLEMEN,

I BEG to submit to your consideration for publication the accompanying memorandum of experiments made by me with the object of determining the correction to be added to the length of a cylindrical resonant tube to find the true wave-length, and the velocity of sound in small tubes.

They give results, as regards the first point, in the main confirmatory of those already arrived at by Lord Rayleigh and Mr. R. H. M. Bosanquet; but the methods of experiment followed by me being different to theirs, the notes may be of interest. As regards the second point I am not aware of any experimental results having been published.

I am, Gentlemen,

Yours obediently,

D. J. BLAIRLEY.

5 Compton Place,  
Canonbury, N.  
March 22, 1879.

The first method of experiment adopted by me is applicable only to tubes closed at one end, and consists in determining in a tube of indefinite length the positions of the first and second nodes of a wave excited by a fork held over its mouth, the tube having one end sunk in water, and the water-level at the position giving maximum resonance determining the position of the node. The tube used was made of thin brass, and had an internal diameter of 2·08 inches; it was slung with a pulley and counterweight over a deep vessel of water, and its height out of the water at each observation read off by a scale. The advantage of the use of water instead of a sliding solid plug is that there is not the slightest noise to interfere with the appreciation of the point of maximum resonance.

The forks used were  $c'$  253·55 vibrations,  $e'$  317·3,  $g'$  380·625,  $b'$  444·5, and  $e''$  507·2. Of these I had an opportunity of comparing all but the  $e'$  with Scheibler's standards in August last, through the kindness of Mr. A. J. Ellis.

The pitch of Scheibler's forks being determined at about 70° F., and the variation being about ·00005 per vibration