

## ANGULAR VELOCITY IN STEAM ENGINES IN RELATION TO PARALLELING OF ALTERNATORS.

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Certain characteristics of the steam engine which are of no importance in most applications of power other than electrical and even in electrical when d. c. generators are used, become of considerable importance when the engine is direct connected to an alternating current generator. These are the peculiar variations in tangential effort on the crank, and the consequent variation in speed of the shaft and fly-wheel.

The angular velocity of the fly-wheel varies in two entirely different ways, due to two different causes. The first and best known is that due to a change of load, then the average velocity changes, and there is a change in the number of revolutions per minute.

The second effect is due to irregularities in the applied force and transmission of that force, and consists in a variation of the instantaneous value of the angular velocity, while the average value and the revolutions per minute remain the same and constant.

The first is altogether a function of the governor of the engine. The latter phenomenon, which is the one to be discussed in this article, is more complicated both in its causes and effects, as it introduces the distribution of the steam pressure in the cylinder, inertia of reciprocating parts, angularity of connecting rod and moment of fly-wheel. It is of importance where two or more alternators are to be run in parallel, or where synchronous motors

are to be run from one or more alternators. Such machines tend to run in exact synchronism, and their inertia gives them a tendency to run at constant angular velocity. If now an irregular pulsation is impressed upon the system by the steam engine, there is a continual effort in each machine to keep in exact step with this pulsation, which causes an exchange of cross-currents and a liability of falling out of step.

When two alternators are running in parallel, if they differ in angular position, even if revolving at the same speed, a cross-current will flow between them, tending to pull them together. This cross-current may be wattless but even then it consumes some energy in the form of  $I^2 r$ .

Assuming that we wish to limit this cross-current to 10% of the full load current of one generator, what angular displacement will be allowable?

Taking several representative alternators having an average regulation of 9%, we find the average short-circuit current is 2.5 times full load current, thus the synchronous impedance is

$$z = e/i = E/2.5.$$

The difference between any two sine waves having an amplitude  $E$  and displaced in phase by an angle  $\beta$  is  $2 E \sin \beta/2$ . This E. M. F. is short-circuited through the impedance of the two alternators in series.

$$\text{Thus cross-current} = e/z = \frac{2 E \sin \beta/2}{2 \times E/2.5} = 2.5 \sin \beta/2$$

placing this equal to .10, we have  $\sin \beta/2 = .04$ ,  $\beta/2 = 2.3^\circ$  displacement between one wave and mean.

If we say  $2.5^\circ$  displacement for round numbers, we have a cross-current of .109.

(A displacement of  $2.5^\circ$  in the electrical circuit is caused by a much less displacement of the fly-wheel. There is one complete cycle of E. M. F. generated per pair of poles. Thus if  $p$  is the number of pairs of poles on the generator, there are  $p \times 360$  degrees of E. M. F. generated in one revolution of the fly-wheel. That is, one degree on the fly-wheel equals  $p$  degrees of E. M. F.)

The causes of the irregular effort are: The pressure of the steam on the piston due to cut-off and expansion is uneven, being great during the first part of stroke, and small and even negative, due to compression, at the end. In the vertical engine the weight of the piston, rod, cross-head and part of connecting rod have to be lifted on the up-stroke, but are additive to the steam pressure on the down stroke. This makes a difference in

the power of each stroke, raises the peak of one and increases unbalanced energy.

This pressure is transferred to the crank-pin through a varying angle so the effort is roughly equal to  $P \sin \phi$ , varying between zero at beginning and end of stroke and  $P$  at middle. Thus there is a combination of maximum values at the point of cut-off, as that comes near the middle of the stroke, and we get quite a pointed peak here.

The angularity of the connecting rod has a different effect in the forward and backward strokes, as in the forward stroke during the early part while the pressure is high anyway, it increases the effect of that pressure on the crank pin, and in the latter part it diminishes that effect. On the return stroke it has the reverse effect, diminishing in the early part and increasing in the latter. So that the final effect is to increase the peak of one impulse and diminish the peak of the other.

The inertia of reciprocating parts absorbs energy at the beginning of the stroke, and gives it up at the end, so it has a beneficial effect on the diagram, smoothing the peak and raising the low values. The combination of two or more cylinders on the shaft naturally tends to even the effort and also to diminish the length of time one impulse lasts, thus diminishing its effect.

Combining the effort of three cylinders would seem to give a more even torque than two cylinders, but the actual diagrammatic combination shows there is very little choice between a two-cylinder or three-cylinder engine.

In a single cylinder, double-acting engine there are two impulses per revolution, and in a cross-compound engine there are four impulses. Each impulse is more or less like any other. By analyzing the variation in speed during one impulse we can derive our conclusions. Though as all the impulses are not equal, it is as well to select that impulse having the greatest amount of unbalanced energy so as to know the worst conditions.

During the first quarter of impulse, the effort is less than average, during the second and third quarter (approximately) the effort is greater, and during the last quarter it is low again. The unbalanced effort represents a force which, acting on the mass of the fly-wheel, produces an acceleration,  $a = f/m$ . Thus by referring the torque diagram (Fig. 1.) to the line of average torque as base, we get a curve of acceleration.

Integrating this acceleration, we get a curve of velocity  $v$ .

This curve lags  $90^\circ$  behind the acceleration curve. Thus starting from a point of minimum torque  $a$ , the acceleration will be negative. From  $a$  to  $b$  the torque is less than average, the acceleration negative, the speed will keep dropping in value until  $b$  is reached, where, since the acceleration changes sign,  $v$  is a minimum, and commences to increase again;  $v$  will keep on increasing as long as the acceleration is positive, or to  $d$ , where  $v$  has its maximum value.

If we assume the variation is sinusoidal, then the average value of  $v$  from  $a$  to  $c$  is  $v_1$ , and of  $s$ , the percentage variation in speed, is  $2s/\pi$ .

The speed is low from  $a$  to  $c$ , or while the crank is passing  $\theta^\circ$  of the crank circle; during this time the crank continually falls behind the position it would maintain at constant angular veloc-

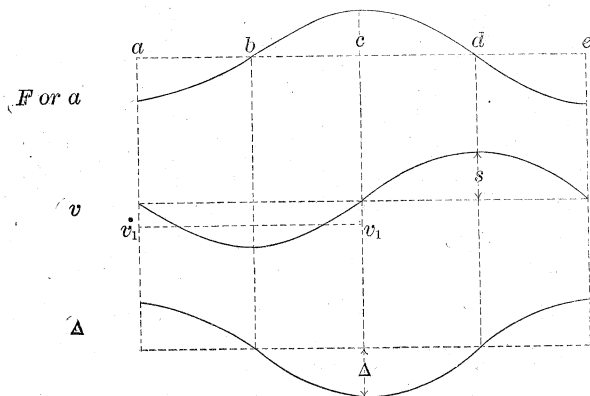


FIG. 1.

ity, and at  $c$  the maximum displacement occurs, for there the speed again becomes greater than average.

If the maximum displacement from mean is  $\Delta$ , then during this time the change is from  $+\Delta$  to  $-\Delta$  or  $2\Delta$  for an average change in speed of  $2s/\pi$ .

That is, if speed is an average of  $2s/\pi$  low during this time, the number of degrees passed will be  $\theta(1 - 2s/\pi)$  and the change will be  $2\Delta = \theta \times 2s/\pi$  or  $\Delta = s\theta/\pi$ , where  $\theta = 360/2n$  approximately.

$a$  the electrical displacement will be:

$$a = \frac{sp\theta}{\pi} \text{ or } \frac{sp360}{2\pi n} \text{ and } s = .0174 \frac{an}{p}$$

where  $p$  = number of pairs of poles

$n$  = number of impulses per revolution.

It is not always safe to assume  $\theta = 360/2n$ , as in cross-compound engines one impulse is frequently very much suppressed (due to reciprocating parts) and another prolonged and enlarged, and the longer an unbalanced torque lasts, the greater the displacement even if the total unbalanced energy is no greater.

It is interesting to note in this formula for  $\alpha$  that the greater  $p$ , the greater the displacement angle and the greater  $n$ , the less the displacement. Hence the advantages of low frequency and multi-crank engines.

Substituting for  $\alpha$  the value  $2.5^\circ$  which we have assumed, we find the allowable speed variation to be  $s = .0435 n/p$  where  $s$  is per cent. variation from mean.

A very good approximation of the fly-wheel necessary may be obtained quickly, thus:

The change of energy stored in fly-wheel for any change in speed is:

$$P_1 = w/2g \times (v_2^2 - v_1^2).$$

If  $v_0$  is the mean linear velocity of  $w$  and  $2s = (v_2 - v_1)/v_0$ , then  $v_2^2 - v_1^2 = v_0^2 \times 4s$ , thus:

$$P_1 = w/2g \times v_0^2 \times 4s.$$

$P_1$  is the unbalanced energy of one impulse in foot pounds given by area  $P_1$  in Fig. 3 and  $k$ , the unbalancing factor, is the ratio of this excess area to the energy of one revolution given by area enclosed by line of average torque.

Hence:

$$P_1 = k \times P = k \times \frac{\text{K.-W.} \times 33,000}{.746 \times \text{eff} \times \text{R. P. M.}}$$

where  $P$  is the energy per revolution of crank.

K.-W. = output of generator.

eff = efficiency of generator.

Equating these two values of  $P_1$

$$\frac{w}{2g} \times v_0^2 \times 4s = k \times \frac{\text{K.-W.} \times 33,000}{.746 \times \text{eff} \times \text{R. P. M.}}$$

and putting for  $s$  its value in terms of displacement  $a$

$$w v_0^2 a n \times \text{eff} \times \text{R. P. M.} = k \times \text{K.-W.} \times p \times 41 \times 10^6$$

$$w v_0^2 a \times \text{eff} \times \text{R. P. M.} = k \times \text{K.-W.} + p \times \theta \times 226 \times 10^3.$$

In this we have three quantities— $w$ , weight of fly-wheel;  $k$ , unbalancing factor, and  $a$ , displacement—two of which may be varied to obtain a satisfactory value of the third.

The exact calculations are much longer and involve integrating first the torque curve (which is also an acceleration curve to another scale since  $a = F/I$  to get a velocity curve and integrating this velocity curve to get the space curve.

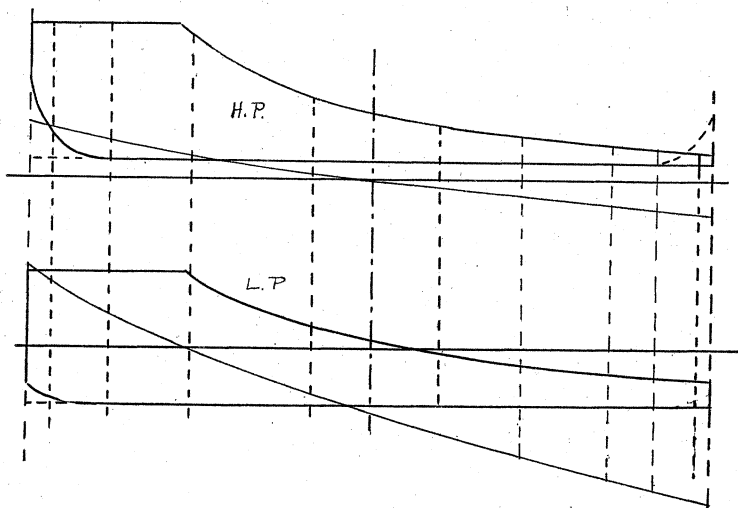


FIG. 2.

Let us take the two indicator cards shown in Fig. 2. These are the theoretical cards of a vertical cross-compound engine of 600-horse power at 150 R. P. M. to be direct connected to a three-phase 400-K. W., 48-pole, 60-cycle alternator to be run in parallel with other generators driven by water-wheels. These cards and data were submitted by one of our largest engine building companies with their proposition in answer to a request for bids.

The cylinders are 17" and 36" in diameter, stroke 30", connecting rod 75" or 5 cranks, boiler pressure 150 lbs., vacuum 26", 145 lbs. initial pressure in cylinder, and 22% cut-off in H. P. cylinder. Both cards are reduced to L. P. piston area and the spring scale is 30 lbs. per inch.

Weight of piston.....	284	H. P.	1591	L. P.
“ “ piston rod.....	173	“	173	“
“ “ cross-head.....	284	“	284	“
“ “ connecting rod.....	571	“	571	“

The fly-wheel weighs 34000 lbs. and the outside diameter is 12', radius of gyration 4.67'.

TABLE I.  
HIGH PRESSURE CYLINDER.

A Crank Angle.	B Stroke.	C Steam Pressure.	D Weight Parts.	E Inertia.	F Net Pressure.	H Sin $\phi$ .	I $x$ .	K Tang. Effort.	L Moment.
....	....	28.8	+7	-11.1	18.4	....	....	....	.....
18	.03	28.8		-10.5	19	.309	.368	7	8,900
36	.115	28.8		-7.8	21.7	.588	.687	14.9	19,000
54	.237	26.4		-4.8	22.3	.809	.905	20.2	25,800
72	.415	14.1		-.6	14.2	.951	1.01	14.3	18,200
90	.600	7.5		+1.8	10	1.00	1.00	10	12,750
108	.725	5.1		+3.3	9.1	.951	.89	8.1	10,300
126	.833	3.6		+5.1	9.4	.809	.712	6.7	8,550
144	.922	2.4		+6.0	9.1	.588	.49	4.5	5,750
162	.980	-4.5		+6.6	2.8	.308	.248	1.6	2,040
180	1.00	-12.9		+7.5	-4.7	...	...	....	.....
LOW PRESSURE CYLINDER.									
90		29.5	+2.29	-32.	-.2		....	....	.....
108		29.5	2.3	-25.8	+6		.368	20.2	2,800
126		29.5		-18.9	12.9		.687	8.9	11,300
144		27.6		-11.7	18.2		.905	16.5	21,000
162		17.4		-2.1	17.6		1.01	17.8	22,700
180		11.4		+6	19.7		1.00	19.7	25,100
198		9.6		+12	23.9		.89	21.3	27,200
216		7.5		+15.6	25.4		.712	18.1	23,100
234		6.6		+18	26.9		.49	13.2	16,800
252		3.6		+20	25.9		.248	6.4	8,200
270		....		+21.3	23.6		....	....	.....

Dividing the crank circle into a number of equal parts, say 20, each division representing  $18^\circ$ , we now find the position of piston when the crank passes each division; this is usually done graphi-

cally by striking circles of a radius =  $l$  = connecting rod through each point on circle; this divides the diameter in a ratio equal to per cent. of stroke.

Analytically we get this<sup>1</sup> as

$$s = r(1 - \cos \varphi) + l[1 - \sqrt{1 - r^2 \sin^2 \varphi / l^2}]$$

where :

$s$  = stroke in feet.

$r$  = crank length in feet.

$l$  = connecting rod length in feet.

$\varphi$  = crank angle assumed.

The net pressure on piston (difference in pressure on two sides) for each value of  $s$  is taken from indicator card and tabulated in Column C, Table 1. For convenience, pressure per square inch of low pressure cylinder is used.

The total weight of reciprocating parts divided by area of piston ( $A = 1020$ ) is given in Column D. These are added on down stroke and subtracted on up stroke. In this instance only the down stroke is considered.

The inertia of the reciprocating parts is next taken as in Column E. This<sup>2</sup> is found from :

$$F = \frac{w v^2}{g r} (1 + r/l) \text{ and } F = \frac{w v^2}{g r} (1 - r/l)$$

the accelerating force at the beginning and end of the stroke respectively. In this  $w$  = weight of piston, piston-rod, cross-head and half the weight of connecting rod,  $g = 32$ ,  $v$  = linear velocity of crank pin,  $r$  = crank radius,  $l$  = length of connecting rod.

The correct curve of inertia is very complicated, but is best approximated by plotting the two values given above and the point at which the acceleration is 0 and drawing a curve. This zero point is accurately enough given by  $\tan \varphi = l/r$ . This is given in Column E.

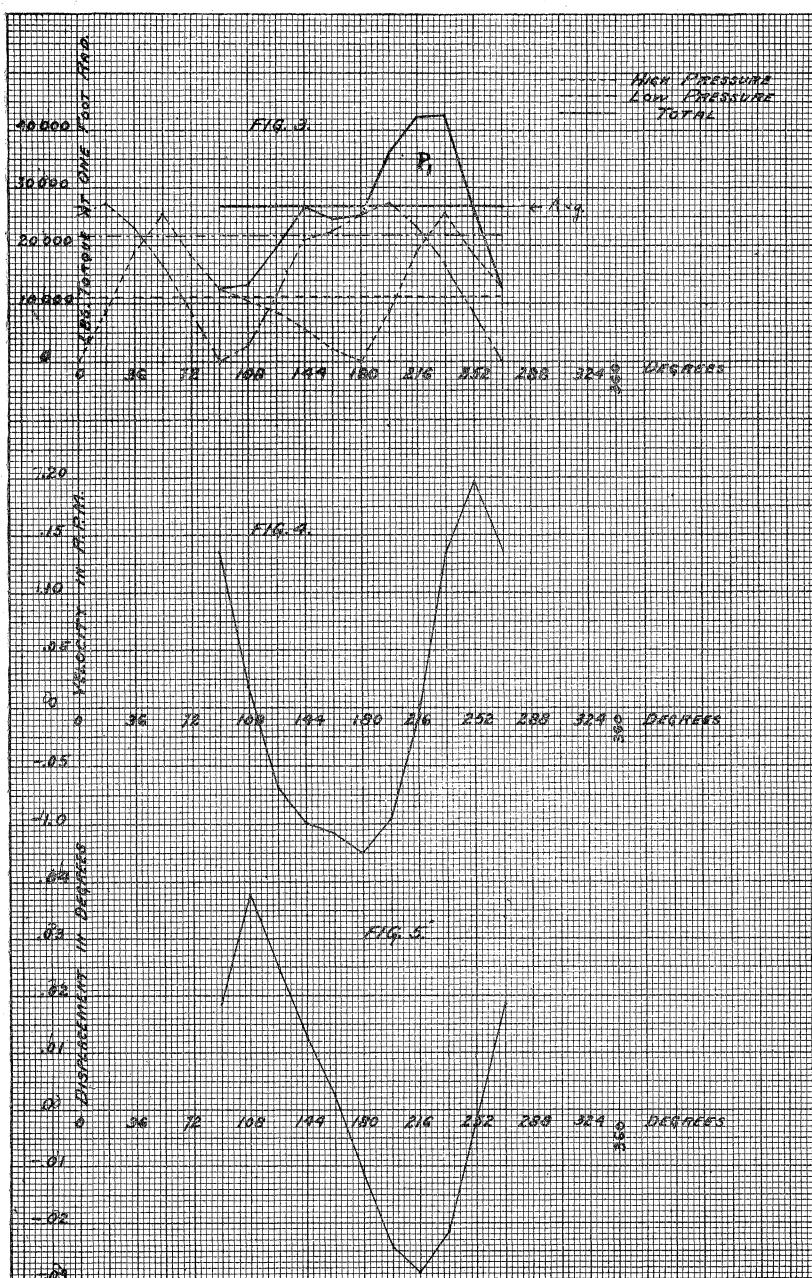
The acceleration of connecting rod used above differs very slightly from the correct value which is very complex, due to its peculiar motion.

Divide  $F$  by area of piston, and we have  $f$ , the pressure per square inch on piston.

1. Perry "The Steam Engine."

2. Unwin, "Machine Design," p. 68.





This is negative, or absorbs energy during the first half of stroke, and positive in the second half.

On the high pressure cylinder the effect of inertia is not very noticeable, but on the low pressure cylinder, due to its larger size, the effect is quite marked. It almost completely suppresses the peak of the impulse curve. (See Fig. 3).

These values in Columns C, D and E are combined algebraically, giving net pressure per square inch of piston, in Column F.

This has to be transferred to crank pin through an angle  $\varphi$  and would give  $P \sin \varphi$  were it not for the shortness of the connecting rod. This effect causes the resolution of forces to be more complicated, and we have<sup>1</sup> the tangential effort  $T$  on crank. Column K.

$$T = P \sin \varphi \left( 1 + \frac{r \cos \varphi}{\sqrt{l^2 - r^2 \sin^2 \varphi}} \right)$$

For comparison the value of this with  $P = 1$  and  $l/r = 5$ , are given along with that of  $\sin \varphi$ , Columns H and I.

We have now the crank effort which, by multiplying by  $A$ , area of piston, and  $r$  the crank radius, gives the actual moment  $F \times r$  in foot pounds. Column L. This we must also find for the L. P. cylinder and combine in proper phase relation, usually  $90^\circ$  behind the high pressure. Column A, Table II.

The next step is to find the average of the resultant moment. This should be averaged for one revolution, but if we take it for a half a revolution, assuming that the energy of two halves are equal, it will be satisfactory. This assumes that the effort is the same at the beginning and end of the half revolution. Column B is unbalanced moment at each period.

Find the average unbalanced force for each period (Column C, Table II) this acting on the fly-wheel produces an acceleration.

$$F = Ia$$

$$a = Fg/wr^2$$

in radians per sec. per sec.

where  $a$  is acceleration.

$w$  = weight fly-wheel,

$r$  = radius gyration.

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<sup>1</sup> Ewing, "The Steam Engine," p. 310.

As the speed does not vary to a great extent, we can assume without much error that equal angles are passed in equal times, therefore the time  $t$  in passing  $18^\circ$ , or  $1/20$  revolution

$$t = \frac{60 \text{ sec.}}{\text{R. P. M.} \times 20}$$

and at the end of the period the velocity will be changed  $a \times t$  in radians per sec. or multiplying by  $60/2\pi$  we get the change in R. P. M., Column D, Table II.

These values are the change in turns per minute for each period so at any point the actual velocity will be the progressive sums of these preceding gains  $+c$  (the velocity at the beginning of the stroke) since  $\int dv = at + c$ .

To find the velocity at the beginning of the stroke we sum all the values in Column E and get:

$$+.062 - 1.1415 + 10c = 0$$

This is equal to 0, since we assumed  $180^\circ$  to be a complete period beginning and ending with the same point, hence all the positive values must equal all the negative values. Having found  $c$ , we add this to each value in Column E, and get the actual velocity in Column F.

We must now integrate the velocity curve to get displacement curve. This is done in the same way as we obtained velocity from acceleration, thus: Find average velocity for each period. Multiply the average velocity for each period by

$$t = \frac{1}{150 \times 20}$$

the time in minutes to pass  $18^\circ$ , and we get the displacement in turns, and multiply this by 360 and get the displacement in degrees of the crank circle. This is given in Column H as change of position. Summing these as we did the acceleration, we get  $(A + c)$ , the relative position.

Eliminating  $c$  we get in Column K the final displacement from mean on crank circle.

To find the electrical displacement select the maximum value and multiply by  $p$ , the number of pairs of poles on the alternator.

$$\text{This is: } .0379 \times 24 = .91^\circ$$

TABLE II.

Crank Angle.	A	B	C	D	E	F	G	H	Relative Position.	Displacement.
96	12750	-14200	-14025	.....	0	+135	+077	.....	0	+0187
108	13100	-13850	-10475	-0116	-0116+C	+019	-025	+0092	+0092+C	+0379
126	19850	-7100	-3650	-087	-0293+C	-068	-083	-0030	+0062+C	+0249
144	26750	-200	-1205	-03	-0233+C	-098	-103	-010	-0038+C	+0129
162	24740	-2210	-2030	-01	-0243+C	-108	-116	-0124	-0162+C	+0025
180	25100	-1850	+3650	-017	-0260+C	-125	-110	-0139	-0301+C	-0114
198	36100	+9150	+12150	+03	-0230+C	-095	-044	-0132	-0433+C	-0246
216	42100	+15150	+15400	+101	-0199+C	+006	+070	-0053	-0486+C	-0289
234	42600	+15650	+7550	+128	-001+L	+134	+165	+0084	-0402+C	-0215
262	26400	-550	-7350	+063	+062+C	+197	+166	+0198	-0204+C	-0017
270	2750	-14200	-7350	+061	+062+C	+135	+166	+0199	-0005+C	+0187

This engine has a very small unbalancing factor and large fly-wheel, and is a very good one for the purpose. The fly-wheel could be made lighter. I might say that these particular engine builders claim to make a point of using a safety factor of 50%, at least in meeting our requirements.