hence $v^2 = \frac{A}{y^2}$, A being constant, and consequently the force is attractive and varies as $\frac{A}{y^3}$.

The principle of vis viva gives us

$$v^2 = \frac{A}{y^2} - \frac{A}{y'^2} + \beta^2,$$

therefore

$$\beta^2 = \frac{A}{y'^2}.$$

Hence:—*A particle constrained to move on the surface under the action of an attractive force varying as* $\frac{A}{y^3}$, *and projected perpendicularly to an umbilicar geodesic with the initial velocity* $\frac{\sqrt{A}}{y'}$, *will describe the curve traced on the surface by the extremity of an umbilicar geodesic of constant length.*

On the Points and Tangents common to Two Conics.

By Professor GENESE, M.A.

[*Read May 10th*, 1883.]

Taking the triangle ABC formed by three common tangents to two conics as that of reference in any system of point coordinates, let the equations to the conics be

$$\sqrt{l\alpha} + \sqrt{m\beta} + \sqrt{n\gamma} = 0 \quad \text{.......................(1)},$$

$$\sqrt{l'\alpha} + \sqrt{m'\beta} + \sqrt{n'\gamma} = 0 \quad \text{.......................(2)},$$

each radical being capable of the double sign; we may, without loss of generality, make the convention that the first term in each is to be taken positively.

For the points common to the two conics, we have

$$\frac{\sqrt{\alpha}}{\sqrt{m}\sqrt{n'} - \sqrt{n}\sqrt{m'}} = \frac{\sqrt{\beta}}{\sqrt{n}\sqrt{l'} - \sqrt{l}\sqrt{n'}} = \&c.,$$

or

$$\frac{\alpha}{mn' + nm' - 2\sqrt{m}\sqrt{n}\sqrt{m'}\sqrt{n'}} = \&c.,$$

or, say,

$$\frac{\alpha}{p - (p')} = \frac{\beta}{q - (q')} = \frac{\gamma}{r - (r')} \quad \text{....................(3)},$$

where p', q', r' may take the double sign; but, since $q'r' = ll'p'$ always, the product $p'q'r'$ must be positive. Thus p', q', r' being certain numerical quantities, the coordinates of the points P, Q, R, S common to the two conics are

$$(p-p',\ q-q',\ r-r') \quad\ldots\ldots\ldots\ldots\ldots\ldots \text{P},$$
$$(p-p',\ q+q',\ r+r') \quad\ldots\ldots\ldots\ldots\ldots\ldots \text{Q},$$
$$(p+p',\ q-q',\ r+r') \quad\ldots\ldots\ldots\ldots\ldots\ldots \text{R},$$
$$(p+p',\ q+q',\ r-r') \quad\ldots\ldots\ldots\ldots\ldots\ldots \text{S}.$$

The equation to AP is $\dfrac{\beta}{q-q'} = \dfrac{\gamma}{r-r'}$;

to BS is $\dfrac{\gamma}{r-r'} = \dfrac{\alpha}{p+p'}$;

to CR is $\dfrac{\alpha}{p+p'} = \dfrac{\beta}{q-q'}$;

thus these straight lines intersect at the point

$$(p+p',\ q-q',\ r-r'),$$

that is, the triangles ABC, PSR are homologous. In the same way, it may be shown that the triangle ABC is homologous with each of the triangles QRS, RQP, and SPQ (the positions of the letters indicating the corresponding points).

The four centres of homology are the points

$$(p+p',\ q-q',\ r-r'),$$
$$(p+p',\ q+q',\ r+r'),$$
$$(p-p',\ q-q',\ r+r'),$$
$$(p-p',\ q+q',\ r-r').$$

It follows, from the above, that *each of the triangles determined by the common tangents to two conics is homologous with each of the triangles whose vertices are at the points common to the conics.*

The theorem may be restated thus :—*If two triangles be such that more than one conic can be simultaneously described about one and inscribed in the other, then the two triangles are homologous.*

A direct analytical proof of this theorem seems to be exceedingly difficult.

It may be added, that the equations to the axes of homology take the forms

$$\frac{\alpha}{p'} \pm \frac{\beta}{q'} \pm \frac{\gamma}{r'} = 0.$$

If the conics be traced, the points P, Q, R, S may be determined by the aid of the following lemma. Let L, M, N be the points of contact of the conic (1) with BC, CA, AB respectively. Then, whatever be the nature of the conic, for the part of the curve from M to N not passing through L (including the case of a passage through a point at infinity) the signs of the radicals in (1) are given by

$$\sqrt{m\beta}+\sqrt{n\gamma}-\sqrt{l\alpha}=0.$$

From N to L the equation is

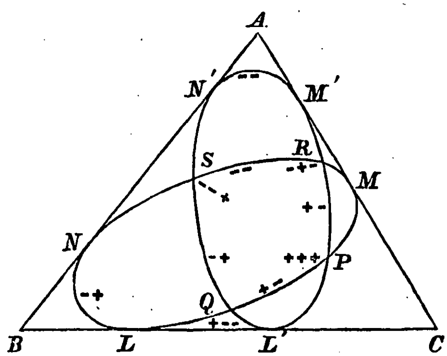
$$\sqrt{n\gamma}+\sqrt{l\alpha}-\sqrt{m\beta}=0;$$

and from L to M $\quad \sqrt{l\alpha}+\sqrt{m\beta}-\sqrt{n\gamma}=0.$

Hence, since it is convenient to give the positive sign to $\sqrt{l\alpha}$, the signs to be taken for $\sqrt{m\beta}$, $\sqrt{n\gamma}$ are—

from M to N, $-\ -$;
from N to L, $-\ +$;
from L to M $+\ -$.

Treating equation (2) similarly, there is no difficulty in determining the proper signs to be given to p', q', r' in (3) for a particular point of intersection of (1) and (2). The figure shows the determination in the case of two ellipses.



Two signs, as $-\ -$, are according to the lemma.

Three signs, as $+\ -\ -$, are those to be given in (3) to p', q', r' respectively; they are to be changed on clearing the brackets.

The theorems admit of easy geometrical verification on projecting the quadrilateral $PQSR$ into a square.

Thursday, June 14th, 1883.

Prof. HENRICI, F.R.S., President, in the Chair.

Prof. W. Woolsey Johnson, of Annapolis, was admitted into the Society. Prof. Cayley made a few remarks "On Seminvariants and Mr. Hammond's recent Discovery." Prof. Sylvester (who was warmly received) and Mr. Hammond spoke on the same subject. Mr. Tucker read parts of papers: by Prof. H. Lamb, "On the Mutual Potential of

Two Lines in Space"; by Mr. H. M. Jeffery, "On Bicircular Quartics with Collinear Foci"; and spoke on the subject of "Inverse Coordinate Curves" (*i.e.*, such that $xx' = a^2 = yy'$).

The following presents were received:—

"Educational Times," for June, 1883.

"Cambridge Philosophical Society—Proceedings," Vol. iv., Parts 2, 3, 4, and 5; "Transactions," Vol. xiii., Part 2.

"Proceedings of the Physical Society of London," Vol. v., Part 3.

"Great Trigonometrical Survey of India," Vols. vii. and viii.

"Johns Hopkins University Circulars," Vol. ii., No. 22.

"Sitzungsberichte der Königlich-Preussischen Akademie der Wissenschaften zu Berlin," 1883, Parts 1 to 21; 8vo, Berlin, 1883.

"Atti della R. Accademia dei Lincei," Vol. vii., Fasc. 9 and 10.

"Journal de l'École Polytechnique, &c.," 52 Cahier.

"Bulletin des Sciences Mathématiques, &c.," Tome vi., Index.

"Bulletin Astronomique et Météorologique de l'Observatoire de Rio de Janeiro," No. 3.

"Grundzüge einer Arithmetischen Theorie der Algebraischen Grössen," Festschrift zu Herrn Ernst Eduard Kummer's Fünfzigjährigem Doctor-Jubiläum, 10 Sept., 1881, von L. Kronecker; 4to, Berlin, 1882.

"Zur Theorie der Abel'schen Gleichungen," von Kronecker; 4to.

"Sur les Unités Complexes," par M. L. Kronecker; 4to, Paris, 1883.

The following pamphlets by M. Kronecker:—

"Ueber Potentiale n-facher Mannigfaltigkeiten."

"Die Composition Abel'scher Gleichungen."

"Die Kubischen Abel'schen Gleichungen des Bereichs $(\sqrt{-31})$."

"Ueber den vierten Gauss'schen Beweis des Reciprocitätsgesetzes für die Quadratischen Reste."

"Ueber die Irreductibilität von Gleichungen."

"Ueber die Symmetrischen Functionen."

"Zur Theorie der Elimination einer Variabeln aus zwei Algebraischen Gleichungen."

"Zur Theorie der Elliptischen Functionen."

"Aus der Theorie der Algebraischen Gleichungen."

"Beiblätter zu den Annalen der Physik und Chemie," Band vii., St. 4 and 5.

"Mémoires de la Société des Sciences Physiques et Naturelles de Bordeaux," Tome v., 2 Cahier.

"Jornal de Sciencias Mathematicas e Astronomicas," publicado pelo Dr. Francisco Gomes Teixeira, Vol. iv., Nos. 1—6; 8vo, Coimbra, 1882.

"Atti del Reale Istituto Veneto di Scienze, Lettere, ed Arti," Tomo Primo, Serie Sesta, Dispensa 1, 2, 3, Tomo Ottavo; Serie Quinta, Dispensa 7, 8, 9, 10; 8vo, Venice.