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To cite this article: Rev. Baden Powell M.A. F.R.S. (1835) XVII. An abstract of the essential principles of M. Cauchy's view of the undulatory theory, leading to an explanation of the dispersion of light; with remarks , Philosophical Magazine Series 3, 6:32, 107-113, DOI: [10.1080/14786443508648544](https://doi.org/10.1080/14786443508648544)

To link to this article: <http://dx.doi.org/10.1080/14786443508648544>



Published online: 01 Jun 2009.



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The cabinet of Natural History under the direction of M. Voltz, at Strassburg, contains an alloy of copper and tin in acicular crystals, the composition of which, according to the analysis of M. Roth, is expressed by the formula 2 Sn Cu . Some of these crystals, which were given me for the purpose of having their form determined, are regular six-sided prisms, cleavable with some difficulty in a direction perpendicular to the axis of the prism. No rhombohedral faces or cleavage could be detected.

XVII. *An Abstract of the essential Principles of M. Cauchy's View of the Undulatory Theory, leading to an Explanation of the Dispersion of Light; with Remarks. By the Rev. BADEN POWELL, M.A., F.R.S., Savilian Professor of Geometry, Oxford.*

[Continued from p. 25.]

Integration of the Equations of Motion.

IN order to proceed to the integration of the equations (12.), M. Cauchy adopts the principle, (or at least one which comes to the same thing,) that whatever may be the values of the functions $\xi \eta \zeta$ which will verify those equations, they may always be developed in some serieses of algebraic functions of xyz , which may be considered as formed by adding together the serieses for the sine and cosine of quantities involving those functions of xyz , with certain arbitrary quantities uvw , and certain coefficients $d e f g h i$, functions of t . Using the symbol Σ to signify the sum of a number of such terms, we shall thus have

$$\begin{aligned}\xi &= \Sigma \{d \cos (u x + v y + w z) + g \sin (u x + v y + w z)\} \\ \eta &= \Sigma \{e \cos (u x + v y + w z) + h \sin (u x + v y + w z)\} \\ \zeta &= \Sigma \{f \cos (u x + v y + w z) + i \sin (u x + v y + w z)\}\end{aligned} \quad (13.)$$

Now the arbitrary quantities $u v w$ may be assumed, so that we have

$$\left. \begin{aligned}\frac{u}{k} &= a, \quad \frac{v}{k} = b, \quad \frac{w}{k} = c, \\ \text{and} \quad a^2 + b^2 + c^2 &= 1;\end{aligned} \right\} \quad (14.)$$

that is, these quantities may represent the cosines of the inclinations to the three axes, of a line passing through the origin, which we will call O P.

And if we write, for abbreviation,

$$a x + b y + c z = g, \quad (15.)$$

then, ϱ will be the perpendicular distance of the point $x y z$, from a plane passing through the origin whose equation is

$$a x + b y + c z = 0. \quad (16.)$$

The equations (13.) will by this notation become

$$\begin{aligned} \xi &= \Sigma [d \cos k \varrho + g \sin k \varrho] \\ \eta &= \Sigma [e \cos k \varrho + h \sin k \varrho] \\ \zeta &= \Sigma [f \cos k \varrho + i \sin k \varrho] \end{aligned} \quad (17.)$$

Now to determine the coefficients $d, e, \&c.$, in functions of the variable t and the arbitrary constants $k a b c$, we may proceed as follows:

Let δ be the angle formed by r with $O P$, then $\cos \delta = a \cos \alpha + b \cos \beta + c \cos \gamma$. (18.)

Also from the values of Δx , &c. (2.) joined with that of ϱ (15.) we have, taking the corresponding small increments,

$$\Delta \varrho = a \Delta x + b \Delta y + c \Delta z = r \cos \delta. \quad (19.)$$

Also we shall find by a simple trigonometrical process

$$\left. \begin{aligned} \Delta \cos k \varrho &= -2 \left(\sin^2 \frac{k r \cos \delta}{2} \right) \cos k \varrho \\ &\quad - \sin (k r \cos \delta) \sin k \varrho \\ \Delta \sin k \varrho &= -2 \left(\sin^2 \frac{k r \cos \delta}{2} \right) \sin k \varrho \\ &\quad + \sin (k r \cos \delta) \cos k \varrho. \end{aligned} \right\} \quad (20.)$$

And in exactly the same manner

In order to simplify the subsequent investigation, we will in the first instance consider the sums of terms (17.) as each reduced to a single term, or take

$$\xi = d \cos k \varrho + g \sin k \varrho,$$

which on differentiating with respect to ϱ gives

$$\frac{d \xi}{d \varrho} = k [-d \sin k \varrho + g \cos k \varrho];$$

and substituting these values in the corresponding formula

$$\Delta \xi = d \Delta \cos k \varrho + g \Delta \sin k \varrho,$$

we shall find the value of that quantity, and by similar means those of the others analogous to it,

$$\left. \begin{aligned} \Delta \xi &= -2 \xi \sin^2 \left(\frac{k r \cos \delta}{2} \right) + \frac{\sin (k r \cos \delta)}{k} \frac{d \xi}{d \varrho} \\ \Delta \eta &= -2 \eta \sin^2 \left(\frac{k r \cos \delta}{2} \right) + \frac{\sin (k r \cos \delta)}{k} \frac{d \eta}{d \varrho} \\ \Delta \zeta &= -2 \zeta \sin^2 \left(\frac{k r \cos \delta}{2} \right) + \frac{\sin (k r \cos \delta)}{k} \frac{d \zeta}{d \varrho} \end{aligned} \right\} \quad (21.)$$

We should now have to substitute these values in the fundamental equations (12.), and thus obtain expressions involving $\xi \frac{d\xi}{d\eta}$, &c., which would obviously extend to some length.

But even without actually going through this process at length, we shall easily perceive a principle of simplification arising out of the form which we shall at once see certain parts of the expressions must take, as follows:

1st. In the forms (21.), all the terms involving $\xi \eta \zeta$ have in their coefficients the *square of the sine* of a function of δ , and these terms, when introduced as multipliers in (12.), are in the first member uncombined with any other functions of the angles $\alpha \beta \gamma \delta$; and in the second members are combined with the *squares of the cosines* of the angles; that is, in every case these terms are of *even dimensions*.

2ndly. All the terms involving the differential coefficients of $\xi \eta \zeta$ have, in (21.), for coefficients the *sine* of a function of δ ; and these in the multiplication also, in the first member, stand uncombined with any other such function; and in the second, combined with the *squares of the cosines*; that is, in every case they are of *odd dimensions*.

Also, it appears from the original construction and from (18.) that the cosines of $\alpha \beta \gamma \delta$ are all positive or all negative together.

Now, in taking the sum of a number of terms (indicated by the symbol S), it is evident in the former of the above two cases that all those terms will be positive whatever be the signs of the cosines. In the second case, for the same reason, the terms will be positive or negative according to the change of sign in the cosines.

If, then, we suppose in such a sum, half the terms corresponding to positive, and half to negative values of the cosines, we shall find that *the coefficients of all the terms in the second case will disappear*, whilst in the first case they will remain. The whole expression will thus be reduced to the terms involving $\xi \eta \zeta$ only.

This last supposition is precisely that of a physical condition which we shall have no difficulty in allowing, viz. that in the state of equilibrium the masses of the molecules $m m' m''$, &c., are two and two equal, and distributed symmetrically on each side of the molecule m on straight lines passing through m . This obviously gives the cosines for half the molecules positive, and for half negative.

In such a case then we shall have the general equations of motion reduced to a considerably simplified form; or, for ab-

breviation, writing the sums of the coefficients derived from those terms of (12.) which involve $\cos^2 \alpha$, $\cos^2 \beta$, $\cos^2 \gamma$, respectively L, M, N; and those which involve $\cos \beta \cos \gamma$, $\cos \gamma \cos \alpha$, $\cos \alpha \cos \beta$, respectively P, Q, R, those equations are reduced to the following:

$$\left. \begin{aligned} \frac{d^2 \xi}{dt^2} &= - [L \xi + R \eta + Q \zeta] \\ \frac{d^2 \eta}{dt^2} &= - [R \xi + M \eta + P \zeta] \\ \frac{d^2 \zeta}{dt^2} &= - [Q \xi + P \eta + N \zeta] \end{aligned} \right\} \quad (22.)$$

These equations enable us at the end of the time t , to determine the three functions $\xi \eta \zeta$; which is, in fact, done, if we have determined the six functions $d e f g h i$; and this we can effect by means of the *initial values* of these functions, and their differential coefficients with respect to t . Writing these initial values of the functions by subjoining (o), and those of their differential coefficients by subjoining (i), we shall have by the formula (17.), supposed reduced to a single term,

$$\left. \begin{aligned} \xi_0 &= d_0 \cos k g + g_0 \sin k g \\ \eta_0 &= e_0 \cos k g + k_0 \sin k g \\ \zeta_0 &= f_0 \cos k g + i_0 \sin k g \\ \xi_1 &= d_1 \cos k g + g_1 \sin k g \\ \eta_1 &= e_1 \cos k g + k_1 \sin k g \\ \zeta_1 &= f_1 \cos k g + i_1 \sin k g \end{aligned} \right\} \quad (23.)$$

In order, by means of these values corresponding to $t = 0$, to deduce those corresponding to any value of t , we must proceed to the following considerations relative to the coefficients.

Let the arbitrary quantities A B C be assumed as the cosines of the angles which a certain line O A through the origin forms with the positive semiaxes; or in other words, so that we have

$$A^2 + B^2 + C^2 = 1 \quad (24.)$$

and the line O A is represented by the equations

$$\frac{x}{A} = \frac{y}{B} = \frac{z}{C}, \quad (25.)$$

Also, if we suppose

$$s = A \xi + B \eta + C \zeta, \quad (26.)$$

the value of s will give the displacement of the molecule m in a direction parallel to the line O A, and positive or negative according to the direction of that line.

Again, if the quantities $A B C$ be so chosen that on multiplying the first term of each member of the equations (22.) by A , the second by B , and the third by C , we have

$$\frac{LA + RB + QC}{A} = \frac{RA + MB + PC}{B} = \frac{QA + PB + NC}{C} \quad (27.)$$

and write these equal to s^2 , then, on differentiating the equation (26.), and substituting the values from equation (22.), we shall find the second differential coefficient to be the remarkable form

$$\frac{d^2 s}{dt^2} = -s^2 s. \quad (28.)$$

From (27.) it is evident that we have three values of s^2 corresponding to three systems of values of the ratios $\frac{B}{A}, \frac{C}{A}$; and consequently there are three straight lines with either of which the line OA may coincide; and the same equations enable us to determine these lines, for they evidently coincide in form with those mentioned in the preliminary article. We can deduce the same equation of the third degree, which for reference we will call (29.); and consequently the three lines $OA' OA'' OA'''$ are identified with the three axes of the surface of the second degree represented by the equation there assumed, involving as coefficients the quantities $L M N$, &c., and which we will call (30.). If it be an ellipsoid, the three values of s^2 are equal to the squares of the three semiaxes of the ellipsoid.

These considerations then enable us to assign the displacement of m at the end of the time t , in directions parallel to three determinate lines at right angles. Let these three displacements be expressed by accenting the letters in equation (26.), or let us suppose

$$\left. \begin{aligned} s' &= A'\xi + B'\eta + C'\zeta \\ s'' &= A''\xi + B''\eta + C''\zeta \\ s''' &= A'''\xi + B'''\eta + C'''\zeta \end{aligned} \right\} \quad (31.)$$

In each set of the coefficients the relation (24.) holds good; also, since the lines are at right angles, we have

$$\left. \begin{aligned} A'A'' + B'B'' + C'C'' &= 0 \\ A'A''' + B'B''' + C'C''' &= 0 \\ A''A''' + B''B''' + C''C''' &= 0 \end{aligned} \right\} \quad (32.)$$

Hence we deduce from (31.)

$$\left. \begin{aligned} \xi &= A's' + A''s'' + A'''s''' \\ \eta &= B's' + B''s'' + B'''s''' \\ \zeta &= C's' + C''s'' + C'''s''' \end{aligned} \right\} \quad (33.)$$

Now, writing s_0, s_1 for the initial values of s and $\frac{ds}{dt}$, we have

$$s_0 = A\xi_0 + B\eta_0 + C\zeta_0 \quad (34.)$$

$$s_1 = A\xi_1 + B\eta_1 + C\zeta_1; \quad (35.)$$

or, substituting the values of ξ_0, ξ_1 , &c., from (23.), these forms become

$$s_0 = [d_0A + e_0B + f_0C] \cos k\varrho + [g_0A + h_0B + i_0C] \sin k\varrho \\ = \varpi(\varrho) \quad (36.)$$

$$s_1 = [d_1A + e_1B + f_1C] \cos k\varrho + [g_1A + h_1B + i_1C] \sin k\varrho \\ = \Pi(\varrho), \quad (37.)$$

using the last symbols to designate the forms of these functions of ϱ .

The form of the expression (28.) shows us at once that its integral must be a trigonometrical function, which it will easily be seen must take the following form, involving as coefficients the initial values

$$s = s_0 \cos st + s_1 \frac{\sin st}{s}, \quad (38.)$$

or, what is the same thing,

$$s = s_0 \cos st + s_1 \int_0^t \cos st \, dt. \quad (39.)$$

If we here substitute the values of s_0, s_1 and the trigonometrical values of the resulting products, and also write

$$\frac{s}{k} = \Omega,$$

we shall at length deduce an expression, which, in comparison with equations (36.) (37.), may, by the same notation, be expressed thus, (carefully observing that no greater generality is implied than belongs to (38.) and (39.):)

$$s = \begin{cases} \frac{\varpi(\varrho + \Omega t) + \varpi(\varrho - \Omega t)}{2} \\ + \int_0^t \frac{\Pi(\varrho + \Omega t) + \Pi(\varrho - \Omega t)}{2} \, dt, \end{cases} \quad (40.)$$

the form being the same for each of the values s', s'', s''' corresponding to the three positive values of s^2 , involving respectively $\Omega', \Omega'', \Omega'''$ and A', A'', A''' , &c. If these values of s' , &c., be substituted in equation (33.), we have ξ, η, ζ in functions of ϱ and t , which satisfy the two conditions of the values ξ_0 , &c., when $t = 0$, and of the equations (22.) for any value of t .

Also the *velocity* ω of the molecule at the end of any time t

in the directions of the axes, and of the three lines $o A$ respectively, will be

$$\frac{d\xi}{dt} \frac{d\eta}{dt} \frac{d}{dt} \quad \text{and} \quad \frac{ds''}{dt} \frac{ds''}{dt} \frac{ds''}{dt},$$

and, we have also

$$w^2 = \left(\frac{d\xi}{dt}\right)^2 + \left(\frac{d\eta}{dt}\right)^2 + \left(\frac{d\zeta}{dt}\right)^2 = \left(\frac{ds'}{dt}\right)^2 + \left(\frac{ds''}{dt}\right)^2 + \left(\frac{ds'''}{dt}\right)^2 \quad (41.)$$

[To be continued.]

XVIII. *On the Existence of Titanic Acid in Hessian Crucibles.*

By MR. R. H. BRETT and MR. GOLDING BIRD.

To the Editors of the Philosophical Magazine and Journal.

GENTLEMEN,

WHILE repeating some experiments lately published on the presence of titanium in organic matter, especially in the renal capsules*, we observed that when an alkaline carbonate was exposed to heat in Hessian crucibles, a fused mass was obtained, which was yellow while hot, but white and opaque when cold; on dissolving this fused mass in dilute hydrochloric acid and mixing the solution with hydrosulphuret of ammonia, a deep olive green precipitate was obtained, which, when dried and ignited, yielded a white powder, insoluble in the dilute acids. These reactions so exactly resembling those yielded by titaniferous substances, we were induced to suspect the presence of titanium in the clay of which the crucibles were formed. To determine this with accuracy we undertook an analytical examination of the several varieties of Hessian crucibles usually met with, and we found them all to consist of (in variable proportions) silicic acid, titanac acid, alumina, and peroxide of iron, with traces of magnesia and manganese, and occasionally of lime.

The quantity of titanac acid present differed considerably in different specimens, in some not amounting to more than $3\frac{1}{2}$ or 4 per cent., and in some few even to as much as 25 or 30: it was exceedingly rare to meet with so much as 25 per cent.; those crucibles that contained that quantity were generally small, very thin, brittle, and studded with numerous black semimetallic-looking specks. The quantity of peroxide of iron present was small compared to that of titanac acid, and they were by no means in the proportions in which they exist either in the iserine or in the menachanite, to the presence of which

* See our last volume, p. 398.—EDIT.