



LII. A reply to Earl Stanhope, on his defence of certain principles and facts erroneously stated in his stereotyped "Principles of the Science of Tuning Instruments with fixed Tones."

Mr. John Farey

To cite this article: Mr. John Farey (1809) LII. A reply to Earl Stanhope, on his defence of certain principles and facts erroneously stated in his stereotyped "Principles of the Science of Tuning Instruments with fixed Tones.", *Philosophical Magazine Series 1*, 33:132, 292-299, DOI: [10.1080/14786440908562870](https://doi.org/10.1080/14786440908562870)

To link to this article: <http://dx.doi.org/10.1080/14786440908562870>



Published online: 18 May 2009.



Submit your article to this journal [↗](#)



Article views: 3



View related articles [↗](#)

correct figure to a convex than to a plane speculum ; and it is well known to practical opticians, that the errors of one spherical speculum often correct those of the other.

If any real advantages arise from shortening the tubes of reflecting telescopes, it becomes a matter of importance that refracting telescopes should possess similar properties. By means of the following contrivance the tubes of refractors may be so much shortened as to be only one third of the focal length of the object-glass. A plane speculum CD, whose diameter is two-thirds of that of the object-glass AB, is so placed that CA is one-third of the focal length of AB. By giving a small inclination to CD, the rays are reflected to a second plane speculum EG, equal to one-third of the diameter of AB, which again reflects the incident rays to F, where the image is formed and magnified by the eye-piece. In this construction, the only disadvantage is the loss of light occasioned by two reflections ; but this may be obviated by increasing the aperture of the object-glass, and is by no means such a serious evil as that which arises in M. Burckhardt's contrivance, from the loss of such a large central portion of the great speculum.

I am, dear sir, your most obedient servant,

D. BREWSTER.

To Mr. Tilloch.

LII. *A Reply to Earl Stanhope, on his Defence of certain Principles and Facts erroneously stated in his Stereotyped "Principles of the Science of Tuning Instruments with fixed Tones."* By Mr. JOHN FAREY.

"The difference between a man of *real science*, and one who has the ambition to be thought so, is very great."—EARL STANHOPE.

TO MR. TILLOCH,—SIR,

THE truths and principles of the Mathematical Sciences are not in any instance to be yielded to authority, however imposing its aspect ; neither should we suffer any other considerations, long to restrain our efforts, in defending their
just

just cause. It has been purely out of regard and tenderness to the unfortunate situation of a musician of the very first rank, whose mental aberrations had been much aggravated by the part he was led to take, and made *appear* to act, in explaining and defending a noble Earl's reveries on the subject of Tuning musical Instruments, that I have been so long kept back from replying to such parts of the two Letters of Earl Stanhope, printed in your Magazine (vol. xxviii. p. 144, and xxx. p. 34,) as relate to the scientific principles of Tuning: and similar feelings towards the very respectable individual alluded to, alone induce me to refrain from again touching on the two "Plain Statements," and the "Narrative," further than to declare, as in justice to Dr. C.'s musical reputation I think I ought, that he never, I believe, perused or saw the Stanhopian "Plain Statement," mentioned by His Lordship in vol. xxx. p. 25, *previously to its publication**, except in the hands of Mr. Ferguson, from whom he *refused to take the proof sheets, or look at them*: but, as Mr. F. himself told me, directed him to take them again to the printer; intending, as he (Dr. C.) has often told me, that His Lordship should be responsible for what he had written and got printed, and not suspecting, under the circumstances, that the name of *J. W. Calcott* would be affixed to it when published. After this, there needs no more for me to say at present, than request those who happen to have the two pamphlets, to *compare them together*, as the worthy and unfortunate Doctor intended, by stitching up and distributing them, as I mentioned in a former communication.

There are six *questions* touched upon in His Lordship's two Letters referred to, on each of which I wish to be indulged in saying a few words:—these are shortly,

- 1st, Whether a monochord board should be divided into 120 or 100 parts?
- 2d, Whether the *difference of the lengths* of string, can *accurately measure the interval* between the sounds of two strings, of the same size, weight, and tension?

* Indeed I saw Dr. C. write a Note to you, Sir, to this effect, in February 1808, with an intent that you should *publish* this fact in your Magazine.

294 *A Reply to Earl Stanhope, on his Defence of*

3d, Whether *four* or *five columns* are to be found in pages 7 and 22 of the Stereotype pamphlet?

4th, Whether the intervals called the four tierce wolves are of *the same* or *different magnitudes*?

5th, Whether *equal temperaments* of successive concords of the same kind, produce *equality in the rates of their beating*?

6th, Whether the notation of musical intervals generally, by Σ , f and m , rather than by their most simple ratios, be analogous to substituting a notation by *scores*, *dozens*, and *odd*, in place of the universally received *decimal notation*?

I.—In addition to the reasons I have given at page 192 of vol. xxvii., for preferring a decimal division of the monochord, I have further to remark, on what has fallen from His Lordship (page 144 of vol. xxviii.) that those “important lengths” which His Lordship’s scale is calculated to show *in round numbers*, are perfectly *unimportant*; for, what person using a monochord, other than as a play-thing, wants to use the scale attached to the string at all, in tuning a *perfect concord* of any kind? And does not the use of its scale as a tuning apparatus wholly consist, in either setting or taking off *tempered intervals*? And whether is it easiest, to set a triequal quint for instance, on a decimal scale by my number .6694329, or on His Lordship’s scale of 120, by means of his vulgar fractions $\frac{2,00898,850}{3,00000000}$ (Stereotype page

23,) or $\frac{71,70,247,592}{107,10,927200} + \left(\frac{A}{D} \text{ p. } 21\right)$?

II.—I have maintained (and am backed by all mathematical writers) that it is *ratios* only, and not *lengths*, except of such things as in their nature measure ratios, as logarithm scales &c. do, that can define musical intervals. And though His Lordship expressly asserts (p. 145, vol. xxviii.) that “deviations from perfect intervals are concisely, as well as accurately and conveniently expressed, by means of the *difference of the lengths* of wires,” I shall take the very example which he alludes to, (Stereotype p. 8,) wherein it is said, that 1.44 the *difference* of two strings, of which the
octave

octave length is 120'00, "shows the value" of the tierce wolf; in order to show, that $\frac{144}{12000}$, instead of expressing an interval called the enharmonic *diesis* ($21 \Sigma + 2 m$) as it ought to do, represents an interval exceeding 6 octaves by a superfluous third ($3905 \Sigma + 77 f + 338 m$)!

III.—Five columns certainly appeared to my eyes, when I was commenting on the Stereotype pages 7 and 22, therefore, unluckily it should seem, I mentioned *five*; but have I anywhere said or insinuated, that His Lordship therefore intended to represent *five wolves*, besides that produced by the quints? And I could not myself have intended to represent five such wolves, when his Lordship is severe upon me for saying there are but two in all. His Lordship's sarcasms, about dividing 12 into 5 aliquot parts, might therefore have been spared.

IV.—My arguments for the exact equality of all His Lordship's four tierce wolves, (at page 200, vol. xxvii.) retain their force, and are not invalidated by what His Lordship has advanced at page 149, vol. xxviii.; where, fortunately, His Lordship has let us into the secret of his blunders in this respect, by the mention of "monochord lengths," showing, that when His Lordship argues for *as complete a distinction* between his tierce wolves, *as to magnitudes*, as between half-guineas, half-crowns, sixpences, and halfpence, he had no better ground than their different lengths on the monochord; forgetting what I had endeavoured to impress on him, under the second head above, as to the fallacy of these lengths as a test of the magnitudes of intervals. Could not His Lordship as easily "distribute" or divide *the same* interval in four different ways in his C G D and A columns; as he can so distribute four *different* intervals? unless he confines his idea of equality, to monochord lengths, as then of course, they would only fit where the *octave* and *thirds* are also of the proper proportionate lengths! Absurdities, to which His Lordship surely could not have turned his attention.

V.—I did think it possible, when writing my observations page 201 to 203, vol. xxvii., that slips of His Lord-

ship's pen had occasioned his appearing to advance a doctrine, so opposite to all that had been demonstrated by Dr. Smith, Dr. Robison, and a host of other mathematical writers ; but his defence of the same in pages 150 to 152, vol. xxviii., precludes any such charitable suppositions in future. The *scientific terms*, or rather the "scientific jargon," of His Lordship, I certainly do not understand, if by that he means, that I am *to receive them*, in opposition to the authorities above quoted, by whom His Lordship was certainly *not* "obliged to use" his new terms, for they have uniformly and consistently used pulses or vibrations for what His Lordship would now for the first time call *beats* ; and what he would exclusively call *beatings* they have generally called *beats*, but have sometimes used *beatings* as synonymous therewith.

Before His Lordship took pen in hand on this subject, I well knew that the rate of beating increased along with every increase of the imperfection of a consonance ; but His Lordship is the only one I ever heard assert, that it increases "*As the imperfection increases*," which is no more true, than that the *sine* of an angle increases *as* the angle increases, or that gravity increases *as* the distance decreases. His Lordship refers (page 151,) to an example, and attempts to prove, that the triequal quints DA, one an octave above the other, *beat equally quick* : let us therefore see what evidence *numbers* furnish in this case. By referring to my table in page 5, vol. xxx. it will appear, that the two D's vibrate or excite 134.44 and 268.88 complete pulses in the air in one second of time respectively, and the two A's 200.83 and 401.66 pulses respectively, and by using these in the proper theorem for the purpose, we get 1.666 beats per second made by the lower, and 3.333 beats per second by the upper of these tempered or tri-equal quints ; the one just *double* of the other, instead of their being *equal* as our noble author has maintained ; and thus we see, that no "beating between the two beatings" could in this case happen even *in theory*, and certainly none *in practice* could be expected ; for who besides Earl Stanhope ever talked of hearing *beatings*, between two noises which themselves occur but $1\frac{2}{3}$ and $3\frac{1}{3}$ times

times per second ! or not above one-eighth of the rate necessary to constitute continuous or musical sound? In the case of *equally tempered* intervals, situate at the exact distance of *any of the concords* from each other, it is generally true, that no "beating between the beatings" either in theory or practice can happen. Suppose for instance, His Lordship's minor sixth C A which is flattened about $\frac{21}{22}$ parts of a comma (not $\frac{20}{21}$ as printed by mistake p. 195, vol. xxvii.) ; this in the middle septave beats 22·6335 times per second (or rather, in practice won't beat at all, but produce a continuous third discordant note) : if we tune another similar or equally tempered sixth, on a note, a true minor sixth below C the bass of the former one, that is, on His Lordship's first bass E : we shall find, that this will beat just 5-eighths as fast as the above, or 14·1459 times per second, but no "beating between the beatings" will take place, although each are quick enough to produce them, owing to their having the true relation of minor sixth between them, and not because they are *unisons* as His Lordship would contend. Let us, however, abandon the supposition of the tempered sixths having basses that are *exactly* at concordant distances, and tune just a similar minor sixth below C to that which His Lordship has above C, that is, take two of these 6ths *in succession* ; then we shall find, the lower note E making 151·79 vibrations per second, and the 6th EC will beat at the rate of 14·3144 times in a second : which not bearing a true concordant relation to the beating of the upper 6th, the sounding of the two together will be found by calculation to occasion a "beating between the beatings" at the rate of 1·3477 per second : thus we see, that a "beating between the beatings" *may* happen to *equally tempered* concords : and the same will indeed *always* happen, in theory at least, to the tempered concords of which His Lordship treats (although His Lordship asserts the contrary) ; for all his tuning is to be performed by *perfect* intervals except two *successive biequal thirds*, and three *successive tri-equal quints*, all of which will have such a "beating between the

the

the beatings," and of course so acute an observer as His Lordship cannot fail of perceiving them: and will be necessitated to "beat" a retreat, out of the labyrinth of error into which he has with temerity advanced, instead of thinking to "beat" his pretended "facts" and "important musical truths" into me, or any one else who has the least pretensions to mathematical knowledge.

VI.—I have here to complain of the same superficial view of the subject, as His Lordship took when commenting on decimally divided monochords: the object of any *general* notation of musical intervals cannot be to represent the perfect concords, as $\frac{1}{2}$ $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, &c., *more simply* than they are already expressed, but for comparing inconcinuous intervals, such as His Lordship's biequal third for instance, with any other intervals: if we examine the "important musical truths" in *Stereotype* with this view, what do we find, more than that the biequal third has an *approximate* ratio of $\frac{2,371,708,245 +}{3,000,000,000}$ (page 23)? If we wish to compare this with the triequal quint for instance, whose ratio is stated in the same page, viz. $\frac{3,008,298,850 +}{3,000,000,000}$, and are desirous to learn their *difference* or the interval remaining after the former is taken from the latter; in vain do we search the records of "musical truths" for the mode of accomplishing this. A novice, misled by the term "difference" in the last column of this page, might think his work easy, and attempt to give us the *difference* of these fractions, already reduced to a *common denominator*, for the purpose; but on discovering that the least interval had the largest numerator, here our tyro's exertions would probably end. One a little more experienced would discover, that it is a *ratio* which is to be deducted, and recollecting his school rule for the division of fractions, would proceed to multiply the denominators and numerators together reciprocally, when after proper reduction, $\frac{2,008,298,850 +}{2,371,708,245 +}$ would appear as the ultimate "truth" to be come at.

Now those who have done me the honour, of attending to
the

the *new notation* vol. xxviii. p. 142, would at once discover, that $354\frac{1}{3} \Sigma + 7 f + 30\frac{2}{3} m$, and $207\frac{1}{6} \Sigma + 4 f + 18 m$, represent the tricqual quint and biequal third respectively, and that the difference of these, or $146\frac{5}{6} \Sigma + 3 f + 12\frac{2}{3} m$, admits of an immediate comparison with all the various intervals in the table in plate V. of the same volume. One simple subtraction would further show it to be, a minor third flattened $14\frac{1}{6} \Sigma + 1\frac{1}{3} m$, or $2\frac{1}{6} \Sigma + \frac{1}{3} m$ more than the diaschisma or quint-wolf of our noble author: and hundreds of instances might be shown, wherein this notation gives still greater facility to the comparison of intervals with very complex ratios, than it does in the above case; but can His Lordship show a single instance (except the well-known and useful process of reducing large numbers of *pence* to pounds, shillings, and pence, for *some* purposes be considered such,) wherein *his* ingenious notation by scores, dozens, and odd would possess any advantage over decimal arithmetic? analogous to the conversion of simple ratios by the new notation into three elementary ratios (or two in some cases) which I have effected for the general comparisons of intervals? or, can his sapient approvers make out, similar advantages to result from *their* "cubit" and "measuring rod of Ezekiel," for expressing the Lapland degree?

I have not dropped my design of entering at some future time on a comparison of His Lordship's monochord and equal-beating systems, with the systems of other writers, particularly those which His Lordship has in so summary a way condemned, as I originally proposed, by the help of a table of the *temperaments* and *beats* of *every concord* which can arise in each system respectively: and as I am kindly assisted in the labour of these calculations, by a gentleman of more leisure than myself, with whom His Lordship is well acquainted, he has in the mean time the opportunity through him, of himself anticipating my intended comparisons, and of giving, any further support to his systems, which such comparative evidence will warrant.

I am, sir, your obedient humble servant,

JOHN FARREY.

12, Upper Crown Street, Westminster,
March 14, 1809.

LIII. On

Phil. Mag. Vol. XXXIII. 179.
Horizontal Section of the Geyserometer
at the Bottom part.

