2. Let Q_{2m} denote the quadrinvariant of the $2m$-ic and let C_{2m+1} be the source of a covariant of degree 3 in the coefficients, such that

$$C_{2m+1} = a_0 V_{2m+1} - 2a_1 Q_{2m} \quad\ldots\ldots\ldots\ldots(1)$$

then, if $$\delta = a_0 \frac{d}{da_1} + 2a_1 \frac{d}{da_2} + 3a_2 \frac{d}{da_3} + \ldots,$$

operating with δ on (1), and remembering that $\delta Q_{2m} = 0$, $\delta C_{2m+1} = 0$, we have

$$\delta V_{2m+1} = 2Q_{2m} \quad\ldots\ldots\ldots\ldots(2).$$

From (1) it follows that

$$C_{2m+1} Q_{2p} - Q_{2m} C_{2p+1} = a_0 (V_{2m+1} Q_{2p} - V_{2p+1} Q_{2m}),$$

where $V_{2m+1} Q_{2p} - V_{2p+1} Q_{2m}$ is a source of degree-weight $(4 . 2m+2p+1)$.

In the case before us $m = 3$, $p = 2$, $2m+2p+1 = 11$, and

$$V_{2m+1} Q_{2p} - V_{2p+1} Q_{2m}$$

is the source of a covariant of degree-order (4 . 6) for the binary seventhic.

Replacing the sources by the corresponding covariants, we have

$$(3 . 7)(2 . 6) - (2 . 2)(3 . 11) = (1 . 7)(4 . 6) \quad\ldots\ldots\ldots\ldots(3),$$

a syzygant of degree-order (5 . 13).

Now, since there are four linearly independent covariants of this degree-order, and four compound covariants, of which (3) shows that there are only three linearly independent; it follows that there must be a ground-form of this degree-order to make up the number of independent covariants.

The source of this ground-form is Θ, which is known to be a ground-source [of degree-order (5 . 3) (5 . 8)] for the quintic and sextic.

It may be shown that Θ is *not a ground-source* for any quantic of higher order than the seventhic.

Addition to the foregoing Paper. *By* Prof. CAYLEY.

[*Read Dec. 14th*, 1882.]

The extreme importance of Mr. Hammond's result, as regards the entire subject of Covariants, leads me to reproduce his investigation in the notation of my Memoirs on Quantics, and with a somewhat different arrangement of the formulæ. For the binary seventhic

$$(a, b, c, d, e, f, g, h \between x, y)^7,$$

the four composite seminvariants of the deg-weight 5 . 11 (sources of covariants of the deg-order 5 . 13) are

I.			II.		
1 . 7	4 . 6	... ...	2 . 10	3 . 3	Deg-order.
1 . 0	4 . 11	... ...	2 . 2	3 . 9	Deg-weight.

a	$+1$

.

a^2ch	$+1$
fg	-1
$abdh$	-4
beg	-2
bf^2	$+6$
c^2h	$+3$
cdg	-2
cef	-6
d^2f	$+10$
de^2	-5
a^0b^2ch	0
b^2dg	$+20$
b^2ef	$+57$
bc^2g	-15
$bcdf$	-24
bce^2	-30
bd^2e	-10
c^3f	$+27$
c^2de	-45
cd^3	$+20$

ac	$+1$
b^2	-1

.

ach	$+2$
dg	-7
ef	$+5$
a^0b^2h	-2
bcg	$+7$
bdf	$+22$
be^2	-25
c^2f	-27
cde	$+45$
d^3	-20

III.			IV.		
2 . 6	3 . 7	... ...	2 . 2	3 . 11	Deg-order.
2 . 4	3 . 7	... ...	2 . 6	3 . 5	Deg-weight.

ae	$+1$
bd	-4
c^2	$+3$

.

a^2h	$+1$
abg	-7
cf	$+9$
de	-5
a^0b^2f	$+12$
bce	-30
bd^2	$+20$

ag	$+1$
bf	-6
ce	$+15$
d^2	-10

.

a^2f	$+1$
abe	-5
cd	$+2$
a^0b^2d	$+8$
bc^2	-6

and it is here at once obvious that there exists a syzygy of the form I. = III. − IV.; in fact, if in III. and IV. we write $a = 0$, then the values are each

$$= -2b\,(4bd-3c^2)\,(6bf-15ce+10d^2);$$

hence III.—IV. must divide by a, the quotient being a seminvariant of the deg-weight 4.11, which can only be a numerical multiple of the second factor of I., and is in fact = this second factor, that is, we have the syzygy I. = III.—IV.

Working out the values of the four products, and joining to them the expression for the irreducible seminvariant of the same deg-weight 5.11 (O, x^8 of my tables for the binary sextic), we have the table:

5.10
a^3dh
eg
f
a^2bch
bdg
bef
c^2g
cdf
ce^2
d^2e
ab^3h
b^2cg
b^2df
b^2e^2
bc^2f
$bcde$
bd^3
c^3e
c^2d^2
a^0b^4g
b^3cf
b^3de
b^2c^2e
b^2cd^2
bc^3d
c^5

5.11	O	I.	III.	IV.	II.
a^3eh		+1	+1		
fg		−1		+1	
a^2bdh		−4	−4		
beg		−2	−7	−5	
bf^2		+6		−6	
c^2h		+3	+3		+2
cdg		−2		+2	−7
cef	−1	−6	+9	+15	+5
d^2f	+3	+10		−10	
de^2	−2	−5	−5		
ab^2ch					−4
b^2dg		+20	+28	+8	+7
b^2ef	+1	+57	+12	−45	−5
bc^2g		−15	−21	−6	+7
$bcdf$	−14	−24	−36	−12	+22
bce^2	+11	−30	−30		−25
bd^2e	+1	−10	+40	+50	
c^3f	+9	+27	+27		−27
c^2de	−14	−45	−15	+30	+45
cd^3	+6	+20		−20	−20
a^0b^4h					−2
b^3cg					−7
b^3df	+8		−48	−48	−22
b^3e^2	−9				+25
b^2c^2f	−6		+36	+36	+27
b^2cde	+16		+120	+120	−45
b^2d^3	−8		−80	−80	+20
bc^3e	−3		−90	−90	
bc^2d^2	+2		+60	+60	
c^4d					

I have prefixed to the table the literal terms of the deg-weight 5.10; for the deg-weights 5.11 and 5.10 the numbers of terms are = 30 and 26 respectively; and it is the difference of these 30−26, = 4, which gives the number of asyzygetic seminvariants of the deg-weight 5.11.

On Compound Determinants. *By* R. F. Scott.

[*Read Nov. 9th,* 1882.]

1. Consider the determinant of order $n+m$,

$$D = \begin{vmatrix} a_{11}, & a_{12}, & \dots & a_{1n}, & b_{11}, & \dots & b_{1m} \\ a_{21}, & a_{22}, & \dots & a_{2n}, & b_{21}, & \dots & b_{2m} \\ \dots & & \dots & & \dots & & \dots \\ a_{n1}, & a_{n2}, & \dots & a_{nn}, & b_{n1}, & \dots & b_{nm} \\ c_{11}, & c_{12}, & \dots & c_{1n}, & h_{11}, & \dots & h_{1m} \\ \dots & \dots & & \dots & \dots & & \dots \\ c_{m1}, & c_{m2}, & \dots & c_{mn}, & h_{m1}, & \dots & h_{mm} \end{vmatrix},$$

which we may regard as made up of four blocks of elements. A block of n rows and columns of elements a_{ik}; a block of n rows and m columns of elements b_{ik}; a block of m rows and n columns of elements c_{ik}; and finally, a block of m rows and columns of elements h_{ik}.

Let A stand for the determinant of order n,

$$\begin{vmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{vmatrix},$$

and let the system of first minors of the determinant A be denoted by

$$\begin{vmatrix} A_{11} & \dots & A_{1n} \\ \dots & \dots & \dots \\ A_{n1} & \dots & A_{nn} \end{vmatrix}.$$

If we border the block of elements a_{ik} in D with a row and column from the blocks of elements c_{ik} and b_{ik}, and that element from the block of elements h_{ik} where these rows and columns intersect, we get a system of m^2 elements of the form

$$p_{ik} = \begin{vmatrix} a_{11} & \dots & a_{1n}, & b_{1k} \\ a_{21} & \dots & a_{2n}, & b_{2k} \\ \dots & \dots & & \dots \\ a_{n1} & \dots & a_{nn}, & b_{nk} \\ c_{i1} & \dots & c_{in}, & h_{ik} \end{vmatrix}.$$