

bleaching agent at a moment's notice, and without requiring extensive changes in his present plant; that the bleaching agent so produced is free from any residue or sediment; that it is much more complete in its bleaching action than chloride of lime; that the action on the fibres is rapid, regular, and not in any manner injurious; that it is cheap, and with a constant, or nearly constant, cost, as the raw materials are low in price.

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## ACTION OF A SINGLE-PHASE SYNCHRONOUS MOTOR.

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BY FREDERICK BEDELL AND HARRIS J. RYAN.

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[Communicated by the authors to the Electrical Section of the Franklin Institute.]

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### INTRODUCTION.

The phase relations of the different variable quantities in any alternating current problem are of paramount importance, and have often been the subject of investigation in the study of the action of the transformer. The writers of this paper have recently had occasion to study the synchronous motor with particular reference to these phase relations, and to compare polar diagrams representing the action of the motor, constructed from *a priori* reasoning, with similar diagrams obtained by experiment. The complete discussion of the action of a synchronous motor which follows, is thus verified by experiment. The polar diagrams here given, constructed from experimental data, are now published for the first time; other portions of the work have already been published in some preliminary papers,\* parts of which are here included *verbatim*. The experimental

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\* "The Behavior of Single-Phase Synchronous Motors," by H. J. Ryan; *Sibley Journal of Engineering*, vol. viii, No. 8, May, 1894.

"An Optical Phase Indicator and Synchronizer," by G. S. Moler and F. Bedell, a paper read before the American Institute of Electrical Engineers, May, 1894; *Transactions*, vol. xi, p. 586.

"The Behavior of Single-Phase Synchronous Motors," by C. E. Hewitt and J. Lyman; *Sibley Journal of Engineering*, vol. viii, No. 9, June, 1894.

work was performed in the spring of 1894, under the direction of the writers, at Cornell University, by Messrs. J. Lyman and C. E. Hewitt, who have already published a partial report of the investigation in one of the papers already referred to.

We will first discuss the action of a synchronous motor, and will then describe the method of conducting the present investigation.

#### THE SYNCHRONOUS MOTOR.

The action of the commutator on a direct-current dynamo is to rectify the current induced in the armature; that is, the current from the dynamo after it leaves the brushes flows continually in one direction, although the current in the armature coils is being constantly reversed. The commutator of a direct-current motor performs the converse function. Let us suppose that we have two similar machines of this kind separately excited, and that the one is driving the other as a motor. If it so happens that the two armatures are revolving at the same speed, the two commutators may be done away with, for it is evident that the rectifying of the current by the dynamo commutator is counter-balanced by the corresponding reversal of the current by the motor commutator. The current in the coils of the motor armature flows alternately in opposite directions corresponding exactly to the flow of current in the dynamo armature. We see, then, that when the motor and dynamo are running synchronously the armature circuits may be directly connected without any commutators; that is, we have an alternating current generator driving an alternating current synchronous motor. The operation of a synchronous motor was first described by H. Wilde in a paper\* read before the Literary and Philosophical Society of Manchester, December 15, 1868. The inherent regulation which maintains synchronism was referred to in this paper.

The significance of these observations was not felt at

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\* "On a Property of the Magneto-electric Current to Control and Render Synchronous the Rotations of the Armatures of a Number of Electro-Magnetic Induction Machines;" *Philosophical Magazine*, January, 1869.

the time, and some years later Dr. J. Hopkinson,\* unaware of the earlier work, developed the analytical theory for the operation of two alternating current machines, his conclusions as to synchronous control agreeing with the observations of Wilde.

Although any alternating current generator will thus operate as a synchronous motor, it is not true that they will all thus operate equally well. The old smooth-bodied armature alternator,† with “pan-cake” armature coils, made a very poor synchronous motor, while the modern alternator, with “T-toothed” armature, fitted with machine-wound armature coils, has been found to give excellent results when run as a synchronous motor. This improved performance of the alternator as a synchronous motor is due to the increased self-induction of the armature and to the useful effects of the armature current on its own field. When the current developed by a generator is in unison with the generated E.M.F., such current exerts very little effect upon the field—neither strengthening nor weakening it. When the current lags behind the E.M.F. of the generator, the armature reaction effect that it produces upon the field is such as to weaken the field, and thus to diminish the E.M.F. produced by the generator. The reverse of this action occurs when the generator furnishes a current that is in advance of its E.M.F. This is shown to be true in the experiments which follow by the gradually increasing values of generator E.M.F. (given in the tabular data in the latter part of this paper), as the current, which at first lags behind the generator E.M.F., is brought into phase with it and

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\* “On the Theory of Alternating Currents, Particularly in Reference to Two Alternate-Current Machines Connected to the Same Circuit;” *Jour. Soc. Telegraph Engineers*, vol. xiii, p. 496, 1884. At the same meeting was read and discussed a paper on “The Alternate-Current Machine as a Motor,” by W. G. Adams. Further references relating to the development of the synchronous motor are given by W. M. Morley, in his paper on “Alternate-Current Working;” *Jour. Inst. Elect. Engineers*, vol. xviii, p. 592, 1889. See also, “Long-Distance Transmission for Lighting and Power,” by C. F. Scott; *Transactions Am. Inst. of Elect. Engineers*, vol. ix, June, 1892.

† We here closely follow the first paper above referred to: *Sibley Journal*, May, 1894.

finally into a position of advance. Precisely the same armature reactive effects on the field occur in a synchronous motor. There is this exception, however, that the motor E.M.F. is counter to that of the generator, so that what is a lagging current for the generator is an advance current for the motor, and the current that is in advance of the generator E.M.F. lags behind the counter E.M.F. generated by the motor. The current that strengthens the field of the generator will thus weaken the field of the motor, and *vice versa*, as will be explained later.

The speed, as is well understood, depends only on the periodicity produced by the generator and the number of poles of the motor. Neither variation of the E.M.F. impressed at the terminals of a synchronous motor, nor variation of its field excitation, will change the resulting speed so long as the motor operates at all. The armature circuits of the motor, generator and line always possess some self-induction. When, therefore, the generator pressure is higher than that of the motor at the moment when synchronism is obtained, and the machines are connected, a current will pass between the machines that will lag, because of the self-induction, behind the generator E.M.F., and will be in advance of that of the motor pressure. This current, as was just pointed out, will weaken the field of the generator and lower the E.M.F. that it generates; it will strengthen the field of the motor, and in proportion will raise its counter E.M.F. The result, therefore, is to equalize the developed pressures of generator and motor, and thus prevent a further increase of current. Such a current will have a larger component in unison with the generator pressure than with that of the motor, depending on the proportion of the electrical energy transformed into mechanical energy.

The diagram, *Fig. 1*, illustrates the action of a synchronous motor that develops a motor pressure  $E'$ , equal to the generator pressure  $E$ . In this, and in all the diagrams following, positive rotation is counter-clockwise. In the case of *Fig. 1*, the circuits are assumed to have no self-induction, but to have the usual resistance. At the instant that syn-

chronism is obtained and the connection of the motor completed, there can be no current established through the motor, and, therefore, no power developed. The motor will lag in rotative speed at once to some such position as  $d$ , where the resultant of the motor and generator pressures is  $ac$  or  $E''$ . Since there is no self-induction in the circuit, the resulting current,  $I$ , is in unison with  $E''$ . Such a current has component values in unison with the generator and motor pressures that are equal. No power can, therefore, be developed, and the motor would promptly come to a standstill.

The conditions represented by *Fig. 2*, are, that the generator pressure is greater than the motor pressure, and that the circuits possess resistance but no self-induction. At the moment that the motor is synchronized and connected, a current will be established that is in unison with the generator pressure and opposite to that developed by the motor. The developed power that thus results is, in general, more than sufficient to keep the motor running light at synchronism. The speed position of the armature is, therefore, advanced to a point  $d$ . Here is obtained a resultant E.M.F.,  $E''$ , that, since there is no self-induction, establishes a current which is in unison with the resultant E.M.F.,  $E''$ . The component  $ae$  along  $E$  is here greater than the component  $af$  in the direction of the motor E.M.F.,  $E'$ . A balance occurs at that point where the power developed is just sufficient to keep up the synchronous speed of the motor armature, and no further acceleration takes place. On loading down the motor its armature position is retarded. The maximum load that the motor will stand is that at which the product of the motor pressure into the component of the current that is in unison with it, is a maximum. This the diagram plainly indicates to be at the point of true synchronism. When this point is reached an increase in the load will further retard the armature, and the above product will again diminish; the motor, being overloaded, will come quickly to rest.

In *Fig. 3*,  $E$  and  $E'$  are equal, the armature and line circuits have no appreciable resistance, and self-induction is

present. At the instant of synchronism and connection of the motor to the generator circuit, no current will be established through the motor, because the generator and counter E.M.F.s are equal. The motor armature will lag to a point where the resultant pressure is  $E''$ .  $E''$  will establish a current  $I$ , one-quarter of a period behind itself. Such a current will have a large component that is negative with respect to the motor pressure and in unison with that of the generator. An early point is reached at which the motor will do good work. Later on a maximum component of this current, that is opposite to the motor pressure, will be found, beyond which the motor will lose synchronism.

In *Fig. 4*, the generator E.M.F.,  $E$ , is greater than the motor E.M.F.,  $E'$ . The circuits possess self-induction, but no resistance. At the instant that the motor is synchronized and connected, the resultant pressure,  $E''$ , is the algebraic difference between  $E$  and  $E'$ . Inasmuch as the circuit possesses self-induction with no resistance,  $E''$  will establish a current through the motor at right angles to itself and the motor and generator pressures. From such a current no power can result. The armature will lag to a later position, where the conditions are found to be practically the same as those discussed in connection with *Fig. 3*.

In *Fig. 5*,  $E$  equals  $E'$ , and resistance and self-induction are both present in the circuits. Since  $E$  and  $E'$  are equal, no work can be done until the armature lags to some position,  $\alpha$ , where the resultant E.M.F. is  $E''$ .

When the E.M.F., resistance, self-induction, and periodicity of a circuit, are known, the impedance is known, and the current becomes known from the relation

$$\text{Current} = \frac{\text{Resultant E.M.F.}}{\text{Impedance}}$$

The phase position of the current is determined by the relation between the resistance and the reactance of the circuit. The projection of  $I$  upon  $E'$  is a quantity that is proportional to the developed mechanical power. This projection, as in all other cases, at a certain position attains a maximum beyond which the motor will come to rest.

In the next diagram, *Fig. 6*,  $E$  is greater than  $E'$ , and self-induction and resistance are present. In general, the position of the armature at no load will be in advance of the normal position of synchronism, the position of advance being limited, as in the similar cases cited above, and which the figure fully illustrates.

In practice, the relations of the magnitudes of  $E$  and  $E'$  are determined by the field excitation and the currents that are established through the motor. Armatures that develop powerful reactive effects upon their fields, in action equalize the motor and generator pressures, and such armatures must necessarily possess considerable self-induction. These are the requirements for suitable working, as indicated by the above analysis.

It is well to suggest, also, that the amount of the mass of the revolving parts of a synchronous motor has an additional effect upon the stable operating conditions. During every complete period, there are two short intervals, throughout which the motor, in general, must act as a generator and give back a small amount of power to the generator. The only source of this power is the fly-wheel property of the revolving parts of the motor. Multiphase motors are independent of this fly-wheel effect, because at no instant does the motor do work on the generator, while on the other hand, the amount of mechanical energy developed instant after instant is practically uniform.

#### THE PHASE-INDICATOR.

In the experimental investigation of the synchronous motor, the Bedell-Moler phase-indicator\* was employed to determine the angular difference in the position of the armatures of the motor and generator. The shafts of the motor and generator were placed in line, abutting, but not quite touching. The phase-indicator consists of two metal disks, each fastened to a collar made to slip on the adjacent ends of the two armature shafts, and held in position by

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\* *Loc. cit.*

set-screws. These disks are of zinc, one-thirty-second of an inch thick, nine inches in diameter, and about one-quarter of an inch apart. In the disks are curved slits, about one-twentieth of an inch wide, extending from points near the hub almost to the circumference. In this case—the motor and generator having eight poles—there were four slits in each disk, one slit corresponding to each pair of poles, or to the number of complete periods in one revolution. The direction of curvature of the slits in one disk is opposite to that in the other, so that the slits of one cross those of the other; otherwise, the two disks are in every way similar. The intersection of the slits forms an opening that will allow a beam of light from an incandescent lamp, placed at one side, to pass through. Each slit extends over a complete period, from its inner to its outer end; therefore, one of the armatures must move, with reference to the other, through a complete period, to cause a spot of light to travel through this range. *Fig. 7* makes this more clear. If the two armatures are revolving at the same speed, or synchronously, the intersections of the slits remain at a fixed distance from the center, and we see, in consequence, a ring of light. If one revolves faster than the other, the ring of light moves either towards the center or towards the circumference of the disk. These slits are so constructed that the distance to or from the center, traversed by the intersections of the two sets of slits, is proportional to the change in the relative angular positions of the two armatures.

The curvature of the slit is that of the spiral of Archimedes, and was constructed as follows: Each quadrant was divided into eighteen equal sectors. Nineteen concentric circles were then drawn with a sharp tool on the face of the disk, thus dividing the portion of the disk lying between the extreme limits desired for the slit, into eighteen annuli, each about three-sixteenths of an inch wide. The slit represents a complete period of  $360^\circ$ ; therefore, the widths of the sectors and annuli represent  $20^\circ$ . The points of intersection of the corresponding radii and circles give the location of the slits.



The position of the ring of light indicates the relative positions of the two armatures. This position was observed in a mirror set at an angle of  $45^\circ$  with the plane of the disks, and arranged with a scale reading in degrees corresponding to the scratched circles on the disk. An observer could thus see, in the mirror, the reflected ring of light and the phase position which it indicated. The disks were set so that the outer ends of the slits were adjacent when the two armatures stood on their corresponding neutral points, the positions being determined by the ballistic galvanometer method of field exploration. With the armature on the neutral point, no throw of the needle of a ballistic galvanometer connected to the brushes is obtained, when the field current is reversed.

In operation, the ring of light shown by the phase-indicator would vibrate, more or less, according to the stability of the motor. This vibration was small, when the motor was operated under the most favorable conditions; but with unstable conditions, it would vibrate, at times, as much as  $30^\circ$  on each side of the mean, before the motor would go out of synchronism.

The present investigation has shown the phase-indicator to be, in all respects, an efficient instrument.

#### THE GENERATOR AND MOTOR.

Two small Westinghouse alternators, of the same size and shape, designed to supply ten incandescent lamps, were used as generator and motor. Before being thrown into direct connection with the generator, the motor was brought to speed by means of a direct current-starting motor, to which it was belted. A half horse-power Edison machine was used for this purpose. When synchronism was reached, the direct current supplying this starting motor was broken at the instant that the alternate current motor was connected to the generator.

The magnetization curve for the alternate current motor was found to be practically a straight line, as shown by the following readings, which are corrected for a normal speed of 2,080 revolutions per minute:

DATA FOR MAGNETIZATION CURVE OF MOTOR.

Current in Motor Field.	E.M.F. at Brushes.	Speed.
1'	9.9	2,080
1'5	16.7	"
2'	22.7	"
2'5	28.7	"
3'	34.5	"
3'5	40.5	"
4'	46.4	"

The stability of a synchronous motor depends largely upon the field excitation and the amount of the self-induction in the armature circuit. With no added self-induction in the armature circuit the motor in the present case was found to be quite unstable. Curve *A*, *Fig. 8*, shows the relation between the current flowing through the motor and generator armatures and the exciting current of the motor field, when no added self-induction was in the armature circuit. The motor was driving the small Edison machine on open circuit as a light load. The generator field excitation was kept constant at 3.5 ampères; the speed was 2,080 revolutions per minute. The extreme range through which the motor could be kept in synchronism was from a motor field current of from 1.5 to 3.3 ampères. With a motor field excitation of 3 ampères, the armature current was a minimum. The operation under these conditions, however, was quite unstable, and a light load would throw the motor out of synchronism, and thus cause it to stop.

The stability was increased by the addition of self-induction to the armature circuit, and for this purpose a coil without iron was employed. Curves *B* and *B'*, *Fig. 9*, show the relation between the motor excitation and armature current, as was shown by curve *A*, for the preceding case without added self-induction. The motor ran synchronously with a field excitation varying from .6 to 2.2 ampères, and from 3 to 3.6 ampères, but would not run with an exciting current between 2.2 and 3 ampères. With the weak field excitation, a large current was consumed by the motor, while with the strong excitation current, a much smaller current was taken by the motor. This may be attributed to the phase position which the current takes in the two cases, with reference to

the phase position of the impressed E.M.F., as is shown by the polar diagrams for such cases.

It was found that by gradually increasing the self-induction in the armature circuit, the two parts of the curve  $B$  and  $B'$  were brought together. The self-induction was increased by inserting in the coil an iron core consisting of a bundle of iron wire. At a certain position of this core the self-induction of the armature circuit was such that the motor consumed a much smaller current than at any time before, ran with great stability, and through a wide range of field excitation, as shown by curve  $C$ , *Fig. 10*, which will be discussed later. The final run was made under these conditions.

The coefficient of self-induction of the motor armature was found to be 0.00032 henrys, and that of the induction coil with the iron core, 0.00168 henrys. The total self-induction was, therefore, 0.002 henrys. The total ohmic resistance of the armature circuit and connections measured 0.31 ohms. The self-induction of the armature was measured for different field excitations and for different currents, the mean of all the values obtained being averaged and taken as constant. The excitation of the field seemed to make no appreciable difference in the coefficient of self-induction of the armature. The resistance and self-induction of the armature circuit vary somewhat with the value of the current flowing, on account of temperature changes and the saturation of the iron, but the error introduced by considering these constant is small. From these measurements the reactance and impedance were determined.

#### THE METHOD OF EXPERIMENT.

Throughout the following investigation, the generator was run at a nearly constant speed of about 2,080 revolutions per minute, and with a constant field excitation of 3.5 ampères. The motor was brought up to speed by the small Edison starting motor mentioned above, and thrown into circuit at the instant when its phase position, as indicated by the ring of light of the phase-indicator, remained  $180^\circ$  behind that of the generator. The small Edison machine

was then driven on open circuit as a light load for the alternate current motor, this being the only load throughout all the experiments here described. The exciting current of the motor field was changed by increments of 0.2 ampères between the limits 1.8 and 6 ampères. The motor and generator field currents were obtained from a storage battery, and were accordingly quite constant.

The quantities required were:

The impressed E.M.F. ( $E$ ) of the generator;

The counter E.M.F. ( $E'$ ) of the motor;

The resultant E.M.F. ( $E''$ ), which is the resultant of the impressed and counter E.M.F.s;

The armature current ( $I'''$ );

The angle of lag ( $\phi$ ) of the current behind the resultant E.M.F. ( $E''$ ); and

$$2\pi \times \text{frequency} = \omega = \frac{2\pi \times 4 \times \text{r.p.m.}}{60}$$

The above quantities are required in order to construct the graphical diagrams which show the complete action of the synchronous motor.

The following quantities were likewise obtained:

Generator field current ( $I'$ ).

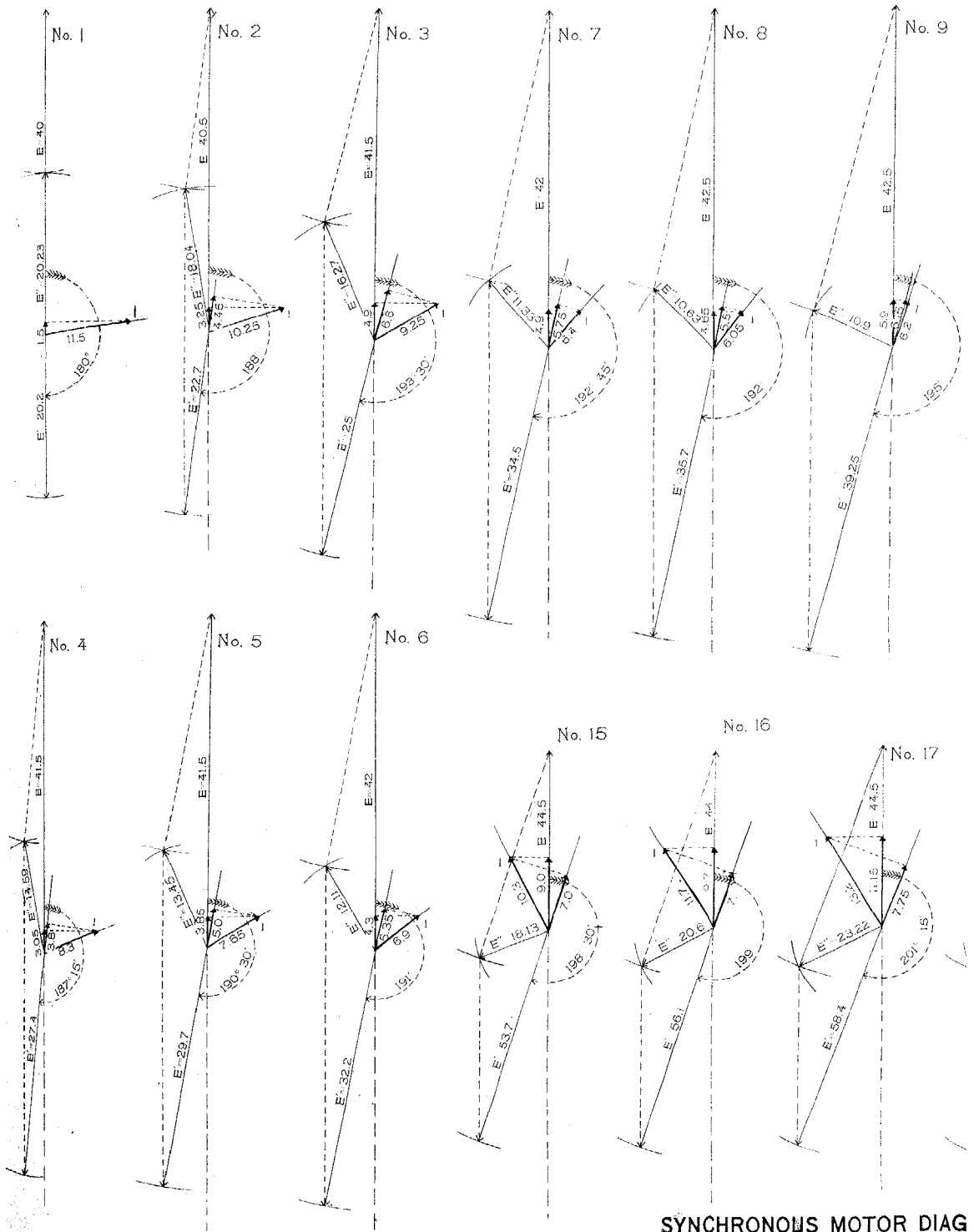
Motor field current ( $I''$ ).

Disk reading, angle  $\phi$ .

The currents were measured directly by means of a Siemens' dynamometer. The impressed E.M.F. ( $E$ ) of the generator was obtained from a voltmeter connected to the generator brushes. The counter E.M.F. of the motor was taken directly from the magnetization curve for the motor plotted according to the table given above, no correction being made for armature reactions.

The electromotive forces  $E$ ,  $E'$  and  $E''$  must all be in equilibrium when the motor is running. The motor, when running in synchronism, will take such a phase position with respect to the generator that this relation is obtained. The resultant E.M.F. ( $E''$ ) is then just sufficient to overcome the product of the current ( $I'''$ ) and the impedance of the armature circuit. Therefore:

$$E'' = I''' \times \text{impedance.}$$



**SYNCHRONOUS MOTOR DIAG**

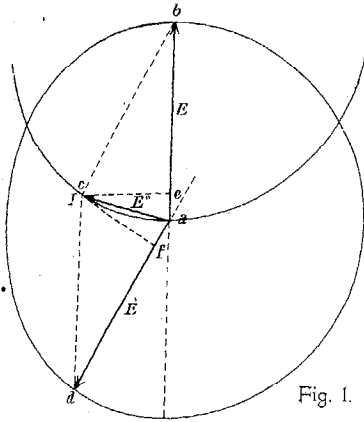


Fig. 1.

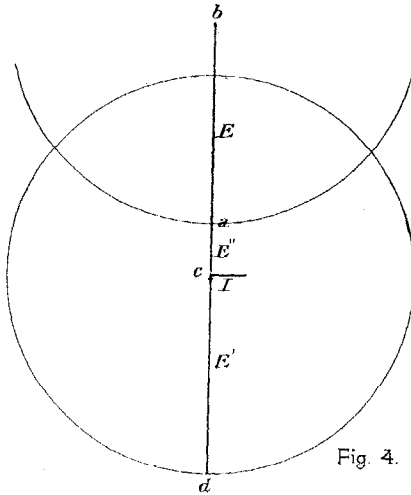


Fig. 4.

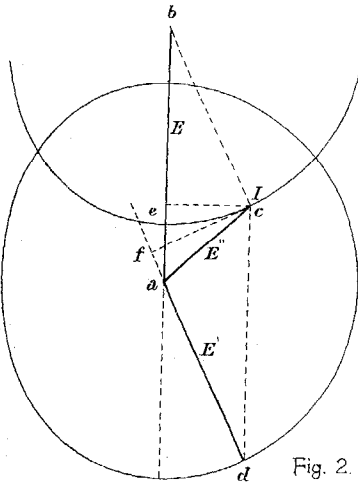


Fig. 2.

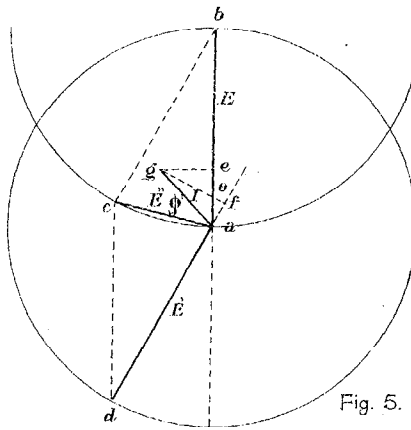


Fig. 5.

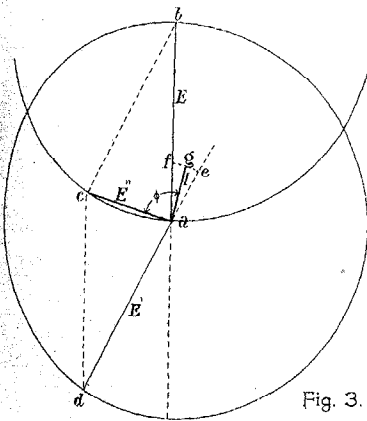


Fig. 3.

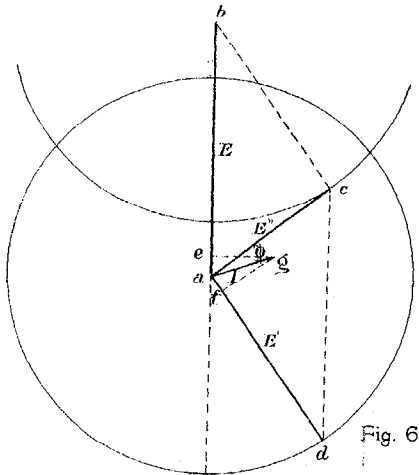
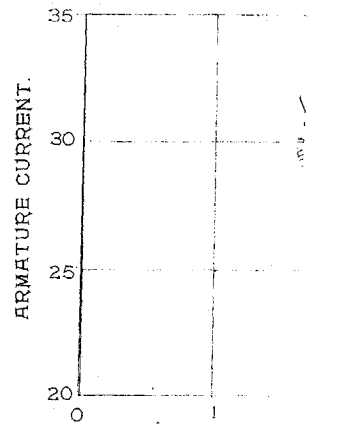
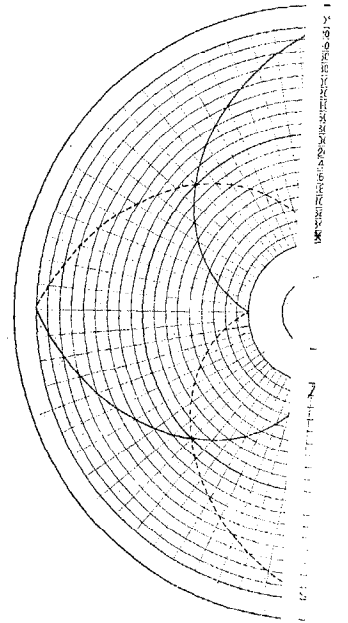


Fig. 6.

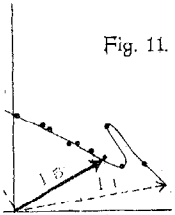


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Fig. 11.



Direction and magnitude  
armature current.

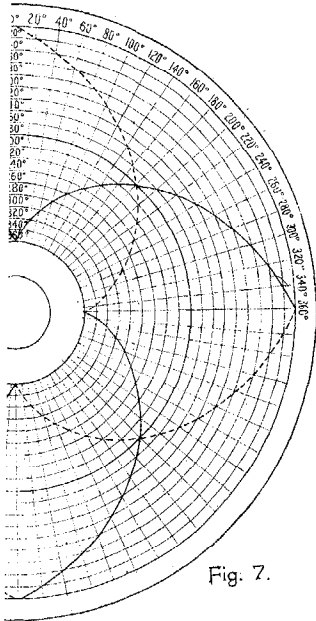


Fig. 7.

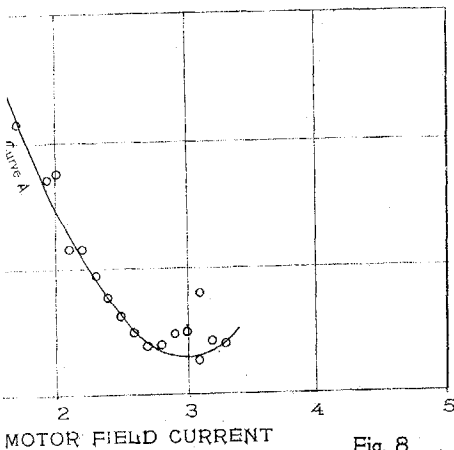


Fig. 8.

MOTOR (Bedell and Ryan.)

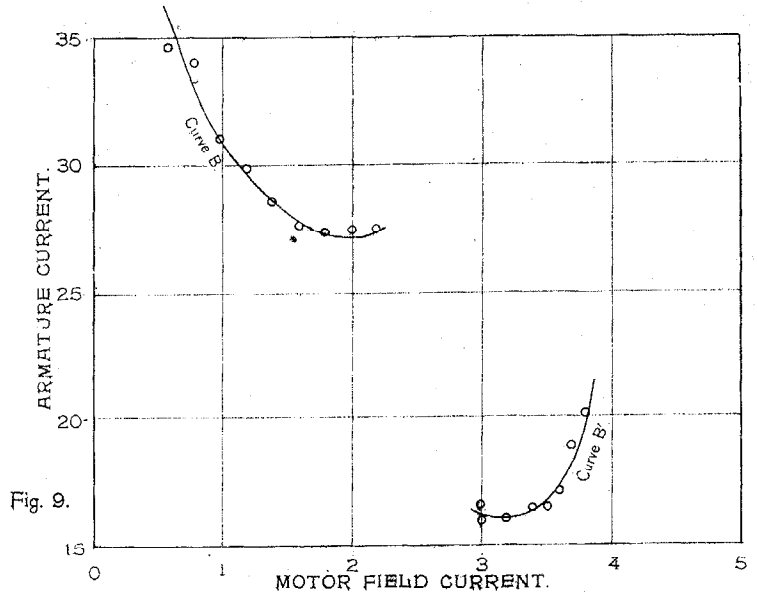


Fig. 9.

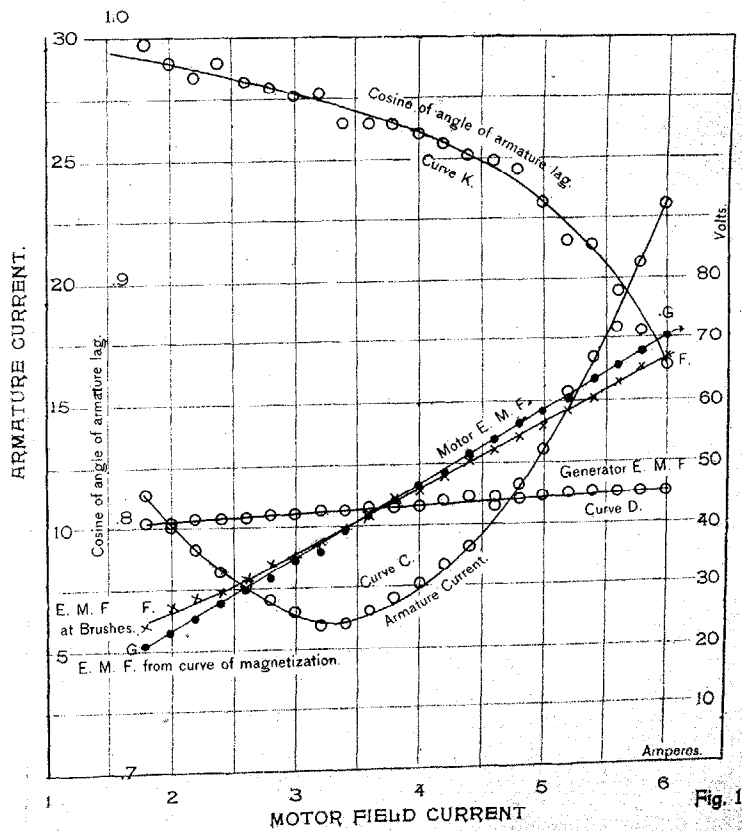
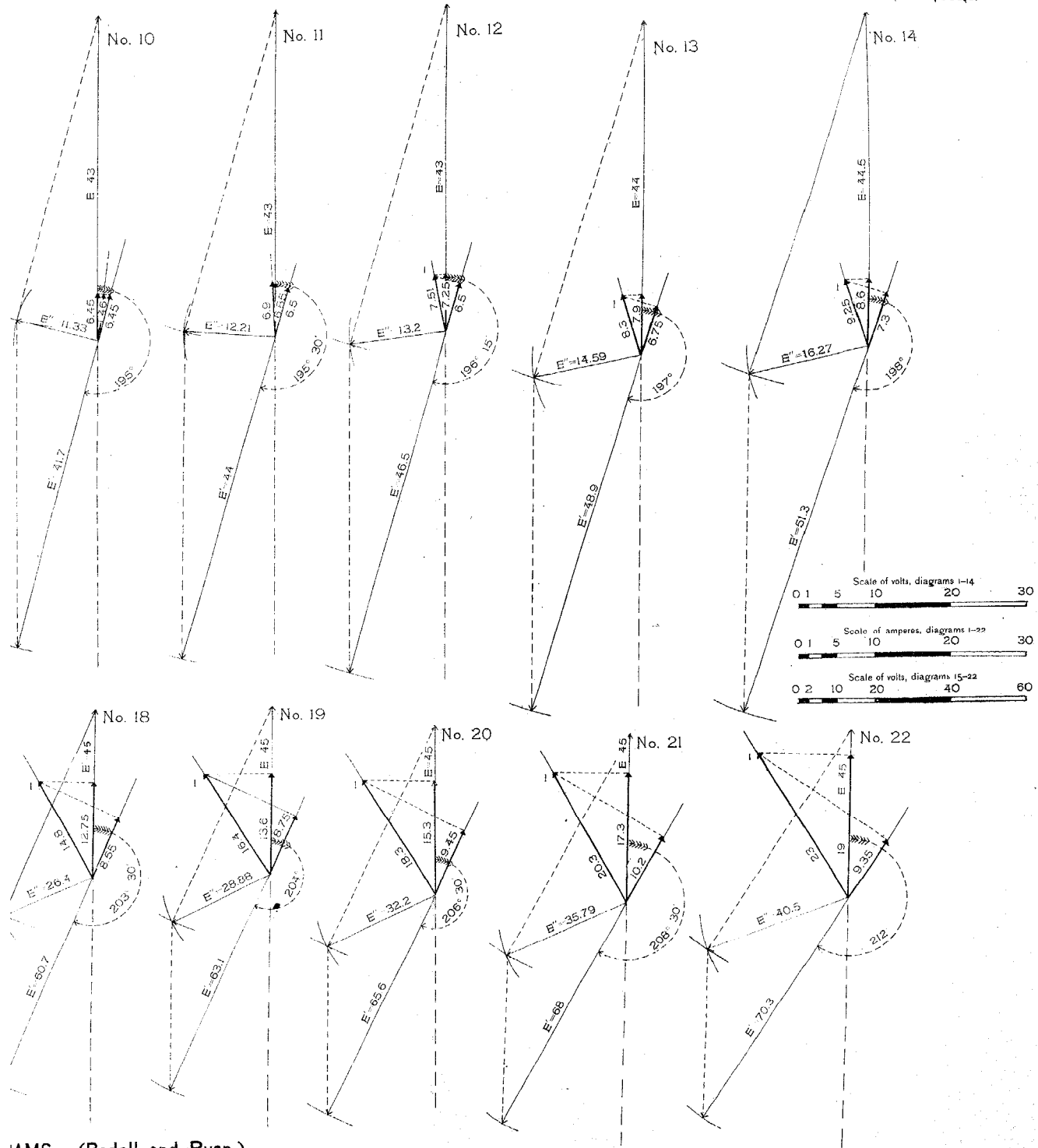


Fig. 10.





The E.M.F. is thus computed: The angle of lag of the current behind the resultant E.M.F. is

$$\phi = \text{arc tan.} \frac{\text{reactance}}{\text{resistance}}$$

The resistance and self-induction being taken as constant, we have, therefore, a constant value for the angle  $\phi$ , viz.:  $82.5^\circ$  (the speed varying little during the run, and  $\omega$  being taken, therefore, as constant).

The phase position of the motor with respect to the generator is represented exactly by the phase positions of their respective E.M.F.s. The relative phase position of these E.M.F.s was determined by graphical construction, as in *Fig. 5*. The generator and motor E.M.F.s and their resultant being known, their position is determined by a construction analagous to the parallelogram of forces. In *Fig. 5*, *ab* represents the direction and magnitude, taken in any convenient scale, of the generator E.M.F.; *ad*, the motor E.M.F., as taken from the magnetization curve; *ac*, the resultant E.M.F. The magnitudes of the E.M.F.'s being known, their relative positions are thus determined. The angle between the resultant E.M.F., *ac*, and the current, being determined as described above, *ag* may be drawn to represent the current. The diagrams, Nos. 1-22, are thus constructed:

The angle *bad* measured backward (or clockwise) shows the phase position of the motor armature with respect to the generator armature, and it should agree substantially with the disk reading of the phase indicator.

The energy in watts delivered by the generator to the circuit is equal to the product of the impressed E.M.F. (*E*) and of *ae*, which is the component of the current *ag* in line with the impressed E.M.F.

The energy in watts transformed into mechanical power by the armature is equal to the product of *E'* and of *af*, the component of the current *ag* in line with *E'*. This product must be negative in value, showing energy consumed and not produced, for the machine to run as a motor. The mechanical power developed is expended in driving the Edison machine on open circuit, and in journal friction, hysteresis and

foucault currents. The iron losses increase as the field excitation is increased. The difference between the watts delivered and the watts transformed into mechanical power is equal to the  $RI^2$  losses of the armature circuit. Thus, from the fifth observation (see Diagram 5), we have :

$$\text{Watts input} = 41.5 \times 3.85 = 160.$$

$$\text{Watts mechanical power} = 29.7 \times 5.0 = 148.$$

$$\text{The } RI^2 \text{ losses} = 160 - 148 = 12 \text{ watts.}$$

In the accompanying table, the observed data are given to the left of the double line, and the computed results to the right. No measurements were made with a motor field current above six ampères, on account of the possibility of springing the armature shaft; it is to be supposed that the motor would run synchronously beyond this point.

The results given in the table are likewise shown in *Fig. 10*, and in the diagrams Nos. 1-22. In *Fig. 10*, curve *C* shows the armature current for different field excitation; the current becomes less and less as it comes into phase with the impressed electro-motive force, and then increases as it takes a position in advance of the electro-motive force. That there is a point of minimum current and a maximum power factor was shown by Mr. Mordey, in a paper read before the Institution of Electrical Engineers,\* in which he gave a curve for the armature current at light load, such as curve *C*, *Fig. 10*. He further showed that the motor E.M.F. could be higher than the generator E.M.F., as is clearly brought out by the present paper. This point was predicted in 1884 by Dr. Hopkinson in his paper† before the Institution of Electrical Engineers, on "The Theory of Alternating Currents." In the discussion, Mr. Kapp‡ showed such curves for several loads, showing the forms these curves would take, both with and without armature reactions. His curves were plotted with abscissæ representing the counter E.M.F. of the motor. The counter E.M.F.

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\* "On Testing and Working Alternators," *Journal of the Institution of Electrical Engineers*, Feb. 23, 1893; see also *London Electrician*, Vol. xxx, p. 545.

† *Loc. cit.*

‡ *London Electrician*, Vol. xxx, p. 575.

OBSERVED.										COMPUTED.				
Number of Observa- tion.	$I$ , Generator Field Current.	$I''$ , Motor Field Cur- rent.	$I'''$ , Main Current.	$E$ , Generator E.M.F.	E.M.F. at Motor Brushes.	Reading of Phase Indicator: Angle $\theta$ .	Speed.	$E'$ , Motor E.M.F.	$E''$ , Resultant E.M.F.	Computed Angle $\theta$ .	Watts Input.	Watts Transformed into Mechanical Power.	$R I^2$ Losses of Line and Motor Circuit in Watts.	Computed $R I^2$ Losses in Watts.
1	3.5	1.8	11.5	40.5	24.	18.5	2,088	20.2	20.23	186°	61	32	29	28.
2	3.5	2.0	10.25	40.5	27.	18.7	2,088	22.7	18.04	186°	131	101	30	23.
3	3.5	2.2	9.25	41.5	28.5	19.0	2,088	25.	16.27	193.5	201	165	30	18.
4	3.5	2.4	8.3	41.5	29.5	19.0	2,085	27.4	14.59	197.3	126	144	22	14.4
5	3.5	2.6	7.65	41.5	31.5	19.0	2,085	29.7	13.45	190.5	186	148	12	13.3
6	3.5	2.8	6.9	42.	34.	19.0	2,095	32.2	12.11	191°	186	172	8	10.
7	3.5	3.0	6.4	42.	35.5	19.5	2,095	34.5	11.33	192.7°	206	197	9	8.6
8	3.5	3.2	6.05	42.5	37.5	19.5	2,080	35.7	10.63	195°	197	196	1	7.7
9	3.5	3.4	6.2	43.	39.5	19.5	2,080	39.25	10.9	195°	250	240	10	8.1
10	3.5	3.6	6.46	43.	41.5	19.5	2,080	41.7	11.33	195°	286	269	8	8.8
11	3.5	3.8	6.9	43.	44.	19.5	2,080	44.	12.21	195.5°	294	286	8	10.
12	3.5	4.0	7.51	43.	45.5	20.0	2,090	46.5	13.2	197.3°	312	302	10	11.8
13	3.5	4.2	8.3	44.	48.	20.0	2,090	48.9	14.59	197°	346	336	12	14.4
14	3.5	4.4	9.25	44.5	50.	20.0	2,075	51.2	16.27	198°	382	374	12	16.
15	3.5	4.6	10.30	44.5	52.	20.0	2,075	53.7	18.13	198.5°	401	392	25	22.4
16	3.5	4.8	11.7	44.5	54.	20.0	2,075	56.1	20.6	199°	427	419	35	28.8
17	3.5	5.0	13.2	44.5	55.8	20.0	2,075	58.4	23.22	201.3°	496	485	41	30.6
18	3.5	5.2	14.8	45.	58.5	20.5	2,070	60.7	26.4	203.3°	568	559	49	46.
19	3.5	5.4	16.4	45.	60.5	20.5	2,070	63.1	28.88	204°	613	602	61	57.
20	3.5	5.6	18.3	45.	63.	21.0	2,070	65.6	32.2	206.5°	689	678	70	67.
21	3.5	5.8	20.3	45.	65.5	21.5	2,070	68.	35.79	208.5°	778	764	84	87.
22	3.5	6.0	23.	45.	67.	22.0	2,070	70.3	40.5	212°	855	837	98	111.

of the motor was not directly proportional to the exciting current, for his curve of magnetization was not a straight line. Mr. Steinmetz\* has determined these curves analytically, plotting them from a quartic equation showing the relation between armature current and motor E.M.F. He assumes a constant reactance and the absence of armature reactions. His theoretical curve for no load (where no load means the absence of friction and of all energy expenditure) meets the X-axis at a sharp point, which would show that when the motor was doing absolutely no work, the armature current would actually be brought to zero, for a particular value of the field current. His curve for light load corresponds approximately with curve *C*. It is, however, constructed for a constant load, whereas the load in the present investigation became greater as the hysteresis losses increased with the excitation.

The increase in the generator E.M.F. on account of armature reactions, although the speed and excitation were constant, is shown in curve *D*, *Fig. 10*. That armature reactions have the opposite effect in the motor, as mentioned earlier in this paper, is shown by curves *F* and *G*, which represent, respectively, the E.M.F. observed at the brushes of the motor, and that obtained from the curve of magnetization. With a small field excitation, the armature current, which is a lagging current with reference to the generator E.M.F., is one of advance with reference to the motor E.M.F.; and the armature reactions are, therefore, such that the E.M.F. at the brushes of the motor is greater than that obtained from the curve of magnetization. With a larger field excitation of the motor, the armature current is one of advance with reference to the generator E.M.F., and a lagging current with reference to the motor E.M.F., which causes the E.M.F. at the brushes of the motor (curve *F*) to be less than that obtained from the magnetization curve, as shown in curve *G*. The rise in the generator E.M.F., due to armature reactions, corresponding to this decrease in

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\* "Theory of a Synchronous Motor," *Transactions of the American Institution of Electrical Engineers*, Oct. 17, 1894.

motor E.M.F., due to the same cause, is shown in curve *D*, as already explained. These curves have not been corrected for the fall of potential due to the impedance of the armature; but the results are quite striking and are in accordance with the explanation given in the early part of this paper, to the effect that the armature reactions in the generator and motor assist in the self-regulation of the plant. The gradual change in armature lag is shown by curve *K*.

Diagrams, Nos. 1-22, are drawn for each corresponding set of readings given in the table. The current, represented by the heavy arrow *I*, lags behind the generator E.M.F. when the field excitation of the motor is weak. The motor thus produces the effect of self-induction in the circuit.

As the excitation is increased, the current comes more and more into phase with the E.M.F., and finally comes into phase with it. The armature current is now a minimum. A further increase in the field excitation advances the armature current ahead of the E.M.F.; that is, the motor now acts the same as a capacity, or condenser, in a circuit, this capacity-effect increasing as the field excitation increases.

In a recent paper\* on "Some Advantages of Alternate Currents," Prof. S. P. Thompson lays particular stress on this condenser action of an over-excited synchronous motor, and emphasizes the fact that synchronous motors may advantageously be operated in parallel with transformers, to overcome the effects of self-induction. We have experimented at length upon the action of condensers in parallel with transformers,\* and the advantages to be obtained by this system of operation. Inasmuch as the transformer tends to make the current lag, and the condenser tends to place the current in advance of the E.M.F., it is possible to obtain such a balance between the two that the line current will be in phase with the electro-motive force. Under these circumstances a certain amount of power is transmitted

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\* British Association, Oxford, 1894.

\* See Hedgehog Transformer and Condensers, *Transactions of Institute of Electrical Engineers*, Vol. x, p. 497.

with a minimum current. The polar diagrams, in the present paper, show conclusively that the condenser action of over-excited synchronous motors will enable them to be operated in the same way as were the condensers in the paper referred to.

The changes in the magnitude and direction of the armature current, as the field excitation changes, are clearly seen in *Fig. 11*. The current for the first observation (compare Diagram 1) for the weakest field excitation, has the position  $I_1$ . As the field excitation increases, it takes successively the positions indicated by points, coming into phase with the E.M.F., and finally in advance of it, as shown in the position  $I_{22}$ , corresponding to the last observation (see Diagram 22). The heavy arrow  $I_5$  represents the current for one particular observation. The locus for the armature current is obtained by drawing the curve through the successive points. With a constant E.M.F. the distance of any point of this curve above the horizontal would be proportional to the power given the motor. With a constant E.M.F. and constant power supplied to the motor, the curve would be a horizontal straight line. The rise in the curve at the left is due to the increased power, necessary in this case as the field current of the motor is increased. The irregularity in the curve at the right is due to the instability of operation during the first few observations.

In constructing the polar diagrams, the values of  $\theta$ , the angle of lag of the motor armature, as obtained by the readings of the phase-indicator, were not employed. From the diagrams constructed as described above, the value of the armature lag was obtained as given in the column headed "Computed Angle  $\theta$ ," which is seen to agree closely with the armature lag as shown by the phase-indicator. The accuracy of the work and the correctness of the reasoning were thus completely verified by the phase-indicator readings.

The polar diagrams, here shown for the first time, thus simply show the complete action of the synchronous motor.