

that I advocate nothing of the sort. What I do advocate is to treat vectors as vectors, and versors as versors, and I show that the products of versors differ essentially from the products of vectors in that the associative rule applies to the former, but not to the latter. Prof. Knott justifies the treatment of quadrantal versors as vectors, because they are compounded according to the parallelogram law. It is true that the components of a quadrantal versor are so compounded, because every versor involves an axis; but the minus comes in, not on account of the axis, but on account of the angle of the versor, the very element which differentiates it from a vector.

I have said that $\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}$ is more consistent with analysis than $\nabla^2 = -\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}\right)$, and I have remarked that in works on mathematical physics, even in Kelvin and Tait's "Natural Philosophy," the minus was dropped. A sign that can be so readily dropped has probably got no good reason for its appearance. In reply, Prof. Knott says that "when $\nabla^2 v$ occurs in ordinary non-quaternion analysis, it is used in the sense of the *tensor*, for only as such can it come in." This explanation does not explain; for "the name *tensor* is applied to the *positive* number which represents the length of a line" ("Hamilton's Elements," p. 164). Now the ordinary analysis is not limited to signless quantities, but embraces quantities which may be positive or negative. Why then is the minus dropped in an analysis where sign is essential? I asked for a proof of the principle that $\nabla(\nabla\omega) = \nabla^2\omega$; it is replied that "in quaternions there is no doubt whatever." Are we permitted, then, to doubt it as a truth in ordinary analysis, being true only in quaternions? If it is a matter of convention, no one desires two contradictory systems of analysis; if it is a matter of truth, it cannot be true "in quaternions" and not in ordinary analysis.

I have said that the rule $ij = k$ expresses what is true in space of three dimensions. Prof. Knott asks: "If a vector cannot be a versor in product combinations, what is the signification of the equation $ij = k$?" Let us first of all remove every ambiguity from the equation. We have then in all three cases: first, i and j both quadrantal versors; second, i a versor and j a vector; third, i and j both vectors. To distinguish between a quadrantal versor and a vector, let the former be

denoted by i^{π} . Then $i^{\pi}j^{\pi} = -k^{\pi}$ means the forward order being taken, that a quadrant round i followed by a quadrant round j is equivalent to a quadrant round the opposite of k .

Again, $i^{\pi}j = k$ means that the vector j , when turned through a quadrant round i coincides with k . Finally, ij means the unit of directed area which has i for base and j for altitude; for some purposes it may be represented by k on the principle that the axis of a plane may be specified by the axis which it wants; but at p. 92 of "The Principles of the Algebra of Physics," I have shown that the several types of products of vectors may be formed independently of that principle. Prof. Knott states that he fails to see what physical considerations have to do with mathematics of the fourth dimension. It is evident, however, that his perception cannot be taken as a criterion of truth, for every type of product of four vectors is geometrically real excepting the one which supposes them all independent of one another.

I have said that the rules for differentiation are much simplified when vectors and versors are not confounded. In proof of this I invite comparison.

I have said that the principles of quaternions can be greatly extended. In my papers will be found for the first time the extension of space analysis to logarithmic spirals and to hyperbolic trigonometry. The connection of the latter with non-euclidean geometry is also pointed out. As further evidence of the fruitfulness of my notation and principles I may mention that I have just read before the Mathematical Congress assembled at Chicago two papers—one on "The Definitions of the Trigonometric Functions," the other on "The Principles of Elliptic and Hyperbolic Analysis." These papers give the trigonometry of the elliptic and hyperbolic surfaces.

As regards Prof. Knott's closing quotation from "Paradise Lost," I feel like the Senior Wrangler who, having read through the poem, remarked that it was all very pretty, but he didn't quite see what it proved. I close with a quotation which is

from as good a book, and possesses more logical force: "Ye shall know them by their fruits. Do men gather grapes of thorns, or figs of thistles?"

ALEXANDER MACFARLANE.

Chicago, Ill., August 26.

Astronomical Photography.

THE letter from Lord Rayleigh in your issue of August 24, on the subject of "Astronomical Photography," will, it is to be hoped, elicit some information from photographic experts.

Meanwhile, accepting what Lord Rayleigh says as to the present possibilities in the preparation of plates, I fail to see where any considerable saving is to be effected in the cost of the apparatus, as he appears to suggest.

For astronomical photography a pair of telescopes are required. The larger of these is employed to take the photographs, and the smaller acts as a guider. Supposing that plates could be obtained which were acted upon by visual rays, while comparatively insensible to the violet and ultra-violet light, this would simply mean that both the objectives would have to be made visually perfect, instead of having one of them as heretofore corrected for violet and ultra-violet light. A photographic objective is no more costly than a visual one of the same aperture; and as to mounting clockwork and dome, there could be no difference in expense.

Of course, if the necessity for a separate guiding telescope could be avoided by the adoption of Lord Rayleigh's suggestion, there would in general be some saving of expense; it should, however, be noted, that even when reflectors are employed for taking the photographs, it has not been always found desirable to dispense with the guiding telescope, though in this case, of course, the question as to the nature of the plates cannot arise at all.

In the particular instance of the instrument now proposed for Cambridge, the guiding telescope is already to hand in the shape of the present Northumberland instrument.

It is certainly easier to test the qualities of an objective corrected for visual rays than for photographic rays (if I may still use language which Lord Rayleigh has pointed out as incorrect). On this account it would, therefore, be desirable to have plates such as he refers to, rendered available for astronomers engaged in photographic work.

ROBERT S. BALL.

Observatory, Cambridge, September 12.

P.S.—Sir Gabriel Stokes, after reading the above, writes: "I would ask whether in an orthochromatic plate the blue and violet are impressed more feebly than the rays which are visually the brightest. It may be so, but I do not happen to know whether it is."

The Constellations of the Far East.

WITH regard to the questions asked by "M. A. B." about the grouping of stars into constellations (NATURE, August 17), I venture to answer the last two, which the limited knowledge of an Oriental may partly meet, hoping thereby to interest some of your readers.

I do not consider that each race necessarily relies on its own plan in the fabrication of constellations. The Coreans and Anamese are said to be still adhering to the Chinese system, and till lately the Japanese were doing so. It is strange to find the latter, replete with so peculiar mythology, on which the national claim for high ancestry rests, possessing very few vernacular constellations.

Undoubtedly the Chinese system is of peculiar aspect. A name is given to a "Seat," which is sometimes a single star, but in general a group of stars, varying in number from two to twenty or thirty; and in one group, the Imperial Bodyguards, they amount to forty-five. Occasionally the same stars are at once named collectively and individually; thus, the first seven stars of Ursa Major are grouped into Peh-tau or the North Ladle, of which the scoop consists of Shu α, Siuen β, Ki γ, and Kiuen δ, and the handle of Yuh-hang ε, Kai-yang ζ, and Yau-Kwang η. With Polaris as the centre, the heavens are radiantly divided into the twenty-eight "Inns" of unequal breadths, each division being denominated after its typical constellation, besides enclosing numerous Seats subordinate to the latter.

The fundamental idea of the plan is enigmatically expressed thus: "Sing (the star) is Tsing (the spirit)." Its solution con-