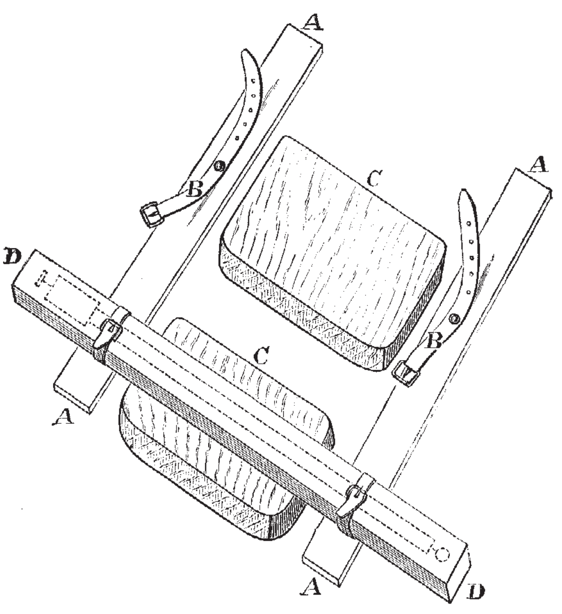
meter directed towards the stern. Barometers can be placed in this manner on ship board by the maker, and can be left to themselves for any length of time. If the person to whom they are consigned is notified of their subsequent arrival at port, he can take them from their hangings on the ship in the best possible condition. Of course this presupposes an arrangement with the officers of the vessel, such that the instruments shall be let entirely alone from the time they are mounted by the consignor until they are received by the consignee.

I think this method of carrying instruments can be very usefully applied in improving our knowledge of the relation of international barometric standards, and at a minimum expense; and I will give a brief outline of a convenient way for accomplishing it. The Deutsche Seewarte at Hamburg, and the Kew Observatory at Richmond (through the London Meteorological Office), are in the best positions for supervising this work, and I venture to express the hope that the matter will be seriously considered.

I will outline the work when carried on from London.

Let two barometers of the best construction, say an Adie Fortin and a Wild-Fuess control barometer, be compared with the Kew normal during a period of a week or more, or long enough to experience considerable variation in the barometer



height. Then let the two barometers be mounted on one of the London Hamburg steamships, in the manner which I have described, and sent to Hamburg, where an employee of the Deutsche Seewarte could be despatched to take down the instruments and carry them to the Seewarte for comparison with the normal barometer. Then the barometers could be taken by a messenger to Lübeck, at an expense of a few shillings, and mounted on a St. Petersburg steamer, which would carry them almost to the door of the Central Physical Observatory, where they could be again taken in charge by a meteorologist, compared for a few days, and then again be mounted on another steamer bound for one of the Scandinavian ports where there is a standard barometer, and finally returned to London by one of the numerous regular steamships. At an expense of a couple of pounds the barometer could be sent from St. Petersburg (or Scandinavia) back to Hamburg *viâ* Stettin and Berlin; thus allowing Berlin to enter into the series. The barometers would probably have to be sent by a messenger from Berlin to Hamburg, thus entailing the just mentioned expense. A second comparison at Hamburg would be desirable, and then the barometers could be returned to London by sea, and again compared at Kew.

Similarly, barometers could be sent to New York for comparison with the sub-standard, by Adie, at the Maritime Exchange; although probably the United States Weather Bureau would assume the expense of the three pounds necessary to carry the instruments to Washington for comparison with the normal there, and then return them to New York and put them on shipboard to be returned to London.

The standard barometers of Australia, India, Brazil, and other countries accessible by sea can be reached from London (or Hamburg) in the same way, and the comparison instruments can be returned to their starting point for additional verification.

My own experience in the transportation of barometers assures me that ship captains would gladly give their hearty co operation to a work of this kind, and there would be no charges for carrying the instruments even half round the world and back again.

In offering this suggestion it is not necessary for me to give the details for the complete organisation of such a scheme; but it may be remarked that if it should be undertaken, the personal experience of those who have been over the ground should be utilised in making plans. A single instance will serve to show why this is advisable. Some years ago I carried two barometers from Hamburg to London by sea. I took the German line of steamers and found myself anchored in the middle of the Thames, and had to get ashore as best I could. I greatly feared that I should never get the barometers ashore in a whole condition, as there was necessitated a great deal of scrambling over lighters, &c., and embarkation in an unsteady row boat in order to make a landing. Had I taken the English steamer, all this worry would have been saved. Other similar instances occurred which could have been avoided by one personally familiar with the routes to be travelled. FRANK WALDO.

Princeton, New Jersey, February 20.

Motion of a Solid Body in a Viscous Liquid.

THERE is perhaps no branch of mathematical physics which has made greater progress during the last thirty-five years than hydrodynamics. During this period numerous important investigations have been published upon the motion of solid bodies in a *frictionless* liquid, upon the theory of discontinuous motion, upon the theory of vortex motion and vortex rings, upon the motion of a liquid ellipsoid under the influence of its own attraction, and upon waves and tides. These investigations constitute an enormous increase in the knowledge possessed by the present generation compared with that of its predecessors; they have to a considerable extent exhausted the field of research in the theory of the motion of *frictionless* liquids; but notwithstanding the importance of the results, the elegance of the methods by which many of them have been obtained, and the skill by which the mathematical difficulties have been surmounted, all the investigations referred to possess the defect of not accurately representing the motion of liquids as they occur in nature.

The reason of this discrepancy between theory and observation is that the ideal substance, which is called a *frictionless* liquid, has no actual existence, for all liquids which occur in nature are *viscous*. The viscosity of the mobile liquids, such as water, alcohol, &c., is a small quantity, being in the case of water equal to a tangential stress of about ·014 dynes per square centimetre; whilst in the case of the sticky and greasy liquids, such as treacle and oil, it is much greater. The viscosity of olive oil is about 3·25 dynes per square centimetre, and is therefore about 232 times as great as that of water.

The mathematical theory of the motion of viscous liquids was elaborated as long ago as 1845 by Sir G. Stokes, in a paper in which he showed that the effect of viscosity might be represented by certain additional terms in the equations of motion of a frictionless liquid, which contain as a factor a new physical quantity called the viscosity. In a subsequent paper, published in 1850, he applied the above theory to calculate the diminution of the amplitude of the small oscillations of a sphere surrounded by water; and by means of experiments in which this quantity was observed, he calculated the numerical value of the viscosity of water, and found that it was in close agreement with the value found by Poiseulle from experiments on the flow of liquids through capillary tubes. An investigation of a similar character was undertaken by von Helmholtz and Piotrowski about 1863, in which the sphere was suspended by a torsion fibre, and made to perform small torsional oscillations about a diameter.

Almost all calculations relating to small oscillations proceed upon the basis that the squares and products of quantities, upon which the disturbed motion depends, may be neglected. This introduces a great simplification into the work, and enables a variety of problems, which would otherwise be exceedingly intractable, to be solved by fairly simple methods. There is, however, another class of problems of great practical importance, in which it is not allowable to neglect these quadratic terms, and towards the solution of such problems theory has as yet made little progress.

When a sphere is constrained to move along a horizontal straight line, but is otherwise free, it is well known that if the surrounding liquid is supposed to be frictionless, its only effect is to increase the inertia of the sphere by half the mass of the liquid displaced. The sphere accordingly requires a larger impulsive force to start it than if the liquid were absent, but when once started it continues to move with its velocity of projection. But when the sphere is surrounded by an *actual* liquid, its velocity gradually diminishes until it ultimately comes to rest; and this fact shows very forcibly the necessity of taking the viscosity of the liquid into account in problems of this character. I obtained a few years ago a mathematical solution, which shows that this effect must necessarily be produced by a *viscous* liquid, but the solution is an imperfect one, as mathematical difficulties compelled me to disregard the quadratic terms.

It is always a great advantage when the solution of a mathematical problem can be made to depend upon a *single* function which satisfies a partial differential equation and certain boundary conditions. This is always the case when a solid of revolution moves along its axis in a viscous liquid which is initially at rest, or has an independent motion which is symmetrical with respect to the axis. In this particular class of problems, the motion can be expressed by means of Stokes's current function in the following manner:—Let z be measured along, and r perpendicularly to the fixed straight line with which the axis coincides during the motion; let w and u be the velocities of the liquid in these directions; then:—

$$u = -\frac{1}{r}\frac{d\psi}{dz},\ w = \frac{1}{r}\frac{d\psi}{dr},$$

$$\left(\nu D - \frac{d}{dt} - u\frac{d}{dr} - w\frac{d}{dz} + \frac{2u}{r}\right)D\psi = 0,$$

where

$$D = \frac{d^2}{dz^2} + \frac{d^2}{dr^2} - \frac{1}{r}\frac{d}{dr},$$

and ν is the kinematic coefficient of viscosity.

So far as I am aware, no serious attempt has been made to obtain a solution of this equation in a suitable form, even when the solid is a sphere. The equation is well worthy of the attentive consideration of mathematicians; and although it is an intractable one, it must be recollected that a general solution is not required, but only a particular one which is suitable in the case of a sphere. It will be quite time enough to consider the possibility of obtaining solutions of a more general character, when the appropriate one in the case of a sphere has been discovered. It is also important to recollect that in most problems which are of practical interest, ν is a small quantity (about ·014 in C.G.S. units for water), and consequently an approximate solution in which ν is supposed to be small would meet the exigences of the case.

When a solid body is moving through a liquid, one of the boundary conditions is that the normal velocity of the solid must be equal to the component along the normal of the velocity of the liquid in contact with it. If the liquid is frictionless, this condition is the only one which has to be satisfied; but when the liquid is viscous, a further question arises as to the law which expresses the effect of the tangential stress exerted by the liquid upon the solid. When the motion is very slow (as in the case of problems relating to small oscillations) the experimental evidence is in favour of the hypothesis of *no slipping;* but when the velocity is considerable, the experimental evidence is not so satisfactory. The partial slipping which takes place under these circumstances must depend partly upon the nature of the liquid, and partly upon that of the surface in contact with it; and the tangential stress to which it gives rise is probably approximately proportional to the square of the relative velocity.

When the motion is symmetrical with respect to an axis, the stresses due to viscosity can be calculated as soon as the value of ψ is known, the resistance which the liquid exerts on the solid can be found, and the equation of motion written down and integrated. This process is, however, an exceedingly tedious one; but it can always be dispensed with in the case of a single solid by employing the principle of momentum. When the motion is not symmetrical with respect to an axis, it cannot be expressed in terms of ψ; but if the velocities of the liquid can be found from the hydrodynamical equations, the components of the linear and angular momenta of the liquid can be calculated, and by applying the principle of momentum to the compound system composed of the solid and the surrounding liquid, the equations of motion of the former can be obtained. Since the momentum of the system is obviously a function of the six coordinates of the solid, this principle furnishes a sufficient number of equations for the determination of the motion.

When there is more than one solid, the principle of momentum is insufficient to determine the motion; but if the velocities of the liquid in the neighbourhood of each solid could be found, the force and couple constituents of the resistance could be calculated, and the equations of motion of each solid written down. Lagrange's equations in their ordinary form cannot be employed, as viscous motion involves a conversion of energy into heat; but problems which can be solved by an indirect method can usually be solved by a direct one, and I feel confident that equations analogous to Lagrange's equations exist, by means of which the motion of a number of solids in a viscous liquid can be found without going through the above-mentioned process. A form of Lagrange's equations has already been discovered, which is applicable when the viscous forces depend upon a dissipation function which is expressible as a homogeneous quadratic function of the velocities; and the circumstance that a dissipation function also exists in the hydrodynamical theory, although it is expressed in a different form, furnishes additional grounds for believing in the existence of equations of this character. The discovery of such equations would constitute an important advance in the theory of viscous liquids.

A. B. BASSET.

SCIENCE IN THE PUBLIC SCHOOLS AND IN THE SCIENTIFIC BRANCHES OF THE ARMY.

ON Friday last Mr. Campbell Bannerman received a deputation on this subject in his room in the House of Commons. There were present Sir Henry Roscoe, the Head Master of Rugby School, the Principal of Cheltenham College, the Head Master of Clifton College, Sir B. Samuelson, Prof. Jelf, and Mr. Shenstone. Lord Playfair, Sir John Lubbock, and Sir Henry Howorth would also have been present, but they were prevented by other engagements. The following is a brief account of the proceedings:—

Sir Henry Roscoe, in introducing the deputation, said that he had introduced a deputation on this subject to Mr. Stanhope about five years ago, and that if the suggestions then made had been adopted the present deputation would not have been necessary. After some remarks which showed the injustice of the present system to the more scientific lads, he pointed out several methods by which this injustice might be removed.

The Head Master of Rugby, Dr. Percival, expressed his strong feeling of the importance of the subject alike to the service, the cadets, and the schools, and said he wished to see both modern languages and science duly encouraged; he thought they might both be made compulsory, as he believed that early education should rest on a wide basis, and that specialising should only be encouraged later. Alluding to the work in science done at the Royal Military Academy, Dr. Percival mentioned that he knew of one cadet who, owing to the absence of any higher teaching there at the earlier stages, was lately learning science which he, the cadet, was well fitted to teach.

The Principal of Cheltenham College, Mr. James, confessed that his own interests and convictions on educational matters were those of a linguist rather than those of a man of science; but practical experience showed him that the present system told most unfairly against scientific boys who entered Woolwich; science was being gradually edged out. Many other head masters of public schools felt with the deputation. He thought also that the present system tended to the disadvantage of the smaller schools, where science was often exceedingly well taught. He hoped that in making any changes the authorities would be careful to consider the interests of linguistic boys, and