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THREE CAMBRIDGE MATHEMATICAL WORKS.

The Algebra of Invariants. By J. H. Grace, M.A., and A. Young, M.A. Pp. vii+384. (Cambridge: The University Press, 1903.) Price 10s. net.

The Dynamical Theory of Gases. By J. H. Jeans, M.A. Pp. xvi+352. (Cambridge: The University Press.) Price 15s. net.

A Treatise on the Analytical Dynamics of Particles and Rigid Bodies. By E. T. Whittaker, M.A. Pp. xiii+414. (Cambridge: The University Press, 1904.) Price 12s. 6d. net.

WHATEVER opinions may be felt as to the desirability of University Presses competing with private firms in swelling the already too large flood of school geometries or issuing cram books for compulsory Greek examinations, there can only be one opinion as to the series of standard treatises on higher mathematics emanating at the present time from Cambridge. In a country which, in its lack of national interest in higher scientific research, particularly mathematical research, stands far behind most other important civilised countries, it necessarily devolves on a University Press to publish advanced mathematical works. We may take it as certain that the present volumes will be keenly read in Germany and America, and will be taken as proofs that England contains good mathematicians, though Englishmen as a nation may be unaware of their existence, with the exception of the senior wrangler of one year, who is forgotten the next.

For years Salmon's "Higher Algebra" has been the treatise which has done most to interest English students in invariants. At the present time a good deal more is wanted in order to bring our knowledge up to date. Messrs. Grace and Young have endeavoured to meet present requirements in a well defined direction. As they state in their preface, the book

"was started as an attempt to meet the need expressed by Elliott in the preface to 'The Algebra of Quantics'—'a whole book which shall present to the English reader in his own language a worthy exposition of the method of the great German masters remains a desideratum.'"

While no book, unless it were written in four languages, could satisfy the patriotic aspirations of every native of our country by appealing to him "in his own language," the production of an English book on a subject largely developed in Germany meets a distinct want.

The subject is practically started *ab initio*. The treatment does not strike us as very hard to follow, although it is difficult for a beginner at first to master the symbolical notation, especially in the definition of transvectants (chapter iii.). In chapter vi. the authors introduce Gordan's theorem, according to which the number of covariants of a binary form is always finite, and in the next chapter they employ his method of proof to obtain the complete irreducible

set of covariants of the quintic. A short chapter on simultaneous systems brings us to Hilbert's theorem, with which the algebra of binary forms may be said to end. Chapters x. and xi. deal with geometrical interpretations, and in particular with apolarity. The sections dealing with ternary forms are less complete, as the authors have considered that "with the methods known up to the present the treatment of ternary forms is too tedious for a text-book."

Mr. Grace has previously been associated with the production of several mathematical text-books of a quite elementary character, and the present book bears many unmistakable traces of his experience as a writer in making a somewhat difficult subject appear relatively easy.

We say "somewhat" difficult, because the subject of Mr. Jeans's new book is incomparably harder than the "Algebra of Invariants." This difficulty arises largely from the fact that the kinetic theory of gases is closely associated with the representation of physical phenomena as they actually exist, and with all the difficulties connected with irreversibility and the existence of temperature. It is only by statistical methods that these phenomena are amenable to the equations of reversible dynamics, and with every method of attack some assumption must be made, since if any motion of a molecular system exists it is equally conceivable that the opposite motion should exist.

Even Willard Gibbs's appeal to experience quoted on p. 167 does not get over the difficulty. If we put red and blue ink together into a vessel and stir them up, it is true that if the inks differ in nothing more than colour the result is a uniform violet ink. But this is because the inks are viscous liquids the motions of which are irreversible. If they were perfect liquids perfect mixing would not take place, and the effect of stirring would merely be to produce vortex motions in which the vortex lines always contained the same particles and remained constant in strength. If we mix counters in a bag, the motions of the counters are retarded by friction; if the counters correctly represented perfectly reversible systems they would never come to rest.

Mr. Jeans in his preface considers that the discrepancy between theory and experiment in connection with the ratio of the specific heats of a gas "is of greater importance than all the others together," and he has endeavoured to emphasise the fact that when account is taken of the interaction between matter and the ether, theory and experiment harmonise well as could be desired. But as soon as this ether is taken into account we have a simple means of obviating the irreversibility difficulty by saddling the ether with the whole responsibility. So long as physicists are contented, in solving the differential equations of wave motion in a medium, to omit the terms which represent waves converging from an infinite distance towards a centre of disturbance, so long will there be an easy way out of the puzzling contradictions arising out of Boltzmann's H-theorem.

But there is really no reason why the presence of a molecule in an indefinitely extended ether which undoubtedly possesses some energy should not bring about the convergence of waves coming in from an

infinite distance in all directions, and gradually increasing in intensity as they approach the molecule. We do not think such cases exist, but we did not expect to discover radium a few years ago.

Let us now see how Mr. Jeans attempts to deal with the difficulties here suggested. In the first seven chapters he follows fairly closely on conventional lines, and deduces the Boltzmann-Maxwell law of distribution, the minimum theorem, the law of partition of energy, and the isothermal equations according to the Boyle-Mariotte and van der Waals's laws. In chapter viii. the author throws over the principle of conservation of energy and assumes that his gas is a dissipative system in which loss of energy occurs by radiation. On this hypothesis he finds that when the rate of dissipation has become very slow probability considerations indicate a tendency to assume a definite statistical specification different from that given by the ordinary theory. It further appears that such a gas has one principal and a number of subsidiary temperatures, a notion which we believe has been previously advanced. In chapters ix. and x. Mr. Jeans considers applications of the theory of a non-conservative gas, particularly in connection with rates of dissipation of energy, and ratios of specific heats.

We thus have a definite attempt to break away from traditional methods and boldly introduce the notion of dissipation into the kinetic theory. The idea is certainly an excellent one. Whether it is free from objection is a matter which cannot be answered as the mere result of a critical examination. Often objections to theories strike the mind of a reader quite unexpectedly.

In the remaining chapters Mr. Jeans deals with "free path phenomena" such as diffusion, conduction of heat, viscosity, and the escape of gases from planetary atmospheres. In this work he is more on the ordinary lines. We notice as an important feature the sections dealing with encounters according to the law of the inverse fifth power. This series of chapters is of considerable use in affording easy access to investigations contained in a much longer form in the original papers of Boltzmann and other writers.

Turning back to the chapter on equipartition of energy, we are led to the following inference:—Mr. Jeans leaves it an open question whether the conventional law of distribution with its attendant consequences of equipartition may represent the ultimate state of a gas, but concludes that in actual gases such as we see around us where dissipation of energy occurs a different distribution holds good.

The second conclusion seems plausible. But the assumption that equipartition of energy holds *even in a conservative system* presents difficulties in connection with Stefan's law of radiation in a black cavity. According to that law the energy of the ether should vary as the fourth power of that of the molecules. It might be said that in the "conservative system" Stefan's law would not necessarily hold good, and that there would be no objection to assuming the energy of the ether to be then directly proportional to that of the molecules, or to the temperature. But the usual thermodynamic investigation—which is more

certain to be valid in the case of the conservative than in that of the dissipative system—would then give a different form for the radiation pressure—apparently $f = \psi (\log \psi + \text{constant})$ —and this result would have to be admitted. On the whole it appears more likely that while distributions satisfying Maxwell's law of equipartition are always theoretically possible, other distributions may exist, and may, indeed, represent a normal and *persistent* state of affairs even in *conservative* systems.

It is remarkable that physicists strain at gnats when put down to study kinetic theory or thermodynamics, and yet they swallow camels with complacency when they read the subject of Mr. Whittaker's book, "Analytical Dynamics." Some writers even go so far as to introduce pages and pages of the most unreal dynamical problems into what they call treatises on physics.

"The soluble problems of particle dynamics" mostly represent things which have no existence. It is impossible for a particle to move on a smooth curve or surface because, in the first place, there is no such thing as a particle, and in the second place there is no such thing as a smooth curve or surface. What constitutes the chief interest of "Analytical Dynamics" is the possibility of forming clear mental pictures of its results by *imagining* bodies capable of performing the motions discussed.

Mr. Whittaker's treatment is essentially mathematical and advanced in character. He opens with sections on the displacements of rigid bodies in which Klein's parameters and Halphen's theorems on composition of screws figure near the commencement. In his chapter on equations of motion physico-philosophical discursions on force and mass are reduced to a minimum. This is as it should be, for there are plenty of people who can write about such matters, but few whose knowledge extends to the more important theorems which follow later. The Lagrangian equations are reached by § 26, which is preceded by a definition of holonomic systems. This distinction might with advantage be put into treatises in physics, for at present students of that subject are apt to assume that Lagrange's equations in their ordinary form are universally applicable, which is far from true. Passing on to chapter v., which deals, *inter alia*, with moments of inertia, our old friend the "principle of parallel axes" is treated generally for a quadratic function of coordinates, velocities and accelerations, readers being doubtless assumed to know the proof for simple cases. Chapter vii. deals with the general theory of vibrations, and the next chapter with non-holonomic and dissipative systems, the first of these two chapters consisting mainly of theory, and the second mainly of examples. The most important chapters are those which follow, dealing with the principles of Hamilton and Gauss, the integral invariants of the Hamiltonian system, and the representation of a dynamical system of equations by means of contact transformations.

Mr. Whittaker some time ago presented a valuable report to the British Association on the problem of three bodies, and he tells us that between 1750 and 1904 more than eight hundred memoirs were

published on this problem. Even at the Heidelberg congress last August further additions were made to this literature. In his chapter on the subject, which is very brief, he discusses the reduction of the equations to a system of the sixth order, thus affording a useful insight into the main features of this difficult investigation. Several other interesting chapters follow.

It will thus be seen that Mr. Whittaker's treatise collects into book form the outlines of a long series of researches for which hitherto it has been necessary to consult English, French, German, and Italian transactions. In recent years Italy has played no small part in the development of dynamics, as may be seen by the number of papers by Levi Civita and other writers which have from time to time appeared in the *Atti dei Lincei*, dealing with integrals of the equations of motion of holonomic systems, particular cases of the problem of three bodies, and allied questions.

The book is thus written mainly for the advanced mathematician. But an interesting feature is the large number of examples both in the text and at the end of the chapters. Of these a good many really contain the substance of minor papers that have been published abroad. Others are followed by the reference "Coll. Exam.," and while it may be taken for granted that Mr. Whittaker has made a judicious selection, some of the questions bearing these references may give foreign mathematicians a little insight into the unpalatable nuts which Cambridge students are expected to waste time in trying to crack for examination purposes. The antics of insects crawling on epicycloids, or the vagaries of particles moving along the intersections of ellipsoids with hyperboloids of one sheet, are of no scientific interest, and the time spent in "getting out" problems of this character might better be employed in learning something useful. Moreover, Cambridge college examiners have a habit of endowing bodies with the most inconsistent properties in the matter of perfect roughness and perfect smoothness. A perfectly rough body placed on a perfectly smooth surface forms as interesting a subject for speculation as the well-known irresistible body meeting the impenetrable obstacle. What the average college don forgets is that roughness or smoothness are matters which concern *two surfaces*, not *one body*.

In our opinion a great deal of the artificiality of the more elementary parts of dynamics might be removed by the more frequent introduction of simple problems in resisted motion. There are plenty of easy ones to be found which would be more helpful to the beginner than problems about ellipsoids rolling on perfectly smooth surfaces formed by the revolution of cissoids or witches about their axes. Those who have the ability to do more difficult work should pass on to the advanced parts of a book like Mr. Whittaker's and learn what foreign mathematicians have been doing; this is much more useful.

It remains to add that the books are neatly bound; the printing and paper are somewhat unnecessarily luxurious in quality, and—most important of all—the Cambridge printers have *not* forgotten to cut the pages with their guillotine.

G. H. BRYAN.

REIN'S "JAPAN."

Japan nach Reisen und Studien. By J. J. Rein. Vol. i. Natur und Volk des Mikadoreiches. Second edition. Pp. xv+749. (Leipzig: Wilhelm Engelmann, 1905.) Price 24s. net, paper; 26s. net, cloth.

THIS is the second edition of a book first published in 1880. The author, now professor of geography in the University of Bonn, was, in 1874, commissioned by the Prussian Ministry of Commerce to go to Japan for the purpose of studying and giving an account both of the trade of Japan and the special branches of industry there carried on to so high a degree of perfection. The writer of this notice had the pleasure of making the acquaintance of Dr. Rein while in Japan, and can testify to the German thoroughness with which Dr. Rein carried out the work for which he was commissioned. The results of that work were two volumes which, from the point of view of the author, have been looked upon as the most scientific and complete of their kind. Some years after their appearance in Germany translations were published in England (Hodder and Stoughton), but both the German and English editions have for some time been out of print, and the author has done well to bring out a new edition, brought up to date in matters both of history and science. For students of Japan it is almost unnecessary to review the work of Dr. Rein, as it has long had an assured position.

The opinion of competent authorities was reflected by Prof. Chamberlain more than fifteen years ago, when, in an edition of his well known book "Things Japanese," he said:—

"At the risk of offending innumerable authors, we now venture to pick out the following works as probably the best in a general way that are accessible to English readers: (1) Dr. Rein's 'Japan,' with its sequel 'The Industries of Japan.'" No person wishing to study Japan seriously can dispense with these admirable volumes. Of the two, that on the "Industries" is the better; agriculture, cattle-raising, forestry, mines, lacquer-work, metal-work, commerce, &c., everything, in fact, has been studied with a truly German patience, and is set forth with a truly German thoroughness. The other volume is occupied with the physiography of the country, that is, its geography, fauna, flora, &c., with an account of the people, both historical and ethnographical, and with the topography of the various provinces.

It is this latter volume which is at present before us, and although it may not be so interesting, from the practical point of view, as its sequel, it is more valuable from a scientific and historical point of view. The book is essentially the same as the first edition, but the author has had the assistance of many friends in Japan in bringing it up to date, both from a scientific and a historical point of view. It is, however, unnecessary to enter into a detailed account or criticism of its contents.

The first part of the book is a very complete and interesting account of the physical geography of Japan; in fact, it is the only systematic account which has been published in a European language. When