



L. On the general quartine, or the incriticoid of the fourth degree

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is about $\frac{1}{2}$ of the length of the bar, there is not much increase of moment due to the increase of the thickness of the bar; that is, the induced magnetic moment will be practically independent of the mass of the iron. This statement, paradoxical as it sounds, is not much to be wondered at if we consider that, when the bar becomes very thick, the substance of the bar itself will be forming a kind of internal armature to the free ends of the bar. Thus it seems likely, where thick bar magnets are used in practice, that there may be found waste of material of iron, although in many cases different shapes and the presence of external armatures will modify the condition from the case of the experiments described.

Glasgow University,
October 10, 1888.

L. *On the General Quartine, or the Incriticoid of the Fourth Degree.* By the Rev. ROBERT HARLEY, M.A., F.R.S.*

CRITICOIDS are those functions of the coefficients of a linear differential equation which remain unaltered when the differential equation is transformed by a change of one of the variables, being analogous in this respect to the critical functions or seminvariants of common algebra. We may divide them into two classes, according as the changed variable is the dependent or independent variable. Sir James Cockle, to whom we owe the discovery of these forms †, calls the first class "ordinary," and the second "differential," but in fact both are differential, because both contain differential coefficients. Professor Malet ‡ describes them as Invariants of the first and second class. The functions, however, are not strictly invariants, and the distinction between first and second class hardly seems marked enough. I propose to give the name Decriticoids to those forms which are unaffected by a change of the dependent variable, and the name Incriticoids to those which are unaffected by a change of the independent variable. A decriticoid of the m th degree may be called an m -ide, and an incriticoid of the same degree an m -ine.

* Communicated by the Author.

† Harley. "Professor Malet's Classes of Invariants identified with Sir James Cockle's Criticoids." Proceedings of the Royal Society for 1884, vol. xxxviii. pp. 45-57.

‡ Malet. "On a Class of Invariants," Philosophical Transactions for 1882, Part III. pp. 751-776.

Using the quantical notation we may write the linear differential equation of the n th order thus—

$$(1, P_1, P_2, \dots, P_n) \left(\frac{d}{d^p}, 1 \right)^n y = 0,$$

$$(1, Q_1, Q_2, \dots, Q_n) \chi_{\frac{d}{dt}}^d (1)^n y = 0,$$
$$\frac{\phi(Q, Q', Q'', Q''')}{Q^{\frac{4}{n}}},$$
$$\frac{\phi(P, P', P'', P''')}{P_n^4},$$
$$\begin{aligned} \phi(Q, Q', Q'', Q''') = & Q_1''' + \frac{12}{n-1} Q_1'' Q_1 + \frac{4}{n-1} (Q_1')^2 \\ & + \frac{44}{(n-1)^2} Q_1' Q_1^2 - \frac{5(n-1)}{2(n-4)} Q_4 + 10 Q_3 Q_1 + \frac{15(n-1)}{2(n-2)} Q_2^2 \\ & - \frac{30(n-2)}{n-1} Q_2 Q_1^2 + \frac{15n^3 - 75n^2 + 120n - 38}{(n-1)^3} Q_1^4 \end{aligned}$$

$$\begin{aligned}
& + \lambda_1 \left\{ Q_2'' - \frac{2(n-2)}{n-1} Q_1'' Q_1 + \frac{10}{n-1} Q_2' Q_1 - \frac{2(n-2)}{n-1} (Q_1')^2 \right. \\
& - \frac{20(n-2)}{(n-1)^2} Q_1' Q_1^2 - \frac{5(n-2)}{3(n-4)} Q_4 + \frac{20(n-2)}{3(n-1)} Q_3 Q_1 \\
& + \frac{5n-7}{n-2} Q_2^2 - \frac{2(10n-27)}{n-1} Q_2 Q_1^2 + \frac{(n-2)(10n^2-37n+17)}{(n-1)^3} Q_1^4 \Big\} \\
& + \lambda_2 \left\{ Q_3' - \frac{3(n-3)}{n-1} Q_2' Q_1 - \frac{n-3}{n-1} (Q_1')^2 + \frac{(n-3)(3n-8)}{(n-1)^2} Q_1' Q_1^2 \right. \\
& - \frac{5(n-3)}{4(n-4)} Q_4 + \frac{5n-9}{n-1} Q_3 Q_1 + \frac{3(n-3)}{2(n-2)} Q_2^2 - \frac{21(n-3)}{2(n-1)} Q_2 Q_1^2 \\
& + \frac{(n-3)(21n^2-57n+26)}{4(n-1)^3} Q_1^4 \Big\} \\
& + \lambda_3 \left\{ Q_1' Q_2 - \frac{n-2}{3(n-1)} (Q_1')^2 - \frac{(n-2)(3n-1)}{3(n-1)^2} Q_1' Q_1^2 \right. \\
& - \frac{3(n-1)}{4(n-2)} Q_2^2 + \frac{3n-1}{2(n-1)} Q_2 Q_1^2 - \frac{(n-2)(3n-1)^2}{12(n-1)^3} Q_1^4 \Big\},
\end{aligned}$$

in which we may assign to the multipliers λ_1 , λ_2 , λ_3 any values as constants, or as functions of n only, that we please. I will only add here that the expression into which λ_3 is multiplied is to a factor the square of a known form of the quadrine; it is equal in fact to

$$- \frac{n-2}{3(n-1)} \left\{ Q_1' + \frac{1}{2} \cdot \frac{3n-1}{n-1} Q_1^2 - \frac{3}{2} \cdot \frac{n-1}{n-2} Q_2 \right\}^2.$$

4 Wellington Square, Oxford,
October 10, 1888.

LI. *On a New Barometer, called "the Amphiscæna."*
By T. H. BLAKESLEY.*

THIS instrument consists of a straight glass tube of uniform internal cross section, closed at one end and open to

* Communicated by the Physical Society: read June 23, 1888.