

The following Communications were read :—

1. On the Dynamical Theory of Heat. Part VII. By
Professor W. Thomson.

This paper commences with a condensed re-statement of the fundamental principles and formulæ of the Dynamical Theory of Heat, from the first six parts of the author's treatment of the subject previously communicated to the Royal Society of Edinburgh, and his articles "On the Thermo-elastic Properties of Matter," in the "Quarterly Mathematical Journal" (April 1855), and on "Thermo-magnetism," and "Thermo-electricity," in Nichol's Cyclopedia (Edinburgh 1860).

The chief object of the paper is the deduction of numerical values in absolute measure for the thermo-electric effects which form the subject of Part VI. of this series ("Transactions of the Royal Society of Edinburgh," 1854; and "Phil. Mag." 1854, second half year, and 1855, first half year), especially for differences of temperature produced by electric convection of heat, and for the changes of temperature due to strain in elastic solids, investigated in the article on thermo-elastic properties of matter above referred to. The very valuable results, recently published, of the experiments of Forbes and Ångström for determining in absolute measure the thermal conductivities of iron and copper, supply a very important element, previously wanting, for definite estimates of those changes of temperature, and are taken advantage of in the present paper. Thus, the author has been enabled to give that practical character to some of his former conclusions, of which, when they were first published, he pointed out the want. In particular, with reference to elastic solids, the *apparent* value of Young's modulus* when the stress is applied and removed, or reversed so rapidly that the loss of thermal effect by conduction and radiation is insensible, is proved to be given by the following formula :—

$$M' = M \left(1 + \frac{e^2 t M}{J s \rho} \right)$$

* The amount of the force divided by the elongation produced by it, when any force within practical limits of elasticity is applied to elongate a bar rod or wire, of the substance, one square centimetre in section.

where M denotes the Young's modulus of the substance for constant temperature, s its specific heat (per unit mass, as usual), e its longitudinal (linear) expansion per degree of elevation of temperature, ρ its density or specific gravity,* and t its actual temperature from absolute zero ("Dynamical Theory of Heat," Part VI., § 100), that is, temperature centigrade with 274 added. Of course, if M is reckoned ("Thomson and Tait's Natural Philosophy," §§ 220, 221, 238), in gravitation measure (weight of one gramme, the unit of mass), J must be reckoned in gravitation measure (grammes weight working through one centimetre), in which case its numerical value is 42,400, being Joule's number (1390), reduced from feet to centimetres. Values of surface resistance to gain or loss of heat in absolute measure, derived from experiments by the author, are used to estimate the effect of radiation and convection in dissipating energy in virtue of the thermo-dynamic change of temperature in a rod executing longitudinal vibrations. The velocity of propagation of longitudinal vibrations (as in the transmission of sound along a bar) being equal to the velocity acquired by a body in falling through a height equal to half the "length of the modulus,"† is, of course, half as much affected as the modulus, by changes of temperature. In iron, for instance, the effect of change of temperature, when there is no dissipation, is an increase of about one-third per cent. on the Young's modulus, and of about one-sixth per cent. on the velocity of sound along a bar. The effect of the conduction of heat in diminishing the differences of temperature in a rectangular bar executing flexural vibrations, is investigated from the solution invented by Fourier for expressing periodical variations of underground temperature. Its absolute amount for bars of iron or copper, of stated dimensions, vibrating in stated periods, is determined from Forbes' and Ångström's conductivities. It is proved that the loss of energy due to this effect at its maximum is not by any means insensible, though it is not sufficient to account for the whole loss of energy which the author has found in experi-

* Which, when the French system (unit bulk of water being of mass unity) is followed, mean the same thing

† The "length of the modulus" is $M \div \rho$, if M be the modulus in grammes weight per square centimetre.—*Thomson and Tait's Natural Philosophy*, § 689.

ments on flexural vibrations of metal springs, which therefore prove imperfectness in the elasticity of flexure, such as he had previously proved for the elasticity of torsion.*

2. The "Doctrine of Uniformity" in Geology briefly refuted. By Professor William Thomson.

The "Doctrine of Uniformity" in Geology, as held by many of the most eminent of British Geologists, assumes that the earth's surface and upper crust have been nearly as they are at present in temperature, and other physical qualities, during millions of millions of years. But the heat which we know, by observation, to be now conducted out of the earth yearly is so great, that if *this* action had been going on with any approach to uniformity for 20,000 million years, the amount of heat lost out of the earth would have been about as much as would heat, by 100° Cent., a quantity of ordinary surface rock of 100 times the earth's bulk. (See calculation appended.) This would be more than enough to melt a mass of surface rock equal in bulk to the *whole earth*. No hypothesis as to chemical action, internal fluidity, effects of pressure at great depth, or possible character of substances in the interior of the earth, possessing the smallest vestige of probability, can justify the supposition that the earth's upper crust has remained nearly as it is, while from the whole, or from any part, of the earth, so great a quantity of heat has been lost.

APPENDIX.

Estimate of present annual loss of heat from the earth.

Let A be the area of the earth's surface, D the increase of depth in any locality for which the temperature increases by 1° Cent., and k the conductivity per annum of the strata in the same locality. The heat conducted out per annum per square foot of surface in that locality is $\frac{k}{D}$. Hence, if we give k and D proper average

* Proceedings of the Royal Society of London, May 1865.—*W Thomson*, "On the Elasticity and Viscosity of Metals."