

A Differentiating Machine. By J. Erskine Murray, D.Sc.

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It was pointed out to me a few months ago, by my friend Professor W. H. Heaton, that our knowledge of the laws of physical variations might be greatly increased if their study were facilitated by the invention of a machine which would automatically deduce the rate of change of a function from the curve representing that function. In cases where the physical law is already known, and is expressible in terms of known mathematical quantities, such a machine is not essential, though it provides an excellent illustration of mathematical laws; there is, however, a vast and ever-increasing mass of numerical results awaiting discussion and co-ordination, and it is in reducing these to law and order that the differentiator should prove a useful tool. As instances of a few cases in which rates of change are of the first importance, I may mention the following:—

- (1) Meteorological observations of Temperature, Pressure, Humidity and Rainfall.
- (2) Terrestrial Magnetic records.
- (3) Experimental results in Physics and Chemistry which involve changes, whether in time or space. The determination of thermal conductivity by Forbes' method is an example.
- (4) Statistics of Population, Mortality, and Migration.
- (5) Statistics of Wages, Prices, and Commerce.
- (6) Medical records.
- (7) Engineering calculations, such as the deduction of Tractive Force from a Time and Space or Time and Velocity diagram.

Up to the present all determinations of rates of change of quantities like those above mentioned have had to be made by laborious arithmetical or graphical methods, involving so great an expenditure of time for their completion that but little has been done. The differentiator reduces enormously the necessary labour,

and even the roughly constructed instrument shown will give results sufficiently accurate for most purposes.

The construction of the differentiator depends on the well-known fact that if the values of a variable quantity be represented on a diagram by the ordinates of a curve, its rate of change, at any point of the curve, is measured by the slope of the tangent at that point.

The machine, then, is guided by hand so that one line on it remains tangent to the curve, while a tracing point describes on a second sheet of paper a curve whose ordinates are proportional to the slope of the tangent. Thus if $y=f(x)$ be the equation to the original curve, the derived curve will have for ordinates the corresponding values of $d(f(x))/dx$. The abscissæ are the same on both curves.

In order that a line may be tangent to a curve it is necessary that two consecutive points on each should coincide. In practice, two black dots on a piece of transparent celluloid are used, the distance between them being about 2 mm.

The plan of the machine is shown in fig. 1. It consists of three parts. Firstly, the large drawing-board $ABCD$, on which the original curve is placed. Fixed to each long side of this board is a metal rail, one, CE , having a plain surface, and the other, DF , a longitudinal groove of V-shaped section. The second part is a smaller board, CHI , having three spherical feet, two of which run in the groove and the third on the plane rail. This arrangement permits free motion of the smaller board in the direction of the length of the larger one, *i.e.* parallel to the Y coordinate. The small board carries the paper on which the derived curve is traced by the machine. Attached to its edge are guides, $JKLM$, which hold the principal part of the mechanism, allowing it free motion in a right and left line.

This part, shown in fig. 2, consists of a frame $ABCD$, at one corner of which is a pin, A , which serves as the vertical axis about which the rod PQ revolves in a horizontal plane. PQ has a slot in it, through which passes the pin R fixed to the rod ST . ST is controlled by guides E and F , so that it can only move in a direction parallel to OY .

Below the arm PQ , and fixed rigidly to it below A , is a small

plate of celluloid, not shown in the diagram, on the under side of which are two dots by which the machine is guided along the curve. The line through the dots is parallel to P Q. The celluloid rests on the paper on which the original curve is drawn, thus supporting the outer end of the frame A B C D.

Since the distance A V between the pin and the centre line

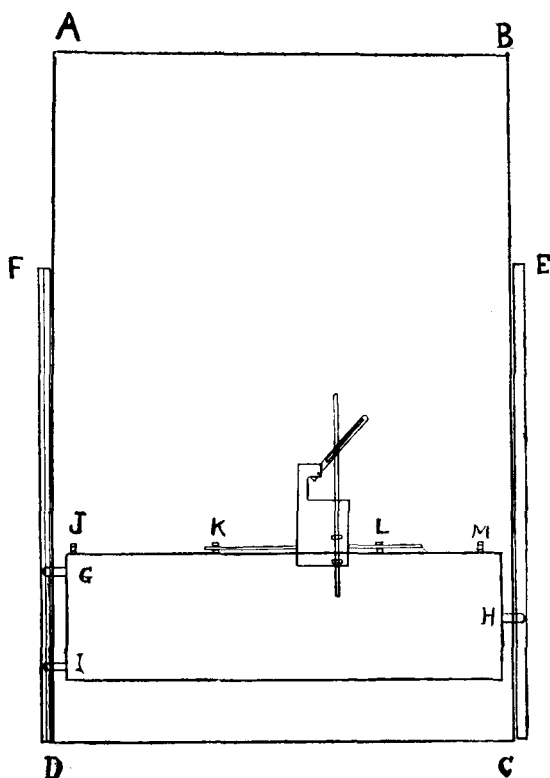


FIG. 1.

of S T is constant, and since $RV/AV \doteq dy/dx$, it is clear that the distance R V which R is displaced above or below the zero line A V measures the tangent of the angle of slope of the curve, i.e. dy/dx . A pen at the end T of S T records the movements of R, and therefore traces a curve of which the ordinates are proportional to the rate of change of the ordinate of the original curve. It should be noticed that the purpose of

