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The iron and manganese were separated from the lime by ammonia after the addition of bromine.

The separation of phosphoric acid was complete; not a trace could be discovered with the bases by means of molybdate of ammonia, nor could a trace of lime be discovered by treating the $2MgO, PO^5$ with sulphuric acid, evaporating and dissolving in alcohol.

The results obtained in two analyses from crystals from different parts of the specimen were in 100 parts,—

	I.	II.
Lime	53·38	52·81
Oxides of iron and manganese	2·96	3·22
Phosphoric acid	41·34	41·80
Fluorine and loss	2·32	2·17
	100·00	100·00

This composition corresponds very nearly to that of fluor apatite, $CaFl+3(3CaO, PO^5)$, in which the lime is partially replaced by the protoxides of iron and manganese. This composition would give per cent. (see Rammelsberg's *Handwörterbuch der Mineralogie*, p. 37),—

Lime	55·88
Phosphoric acid . . .	42·02
Hydrofluoric acid . .	2·10
	100·00

And these analyses of Francolite confirm, by the direct estimation of the PO^5 by an accurate method, the results obtained by Gustave Rose in an elaborate investigation of several varieties of apatite from various localities, published many years ago*, in which the phosphoric acid was estimated from the loss.

XVIII. *On the Equation* $Q=q(w, x, y, z)=w+ix+jy+kz$. *By* WILLIAM SPOTTISWOODE, *M.A., of Balliol College, Oxford*†.

THE theorem expressed by the above equation is of considerable importance in the calculus of quaternions, and indeed essential for the application of that method to geometrical and physical problems. Sir W. R. Hamilton in his researches (Transactions of the Royal Irish Academy, vol. xxxi.), has effected the transformation by means of the symbolical division of numeral sets; but since nothing, which may throw light

* Poggendorff's *Annalen*, vol. ix.; and Berzelius, *Jahresbericht*, 1828.

† Communicated by the Author.

upon the nature and properties of these functions, is entirely without interest, I have ventured to suggest another mode in which the question may be viewed.

It is known that the symbol R_n^1, when prefixed to any quaternion, indicates that, out of the four constituents w, x, y, z, regarded in a definite order, that one is selected which stands in the nth place from the left-hand. If, then, to the complex symbol R_nQ there be prefixed another symbol of selection R_m, it is clear that, since in the expression R_n^1Q there is only one constituent from which the new selection is to be made, the combination R_mR_nQ will vanish for all values of m, except $m=1$. Hence may be formed the following symbolical system:

$$\left.\begin{array}{llll} R^2_0=R_0, & R_1R_0=0, & R_2R_0=0, & R_3R_0=0 \\ R_0R_1=R_1, & R^2_1=0, & R_2R_1=0, & R_3R_1=0 \\ R_0R_2=R_2, & R_1R_2=0, & R^2_2=0, & R_3R_2=0 \\ R_0R_3=R_3, & R_1R_3=0, & R_2R_3=0, & R^2_3=0, \end{array}\right\} \quad (1.)$$

and consequently, by the principles of the calculus,

$$Q=(R_0Q,\ R_1Q,\ R_2Q,\ R_3Q) \quad . \ . \ . \ . \ . \ . \ . \ . \quad (2.)$$

$$\left.\begin{array}{l} =\{(R^2_0-R^2_1-R^2_2-R^2_3)Q, \\ \quad (R_1R_0+R_0R_1+R_3R_2-R_2R_3)Q \\ \quad (R_2R_0+R_0R_2+R_1R_3-R_3R_1)Q \\ \quad (R_3R_0+R_0R_3+R_2R_1-R_1R_2)Q\} \end{array}\right\} \quad . \ . \ . \ . \ . \quad (3.)$$

$$\left.\begin{array}{llll} =(R^2_0Q, & R_1R_0Q, & R_2R_0Q, & R_3R_0Q) \\ +(R_{-1}R_1Q, & R_0R_1Q, & R_{-3}R_1Q, & R_2R_1Q) \\ +(R_{-2}R_2Q, & R_3R_2Q, & R_0R_2Q, & R_{-1}R_2Q) \\ +(R_{-3}R_3Q, & R_{-2}R_3Q, & R_1R_3Q, & R_0R_3Q), \end{array}\right\} \quad (4.)$$

$$\left.\begin{array}{l} =R_{0,1,2,3}R_0Q+R_{-1,0,-3,2}R_0Q \\ \qquad +R_{-2,3,0,-1}R_0Q+R_{-3,-2,1,0}R_0Q \end{array}\right\} \quad . \ . \quad (5.)$$

$$=w+ix+jy+kz \quad . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \quad (6.)$$

The same result might have been obtained by means of the relation

$$(w,x,y,z)=(w,0,0,0)+(0,x,0,0)+(0,0,y,0)+(0,0,0,z), \quad (7.)$$

the second side of which might be at once replaced by (4.); but in some respects the former method is preferable. In either case it appears that the expression (6.) is only one out of an infinite number which might have been obtained, by substituting for the expressions in (3.) any arbitrary combinations of R_0, R_1, R_2, R_3, respectively equivalent to those symbols themselves;

but the advantage of (6.), resulting from the relations between i, j, k, is well known.

It may be further remarked, that although

$$\left.\begin{array}{l}(w,0,0,0)=R_{0,1,2,3}(w,0,0,0)=Q_0\\(0,x,0,0)=R_{-1,0,-3,2}(x,0,0,0)=iQ_0'\\(0,0,y,0)=R_{-2,3,0,-1}(y,0,0,0)=jQ_0''\\(0,0,0,z)=R_{-3,-2,1,0}(z,0,0,0)=kQ_0'''\end{array}\right\}\quad .\quad .\quad (8.)$$

(where the meanings of the expressions Q_0, Q_0', Q_0'', Q_0''' are obvious), and consequently by (7.)

$$Q=Q_0+iQ_0'+jQ_0''+kQ_0''' \quad . \quad . \quad . \quad . \quad (9.)$$

(an expression of the same form as (6.)); yet this would not be sufficient for the present purpose, for all the terms on the right-hand side of this equation are themselves quaternions, while the object of the transformation is to exhibit a quaternion under the form of a series of terms, which admit of being combined by laws in some degree analogous to those of ordinary algebra. That expressions of the form (9.) admit of such combinations is certainly true; but this can be proved only by means of some such formula as (6.).

As I do not propose to enter at present upon the general idea of this calculus, I will only add, that the form of the expression (3.) seems worthy of attention, from the similarity of the combinations of R_0, R_1, R_2, R_3 in it, and those of w, x, y, z,... in the squares and products of quaternions.

XIX. *Observations upon* M. Boutigny's *recent Experiment.* *By* Professor Plücker *of Bonn**.

IT may perhaps be a matter of interest to you to obtain a confirmation of Boutigny's recent experiment. With his usual kindness, he exhibited to me last Easter his former experiments; and whilst admiring his rare perseverance in following up a fertile idea, I then acquired an impression that it referred to a law of nature which was by no means completely revealed, and in which opinion I was further strengthened by the report of his last experiment. In consequence of an oral communication of this experiment, M. Fessel wrote to me from Cologne, stating that on the following day he had dipped his finger into lead heated to its highest point, by

* From Poggendorff's *Annalen*, Dec. 7, 1849.