

Classical Elastic Scattering Probabilities and the Necessity of a Wavelength

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In previous notes, we argued that one may introduce a complex probability into Newtonian mechanics to describe elastic scattering. In particular, even in exercises in Newtonian mechanics, given an elastic two-body collision, energy and momentum are conserved and so one accepts that any E, p sets that satisfy this conservation have equal probability. In other words, deterministic mechanics is not deterministic enough in this case to predict the exact outcome of a collision of two $(e_1, e_1), (p_1, p_2)$ vectors. This justifies the introduction of energy and momentum probabilities.

In previous notes, we suggested $\exp(ip)$ as the momentum probability. A complex number with unit modulus is used because each free particle has the same weight. Here we point out a problem with this definition which was not mentioned in previous notes. Given the periodicity of $\exp(ip)$, $\exp(ip) = \exp(i(p + 2\pi\hbar))$ which means that p has the same probability as $p + 2\pi\hbar$. One, however, wishes to have a unique probability. In previous notes we went on to argue that one ultimately requires $\exp(-iEt + ip \cdot r)$ as this is Lorentz invariant and $\exp(ipx)$ yields the same value for the x -axis pointing in one direction or the other.

If one considers $\exp(ipx)$, however, the same problem mentioned above occurs, because any $px_1 + 2\pi\hbar$ yields the same value for $\exp(ipx)$ and so any $p_{\text{new}} = p + 2\pi\hbar/x_1$ has the same probability as p when what one desires is a unique probability. We note that for changing x_1 values, the $p(\text{new})$ values change. In fact, one may obtain a uniqueness for $\exp(ipx)$ if one considers a range of x values, i.e. the wavelength $= \hbar/|p|$. This leads to the notion of orthogonality, i.e. $\int dx \exp(-ip_1x) \exp(ip_2x) = 0$ if $p_1 \neq p_2$. By sampling x -space, i.e. by taking the notion of wavelength seriously, one may have $\exp(ipx)$ represent a unique p value over a range of x because in such a case one has a unique number, namely the wavelength $\hbar/|p|$. Thus, it is the wavelength and the positive and negative values of $\sin(px)$ and $\cos(px)$ which in fact define the probability associated with p . P represents a momentum impulse hit, but in terms of probability, it is actually the wavelength and the adding and subtracting of probability in space which defines p , we argue. This probability in space with addition and removal of probability then becomes important in a calculation of a probabilistic interaction with a 2-slit apparatus in which the slits are separated by about a wavelength.

Classical Probability for Two Body Elastic Scattering

Newtonian mechanics is called deterministic, but it is common knowledge that two body elastic collisions are often treated in a manner in which any solution which conserves energy and momentum is equally likely. In this specific example, Newtonian mechanics is not deterministic enough to predict the exact energy and momentum outcomes. One might argue that with extra information provided one could actually determine the outcome, but realistically this extra information is seldom provided.

As a result, one may introduce the notion of a probability for E and p in Newtonian elastic scattering. In previous notes, we suggested:

$$\exp(iE) \text{ and } \exp(ip) \quad ((1))$$

We used a complex number with unit modulus because free particles have no real value weight. We note that one cannot use $\exp(i|p|)$ because one must account for p and $-p$ in a collision. Here we point out an immediate problem with $((1))$ which was not mentioned in previous notes, namely that:

$$p_1 = p + 2\hbar k n \quad (n=\text{integer}) \quad ((2))$$

yields the same $\exp(ip)$ probability. One wishes probability to be unique and so this is a problem. We went on (in previous notes) to argue that in fact the full free particle probability is:

$$\exp(-iEt + ip \cdot r) \quad ((3))$$

which is Lorentz invariant and allows $\exp(ip \cdot x)$ to retain the same value if the direction of the x axis is switched. The problem mentioned above, however, remains because for a given x_1 :

$$p_1 = p + 2\hbar k n / x_1 \quad ((4))$$

yields the same $\exp(ipx_1)$ value. We observe, however, that each x_1 leads to a new set of p_1 values. As a result, it seems that for a momentum probability function to be unique in p , one must consider a range of x values, not a single one. This is a very different concept because Newtonian mechanics considers a particle at a given x at a given t . We suggest that if one wishes to consider the probabilistic nature of 2-body elastic scattering, one cannot consider a single x point. In other words, the unique value which characterizes a given $\exp(ipx)$ is not p , but rather:

$$\text{Wavelength} = \hbar / |p| \quad ((5))$$

Thus, spatial ranges, i.e. frequencies in space are the key when using $\exp(ipx)$ as a probability. We note that this means that probability is removed from one region and added to another because:

$$\sin(px), \cos(px) \quad ((6))$$

have positive and negative values (which are symmetric). We also note that:

$$\int dx \exp(-ip_1 x) \exp(ip_2 x) = 0 \text{ if } p_1 \neq p_2 \quad ((7))$$

Even though the value of p is key for describing impulse hits, it is the notion of a wavelength in x which is key in describing a probability based on p which applies to elastic collisions.

Conclusion

In conclusion, in previous notes we argued that one may introduce a probability based on energy and momentum to describe Newtonian 2-body scattering. As a first guess, we suggested $\exp(iE)$ and $\exp(ip)$ because free particles all have the same real value weight. Here we note that there is an immediate problem with this definition because $p_1 = p + 2\pi\hbar/x_1$ yields the same probability value. A momentum probability for p should be unique. In previous notes, we generalized the probability to $\exp(-iEt + i\mathbf{p} \cdot \mathbf{r})$ which is Lorentz invariant, but this does not solve the problem because any $p_1 = p + 2\pi\hbar/x_1$ for a given x_1 has the same probability at x_1 . A hint is that this set of probabilities changes with x_1 . In other words, in order to have a momentum probability one must consider a range of x , i.e. a wavelength $= \hbar/p$ and it is this which is unique. This means that probability is added to regions and removed from others in a symmetric manner to the positive/negative values of $\cos(px)$ and $\sin(px)$. In addition, $\int dx \exp(-ip_1x)\exp(ip_2x) = 0$ if $p_1 \neq p_2$. As result, even though p is the key idea for an impulse, it is actually the notion of wavelength that is key for a probability calculation. This means, however, that the Newtonian concept of a particle at x at t cannot be used if one wishes to describe an interaction in terms of probability. Rather one is forced to consider p being able to hit over an x range which is of critical importance in an interaction with a 2-slit apparatus with slit widths separated by about a wavelength. In the probability scenario, the particle may interact with both slits and this must be treated in a probabilistic manner in which it does as in an OR probability treatment. The extra piece, however, is that the positive/negative values of $\cos(px)$ and $\sin(px)$ allow for building and removing probability from different regions in space in an OR case, which is called interference.

The fact that p seems to be able to strike over a range of x values, i.e. that there exists a wavelength, is based on the notion of a momentum probability. If there were no momentum probability, there would be no wavelength. As a result, momentum probability does not seem to be an approximation to dealing with an elastic collision, but seems to be a physical reality due to 2-slit interference. This, however, does explain the exact physical mechanism which creates this probability with a wavelength. One only describes the result of the mechanism in a probabilistic manner, we argue.