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II. On the rules for algebraic multiplication

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Description of the Drawing.

Plate I. fig. 1, 1, 1, 2, 3, different sand plants, showing how they appear when a breadth of the leaf is cut.

Fig. 2, being the upright hairs, while the figure 1s show the horizontal ones.

Fig. 3, shows the sort of borders that covers each face of the leaf and is often seen with the upright hairs.

Fig. 5, is a leaf which is too small in proportion, but is designed to show that one side of the leaf is generally filled and emptied at a time, as here represented, only that to show them well the hairs are drawn too large.

Fig. 6 and 7, the hairs showing full and empty.

Fig. 8, an upright hair, various figures of this sort.

In Parasite Plants.

Fig. 9, the manner in which the pumps are formed: some have three or four points in one case; some have only one; but they have all the loose part *a*, *a*, which falls over the point, and glues itself to the plant it is designed to live on :—a piece of the ivy, fig. 10.

Fig. 12, a breadth of the leaf as cut from the xylophylla plant.

II. *On the Rules for Algebraic Multiplication.* By Sir H. C. Englefield, *Bart. F.R.A. and L.S.*

To Mr. Tilloch.

Sir,—The following had been several years ago composed, but it is only a few months since the approbation it received from some friends well qualified to appretiate it induced me to send it to you for publication, if you should judge it worthy. Since it was written I have seen the work of Maria Agnesi, the celebrated female professor of mathematics at Bologna. She has in the part relative to subtraction made use of a reasoning similar to that adopted by me; but she has not extended it to multiplication. A friend pointed out to me the work of Mr. Woodhouse, entitled Principles of Analytical Calculation.

In this excellent work, peculiarly valuable for the clearness and precision with which it is written, I found with great pleasure that the mode of treating the subject of multiplication was in effect similar to my own, though expressed with more conciseness, and perhaps not so evident to mere beginners. I therefore have still thought that what I now send you may be of use to that class of readers, and in that hope transmit it to you. I am, sir,

Your obedient servant,

Tilney Street, Nov. 1, 1814. H. C. Englefield.

Numbers are abstract representations of quantity, and have no particular meaning till applied to some object. Thus 10 may mean ten feet, ten apples, ten pulsations, ten distances of the sun from the earth; but they have one clear and distinct idea attached to them, that the unit, whatever it be, is taken ten times. Every sum is therefore in fact a multiple of unity; and in common multiplication, the multiplicand being considered as unity, the primary and fundamental idea is the same as in the simplest figure or figures written down.

But in pure symbolic algebra the abstract is carried much further; for the letters are not only, as in numbers, general signs of quantities, but do not even represent any definite numbers of unities; and therefore are rather receipts or directions for the performance of operations, than real operations themselves: just as a man who has written a perfect receipt for making a pudding cannot be said to have made a pudding, but told how that pudding has been or may be made.

$a+b$ only means, that when instead of those letters some definite quantities are used, they are to be added together, not that they are now incorporated; and to call this by the name of addition, seems an abuse of terms, or rather a confusion of definitions. In like manner $a-b$ means that b, being less than a, is to be taken from it when by the substitution of some real quantity this can be done. And this is so true, that, if improper or dissimilar quantities are used, the expression remaining the same, absolute nonsense ensues.

If, for example, a be supposed to represent ten shillings, and b seven miles, it is obvious that nothing but absolute nonsense is talked when we speak of $a-b$.

In what has been called geometrical multiplication, the abstraction, though very analogous to that of algebraic notation, is not however precisely the same. In this a superficies is considered, having its four angles right angles, and its sides the two given lines which are thus considered as multiplied by, or drawn into, each other; and the results of this operation are rigorously true, and may be as logically reasoned on when the values of them are inexpressible by numbers, or are incommensurable, as when they are commensurable. The properties, for example, of the area formed by the side of a square and its diagonal may be investigated, though no numbers can express them. In this respect the abstraction is very similar to that of pure algebraic symbols, which may be supposed to represent any quantities of the same kind, whether expressible by numbers or not. In another point of view the abstraction of geometrical investigation is very similar to algebraic. The figure drawn is not the

the real representation of that under consideration in the mind: it is only a sort of assistance to the mental process. The lines though crooked, the angles though not right angles in the figure drawn, represent straight lines and right angles, to the reasoner; and still further, the figure which he thus contemplates has no determinate magnitude to his mind, and indeed cannot have any; for, if it had, the deductions would not be universal, but particular. It would, I believe, be of great advantage to the young student to consider these matters attentively on his first outset, as many false ideas would be thereby prevented from entering his mind.

Perhaps too much has been said on this even for a learner:—to proceed to the common rules of what is called algebraic multiplication.

1st. $+ \times +$ is $+$ plus, $a \times b$ evidently means that a is to be taken b times. $a + b \times c$, that the sum of a and b is to be taken c times: but as a and b cannot in this state be really added together, we can only say that when they can, a is to be taken c times, and b is to be taken c times, and the mode of writing this direction shortly is $ca + cb$.

2d. That $+ \times -$ is minus, admits of an equally clear demonstration or rather explanation. We must first remember that no quantity simply considered can be $-$ minus. It must be compared with some other quantity either greater than itself or of an opposite direction. In the common operations of algebra it is only used in the former sense, and $a-b$ merely means that b being less than a, it is when it can be done, to be subtracted from a; $a-b$ therefore really means the difference between a and b: if then $a-b$ is to be multiplied by c, having first taken a, c times, we have evidently done too much, for that would be the product of a only, taken c times; what then is to be subtracted from ca in order to have a true result? evidently b taken c times, cb, and the direction will be, multiply a by c, and from the product take the multiple of b by c. That is in the short-hand of algebra $ca-cb$: nor can this operation be announced in this mode of directing our operations in any other than this circuitous way; but in practice the operator would certainly go a shorter way to work, and having first subtracted b from a, would multiply the remainder by c.

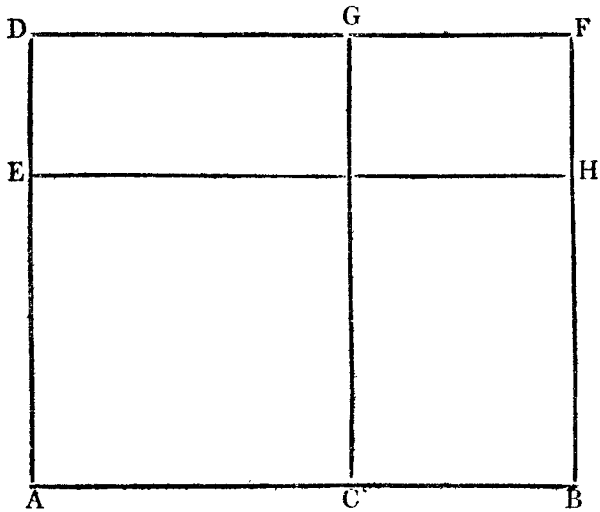
3dly. $- \times -$ produces $+$.

This most perplexing rule to every clear-headed student, and which has been rendered only more obscure by every attempt I have yet met with to explain it, (for demonstrations I cannot call them) may be equally explained by following one step further the above train of reasoning. If $a-b$ is to be multi-

 plied

plied by $c-d$, or taken $c-d$ times, the evident meaning is, that the difference of a and b is to be multiplied by the difference of c and d. Now we have before shown that if $a-b$, be taken c times, the direction for this operation is $ca-cb$. But at present we are to have a less result, as the multiplier c is diminished by d. Something more therefore is to be taken from ca, than in case 2. Now as we can only operate on the symbols separately, we will now take from ca, dg, and the expression will be $ca-cb-da$. But it is obvious that we have now taken away too much from ca, for da is the whole quantity a taken d times; whereas the quantity we ought to have taken away was only a lessened by b, d times. It is therefore obvious that having in the expression $ca-cb-da$, lessened ca too much by db, or be taken d times, we must add to that expression db, and the true direction will be $ca-cb-da+bd$: and as these symbols are of no particular meaning the rule must be universal, and must extend to multinomials as well as binomials, for in all cases the reasoning is precisely the same.

This may also be demonstrated geometrically with great care.



Let the line $AB=a$, $BC=b$, $AD=c$ and $DE=d$. Then will AC be $=a-b$, and $EA=c-d$. Complete the parallelogram ABDF and draw CG and EH through C and E parallel with DA and AB.

Then the rectangle AF will represent ca, the rectangle CF, cb, the rectangle EF, da, the rectangle GH db, and the rectangle CE, CE, the result arising from multiplying $a-b$ by $c-d$. Now in the formula $ca-cb-da+db$ we are directed first to take from AF; CF, and then EF, but the rectangle GH being common to both CF and EF, has been already taken away when we took away CF, and as in the algebraical formula this cannot be told, we take away the whole rectangle EF, and then add the rectangle GH, which if it had been taken out of the rectangle CE would evidently have left the result of the operation too small.

III. *New Outlines of Chemical Philosophy.*

By Ez. Walker, *Esq. of Lynn, Norfolk.*

[Continued from vol. xliv. p. 441.]

The composition of the atmosphere is still but imperfectly understood. It is said that it consists of 21 per cent. of oxygen gas and 79 of nitrogen; but it is well known that hydrogen and carburetted hydrogen gases arise from various species of vegetable and animal decay and putrefaction, and in great abundance from marshes and stagnant pools of water, especially in hot weather. Consequently those gases would soon contaminate the atmosphere to such a degree, as would render a country uninhabitable, were it not for the natural means by which those gases are decomposed and returned to the earth. Hence it is probable that the upper parts of the atmosphere generally contain a large portion of those gases.

The air upon the tops of mountains is much colder than in the valleys, because hydrogen gas, being much lighter than atmospheric air, rises in great abundance into the higher regions; but oxygen gas cannot ascend so high, in consequence of its being many times heavier than the other.

Hence, the temperature of the air in the higher regions cannot be equal to its temperature upon the surface of the earth where those gases are in a more condensed state*.

"In every case of combustion there must be present a combustible and a supporter;" nor can the temperature of a body be increased without these two elements be united.

To illustrate this position by experiment; Let two pieces of dry wood be smartly rubbed together, and an increase of temperature will be generated long before combustion takes place†. Consequently *combustion and increase of temperature are effects of the same cause.*

* Phil. Mag. vol. xliii. p. 103. † Ibid. p. 103.

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