



X. The magnetic field produced by a charged condenser moving through space

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in the furnace experiments. 96 grams of the powder were so enclosed. After more than a month the amount of emanation collected in the tube was investigated by exhausting an electroscope, after first carefully rating it, and filling it with air drawn slowly through the U-tube. The gain in the rate of discharge was less than 3 scale-divisions per hour; barely more than $2\frac{1}{2}$. This represents an amount of radium no more than 1.5×10^{-12} gram. The emanation in the 96 grms is about 211×10^{-12} . Hence the radium content of this powder, as determined by the emanation contained in it, would be underestimated by less than one per cent. The error is, of course, quite negligible.

The method of extracting radium emanation by fusion is applicable to other investigations than rocks and minerals. Natural waters, such as river waters, &c., would certainly be best dealt with by this method; the residues from large quantities, obtained by evaporation, being fused with carbonates in the furnace. For such delicate work small-sized furnaces are easily constructed. In the case of sea waters its use seems particularly desirable. Discrepancies arising from conditions of ebullition, acidification, presence of organic matter, or from whatever source, must assuredly disappear when the dry salts are treated in the furnace. In this case the bulk of the residue left by evaporation may be much reduced by evaporation of the chloride of sodium in an open crucible, and, I believe, without risk of loss of radium.

I hope to be able shortly to give results obtained by these applications of the fusion method.

I desire to thank Mr. W. Tatlow for much kind assistance in arranging for the supply of current required in the rather extensive series of preliminary experiments which I found it necessary to make: also Mr. L. B. Smyth for valuable help in carrying out the experiments.

X. *The Magnetic Field produced by a Charged Condenser moving through Space.* By W. F. G. SWANN, D.Sc., A.R.C.S., Assistant Lecturer in Physics at the University of Sheffield*.

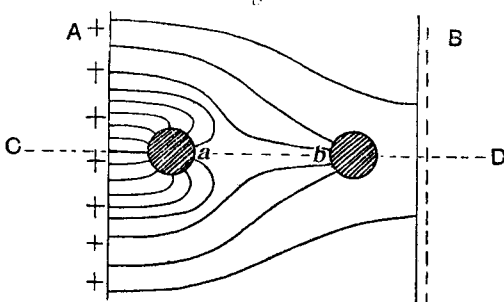
Introduction.

A SYSTEM of charged bodies moving through space with velocity v , should, on the assumption of a quiescent æther, give rise to a magnetic field which at each point is equal to $4\pi vP$, where P is the electric polarization at that

* Communicated by the Author. Read at the Meeting of the British Association, 1910.

point resolved perpendicular to the velocity v . Röntgen endeavoured to detect this field by its effect on a magnetic compass-needle, but no such effect was observed. Sir Joseph Larmor has explained this by showing that the induced charge produced on the surface of the compass-needle is such as by its motion to completely annul the magnetic field produced by the charged bodies, at all points in the interior of the compass-needle. I think this can also be seen to follow from the consideration that, since all the bodies are moving in the same direction, and since all the tubes of force which strike the surface of the compass-needle end there, none of them exist within the compass-needle, to produce by their motion a magnetic field. Now although the magnetic field is incapable of being detected by a compass-needle, it seems at first sight that it might be capable of detection, by means of a rotating coil; for let AB (fig. 1) be two

Fig. 1.



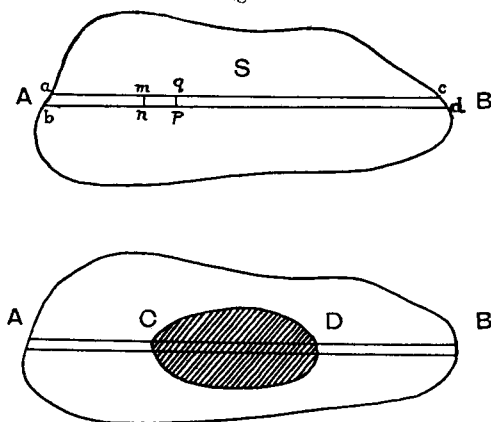
condenser-plates moving with the earth in a direction perpendicular to the plane of the paper, and imagine the coil shown in section to rotate about an axis CD. It is easy to see that the tubes of force will be distributed in the manner qualitatively illustrated in the figure, and it is not obvious that the magnetic flux produced by the motion of those tubes which strike the side b of the coil will be exactly cancelled by the flux caused by the motion of those tubes which bend round, and strike the coil on its inner side at a . Since the flux is certainly zero when the plane of the coil is perpendicular to the line of motion, we might expect on rotating the coil to obtain an alternating current which by suitable commutation could be detected by means of a galvanometer.

Unfortunately it appears on examination, that the flux through the coil is at all times zero, the two opposing portions annulling each other, as will be shown in Section 1, provided that the whole of the space in the vicinity of the coil is filled with material of constant specific inductive

capacity. It will appear, however, (Section 2) that if part of the space inside the coil is filled with dielectric material of one kind, *e. g.* paraffin wax, and the other part with dielectric material of another kind, *e. g.* a vacuum, a resultant magnetic flux should be obtained through the coil, provided that the specific inductive capacity is a quantity which may be looked upon as continuous throughout the medium to which it refers. As will be shown later (Section 4), while the electron theory satisfies the requirements of a hypothetical dielectric of this kind, as far as the field outside it is concerned, nevertheless, inside the dielectric, the condition of continuity is not satisfied; and it will follow that on this view of dielectric action the flux through the coil in all positions will be zero, so that the absence of any effects of the kind sought in the experiments to be presently described affords direct proof of the discontinuous nature of the specific inductive capacity, and shows that dielectric action is to be accounted for by the presence of something of the nature of electric doublets, and cannot be attributed to any other mysterious power of twisting the tubes of force, which the dielectric might be imagined to possess.

Section 1. *The total magnetic flux through a closed circuit kept at constant potential, and due to the motion of a system of charged bodies along with the closed circuit, is zero, when the whole medium in the neighbourhood of the closed circuit is filled with a substance of constant specific inductive capacity K.*

Fig. 2.



Let S (fig. 2) be the circuit (not necessarily a plane circuit), and v its velocity in common with the system of

charged bodies. Draw two parallel planes across the circuit at a distance apart dy . Let these planes cut the circuit in the points $acdb$, and let $mnpq$ be a rectangular element of $acdb$. Let ψ be the angle made at any point by v with the plane of $mnpq$, and let P be the electrical polarization resolved parallel to the long axis of $acdb$. On writing dx for mq the magnetic flux through $mnpq$ is

$$4\pi vP \cos \psi \cdot \overline{mn} \cdot dx.$$

Remembering that $\overline{mn} \cos \psi = dy$ and writing F for the electric intensity corresponding to P , we have

$$\text{Magnetic flux through } mnpq = KF \frac{v}{c^2} dy \cdot dx.$$

Hence the total flux through $acdb$

$$= \frac{Kv}{c^2} dy \int_A^B F dx,$$

c being the velocity of light. Now F , which is the electric intensity corresponding to P , is composed of two parts, and may be written in the form $X - v\gamma$, where X is the part which is derivable from a potential, and which can produce mechanical force on a charge e moving with the system, while $-v\gamma$ is a part whose mechanical force $-v\gamma e$ on a charge e moving with the system is just counteracted by the mechanical force $+v\gamma e$ exerted on the charge owing to its motion in the magnetic field, γ being the magnetic field resolved perpendicular to P . The field γ is of course also perpendicular to v . Since both ends of the strip AB are at the same potential $\int_A^B X dx = 0$, so that

$$\begin{aligned} \text{Flux through strip} &= \frac{Kv}{c^2} dy \int_A^B (X - v\gamma) dx = \frac{Kv^2}{c^2} dy \int_A^B -\gamma dx \\ &= -\frac{Kv^2}{c^2} (\text{Flux through strip}). \end{aligned}$$

Hence the flux through the strip is zero, and since the whole curve S may be divided up into strips in this way, the total flux through it is zero*.

* The absence of any magnetic field in the interior of a conductor, *e. g.* the conducting compass-needle in Röntgen's experiment, due to the motion of charged bodies along with the conductor, is a particular case of this more general theorem.

Section 2. *Case where K is variable.*

Suppose now that a portion of the space within S (the shaded portion) is filled with material of specific inductive capacity K_2 , the rest of the space being filled with material of S.I.C. equal to K_1 . By an argument exactly similar to that given above, we arrive at the expression

$$\text{Total flux through strip} = \frac{v}{c^2} dy \int_A^B K(X - v\gamma) dx.$$

Thus, since K is not the same at all points along the strip, we have

Total flux through strip

$$\begin{aligned} &= K_1 \frac{v}{c^2} dy \int_A^C (X - v\gamma) dx + K_2 \frac{v}{c^2} dy \int_C^D (X - v\gamma) dx \\ &\quad + K_1 \frac{v}{c^2} dy \int_D^B (X - v\gamma) dx \\ &= K_1 \frac{v}{c^2} dy \int_A^B (X - v\gamma) dx + (K_2 - K_1) \frac{v}{c^2} dy \int_C^D (X - v\gamma) dx. \end{aligned}$$

Since $\int_A^B X dx$ is zero, and since, as is very easily verified, the quantity $v\gamma$ which itself arises from the motion of the system is only of the second order in $\frac{v}{c}$ compared with X , the integrals of the form $\int v\gamma dx$ are negligible, and we obtain

$$\begin{aligned} \text{Flux through strip} &= (K_2 - K_1) \frac{v}{c^2} dy \int_C^D X dx \\ &= (K_2 - K_1) \frac{v}{c^2} dy \left\{ \begin{array}{l} \text{Potential difference} \\ \text{between C and D} \end{array} \right\}. \end{aligned}$$

If we now draw an infinite series of lines perpendicular to the axis of y , each line connecting two parts of S, and if V represents the potential difference between the points where these lines enter and leave the intermediate dielectric, we have

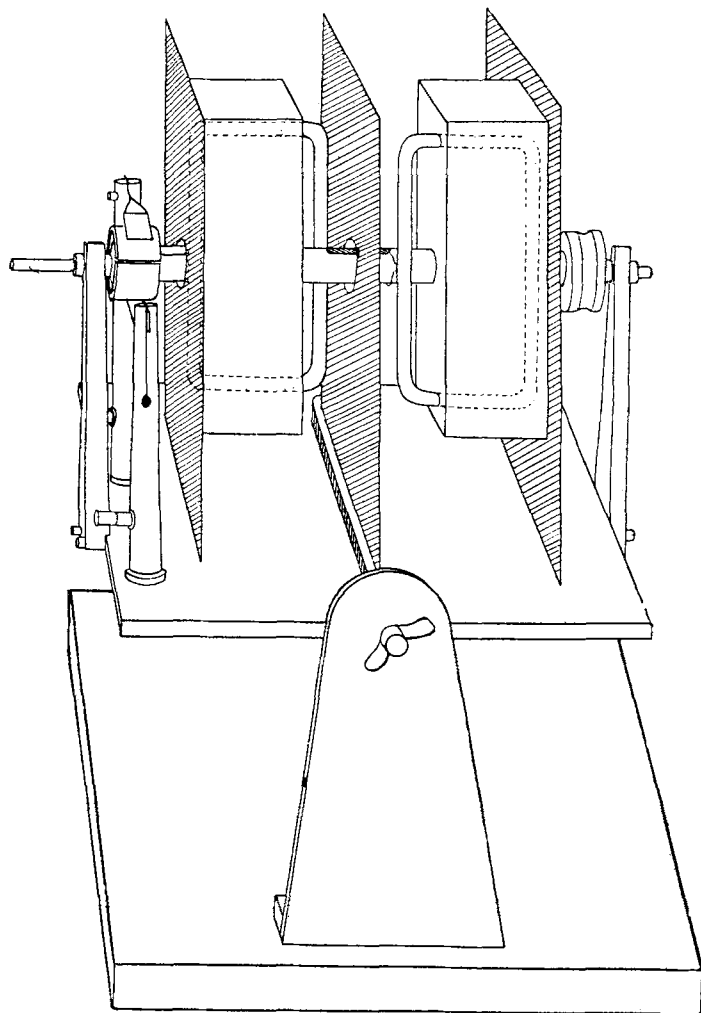
$$\text{Total Magnetic flux through S} = (K_2 - K_1) \frac{v}{c^2} \int V dy,$$

the integral being taken right across the circuit.

Section 3. *The Apparatus and the Experiment.*

After performing several experiments with a preliminary apparatus consisting of a single coil rotating between two condenser-plates, the form of apparatus shown in fig. 3 was

Fig. 3.



finally decided upon. Instead of one rotating coil there were two, each wound with about 300 turns of copper wire to a resistance of 36 ohms, the coils being afterwards bound

with silk, and shellacked. The coils were connected in such a way that on rotating them together the E.M.F.'s due to the earth's magnetic field acted in opposite directions round the circuit, so that the effect of this field was eliminated, and what is more important, any effects due to its fluctuations were automatically eliminated. The three vertical plates shown in the figure formed two condensers; the central plate, which was insulated in sulphur, could be charged, while the other two were fixed to a brass plate which could be earthed; thus any magnetic fluxes due to the earth's motion through space were additive in their effects on the galvanometer. The coils were partially embedded in rectangular blocks of paraffin wax, which also served to hold them in position, and they were arranged symmetrically with respect to the axis of rotation, so that the system was balanced when rotating. The galvanometer used was of the Broca type, and contained two coils each of resistance 32 ohms; these coils were arranged in series, and the point of junction was earthed. By earthing this point, and in virtue of the symmetry of the arrangement, various small leakage effects which caused trouble in the preliminary experiments were successfully overcome; for example, gradual leakage from the charged plate, and consequent alteration of the induced charge on the surfaces of the coils, resulted in electricity coming up from earth, and some of this came through the galvanometer, but in virtue of the symmetry of the above arrangement, equal quantities came through the two galvanometer coils, and consequently their effects on the galvanometer-needle cancelled.

The apparatus was mounted on a base as shown in the figure, in such a manner that it could be turned about, and the base could be clamped to the table in either a horizontal or a vertical plane by means of a nut. The rotating coils were connected by two pieces of copper foil cemented to the axle, which consisted of an ebonite rod with a brass core, and the system could be rotated by a motor placed 2 metres away.

The central plate, which was connected to the inner coating of a leyden-jar, was charged from a Wimshurst machine, one pole of which was earthed, and the potential of the plate was measured with a Kelvin electrostatic voltmeter.

It was necessary to carefully earth one pole of the wimshurst, as otherwise currents were produced through the tables &c., which were sufficient to cause slight kicks in the galvanometer when the condenser was charged or discharged. Kicks of this kind were not of vital importance, since they

did not affect the steady position of the zero, but nevertheless it was convenient to reduce them to a minimum.

Gradual leakage of electricity from the charged plate could not result in any permanent alteration in zero, even when the coil was rotating, for, in view of the fact that the charging connexion was made to the central plate at the top, the magnetic flux through the coil due to such leakage would be parallel to the brass base of the apparatus, and even if its magnitude were worth considering, it could produce no effect on the rotating coil. Only magnetic flux perpendicular to the brass base could produce any effect on the rotating coil, since the commutator was fixed so that when the planes of the coils were parallel to the brass plate, a line joining the commutator slits was perpendicular to it.

Method of making the experiment.

It was first necessary to set the apparatus so that a line parallel to the brass base, and perpendicular to the axis of rotation, should be parallel to the direction of the æther drift.

Professor Trouton, in connexion with his work on the "Couple exerted on a Charged Condenser moving through Space"*, has determined the time of the day throughout the year at which the æther drift is parallel to the earth's surface, and he also gives the azimuth of the drift, and its magnitude, taking into account the Sun's orbital and proper motions. Though his calculations refer to London, it is sufficient for the purpose in hand to take them as applying to Sheffield. It is easy to find from Trouton's data the time at which the drift is vertical, and the experiments in this paper were always performed when the drift was either horizontal or vertical.

The apparatus having been adjusted to the correct position, and the commutator having been well wetted with paraffin oil, the motor was started. Any deflexion produced by want of complete compensation of the coils was balanced by magnets placed slightly asymmetrically, about 1.5 metres away. The central plate was then charged, say positively, to about 4000 volts, as recorded by the electrostatic voltmeter. The zero of the galvanometer having been read, the plates were discharged by two knobs placed some distance away, and any alteration in the galvanometer reading was noted. The experiment was repeated several times, and another series was then taken with the central plate charged negatively. The whole apparatus was then turned through 180 degrees, so that any magnetic field due to the earth's motion was

* Phil. Trans. A. 1904, p. 165.

reversed through each coil, and the experiments were repeated. A measurement of the time of swing of the galvanometer completed the set of observations.

*Magnitude of the effect to be expected due to the
Earth's Motion.*

The expression for the maximum flux through the coil, given on page 154, becomes

$$\frac{1}{9} \times 10^{-12} v (k-1) \int V dy,$$

where k and 1 are the specific inductive capacities of paraffin wax and air respectively, and V is in volts.

If l is the length of the line formed by the intersection of the plane of the coil with the dielectric, and if \bar{V}_1 and \bar{V}_2 are the respective mean values of the potentials over the corresponding lines for the two coils, we have

Maximum flux through the two coils

$$= B = \frac{1}{9} \times 10^{-12} v l (k-1) (\bar{V}_1 + \bar{V}_2).$$

The method of measuring \bar{V}_1 and \bar{V}_2 is explained on page 162.

In order to determine the magnitude of the deflexion which the above flux should produce when the coils rotate, it was best to short-circuit one of the coils (which was done by means of a brass collar connecting the copper strips joining the coils), set the other coil in rotation in such a position as to be affected only by the earth's vertical component, and observe the deflexion δ_2 produced on the galvanometer. A high resistance was put in series with the galvanometer, and the pair were shunted for this purpose.

Let R_2 be the resistance of the circuit containing the single unshort-circuited coil and the shunted system, G , Z , and S the resistances of the galvanometer, the coil in series with it, and the shunt respectively; let a be the effective mean area of the coil, n_2 the frequency of rotation of the coil, and W the vertical component of the earth's field; then the required deflexion which would be produced by the flux B is easily seen to be

$$\delta_1 = \left[\frac{\delta_2 n_1 R_2 (G + Z + S)}{n_2 R_1 W a S} \right] \left[\frac{1}{9} \times 10^{-12} v l (k-1) (\bar{V}_1 + \bar{V}_2) \right] \frac{T_1^2}{T_2^2},$$

where R_1 is the resistance of the two rotating coils and the galvanometer in series, n_1 is the frequency of rotation in the main experiment, T_1 and T_2 are the times of swing of

the galvanometer in the main and subsidiary experiment respectively.

In view of some uncertainty as to the effective resistance of the commutator when rotating*, the precaution was taken of placing in series with the rotating coil in the subsidiary experiment, such a resistance as to make the resistance of the circuit containing the shunted galvanometer and the rotating coil the same as the resistance of the circuit composed of the galvanometer and the two rotating coils in the main experiment. Thus the quantities R_1 and R_2 , which contain the resistance of the commutator, cancelled out.

The value of δ_2 was obtained by reversing the current through the galvanometer, and taking half the deflexion observed.

The accompanying table shows the values of $\frac{\delta_2}{n_2}$ obtained in three successive experiments; the constancy of the numbers in the last column gives an idea of the degree of consistency of the readings.

Speed revs., per min.	Deflexion on reversal.	$\frac{\delta_2}{n_2}$.
735	230	0.156
800	253	0.158
800	248	0.155

$$\text{Mean } \frac{\delta_2}{n_2} = 0.156$$

The time of swing of the galvanometer was 11.0 seconds, and the value of $\frac{R_2(G+Z+S)}{R_1S}$ was 280, so that in view of the fact that $W=0.44$, $k=2.3$, $a=31.3$, $l=8$ cms., we find for the deflexion to be expected in the main experiment, for a case where $v=18$ miles per second (2.9×10^6 cm./sec) †, $V_1 + V_2 = 1200$ volts, $n_1 = 800$, and $T_1 = 11$ seconds,

$$\delta_1 = 10.2.$$

* The commutator was kept well wetted with paraffin oil while running, to keep its effective resistance constant.

† The fact that the resultant velocity of the æther drift happened to coincide with the velocity (18 miles per second) of the earth's orbital motion, is a mere coincidence; the directions were of course not the same.

The Results†.

The following tables show the results obtained in the main experiments. The third column represents the deflexion observed on discharging the central plate of the apparatus, and the fourth column the values calculated in the above manner. 0 is written when the deflexion is less than 0.5 division. Such very slight deflexions as were observed are doubtless due mainly to alterations of zero due to incomplete astaticism of the galvanometer-needle, to lack of compensation of the two coils as far as fluctuations of the earth's field are concerned, and to residual electrostatic effects. There is obviously no evidence of any appreciable deflexion; the very slight irregular deflexions observed are hardly to be wondered at, in view of the facts that the galvanometer had a time of swing of 11 seconds, the scale was 3 metres away, and the central plate was charged to 4000 volts.

Continued repetition of the experiments seemed unnecessary, as the observations below were taken after much experience with the apparatus. The main part of the work

TABLE I.

Experiments performed on July 20, 2.40 A.M. to 3.40 A.M.
Æther drift vertical. $T_1 = 11.4$ seconds.

Central plate charged positively.				Central plate charged negatively.			
Speed revs. per min.	Potential of central plate. Volts.	Deflexion observed.	Deflexion calculated.	Speed revs. per min.	Potential of central plate. Volts.	Deflexion observed.	Deflexion calculated.
810	+4000	0	10.4	810	-4000	+2	10.4
810	...	0	10.4	740	...	+2.5	9.5
800	...	-2	10.3	740	...	+2	9.5
800	...	-3	10.3	750	...	0	9.6
800	...	0	10.3	760	...	0	9.8
800	...	-1.5	10.3	740	...	+1	9.5
800	...	+0.5	10.3	720	...	+0.5	9.2
800	...	-4*	10.3	710	...	+2	9.1
800	...	0	10.3				
800	...	0	10.3				

* The spot of light was unsteady when this observation was taken.

† The calculated values of δ_1 given in the tables are obtained on the assumption that $k=2.3$. The value of k as measured, without taking any precautions to prevent soaking, was higher than this. If the larger value were used the calculated values of δ_1 would be greater; it was considered safer, however, in view of the possibility of soakage, to take the value of k usually given in the tables for the kind of wax used.

TABLE II.—Apparatus turned through 180° .

Central plate charged positively.				Central plate charged negatively.			
Speed revs. per min.	Pot. of central plate. Volts.	Deflexion observed.	Deflexion calculated.	Speed revs. per min.	Pot. of central plate. Volts.	Deflexion observed.	Deflexion calculated.
720	+4000	+1	9.2	720	-4000	0	9.2
730	...	0	9.3	720	...	-3	9.2
720	...	+1	9.2	720	...	0	9.2
700	...	-3	9.0	710	...	0	9.1
680	...	+1.5	8.5	710	...	+2	9.1
680	...	+1.5	8.7	700	...	-1	9.0
690	...	0	8.9	710	...	0	9.1
680	...	0	8.7	700	...	0	9.0

TABLE III.—Experiments performed on July 20, 2.55 P.M. to 4.20 P.M.
Æther drift horizontal. $T_1 = 10.6$ seconds.

Central plate charged positively.				Central plate charged negatively.			
Speed revs. per min.	Pot. of central plate. Volts.	Deflexion observed.	Deflexion calculated.	Speed revs. per min.	Pot. of central plate. Volts.	Deflexion observed.	Deflexion calculated.
690	+4000	+2	7.7	750	-4000	+3	8.4
740	...	-3	8.2	760	...	-1.5	8.5
700	...	+1	7.8	760	...	-1	8.5
700	...	0	7.8	780	...	0	8.7
690	...	-1	7.7	760	...	0	8.5
680	...	+0.5	7.6	700	...	+2.5	7.8
780	...	+1	8.7				
740	...	-1	8.2				

TABLE IV.—Apparatus turned through 180° .

Central plate charged positively.				Central plate charged negatively.			
Speed revs. per min.	Pot. of central plate. Volts.	Deflexion observed.	Deflexion calculated.	Speed revs. per min.	Pot. of central plate. Volts.	Deflexion observed.	Deflexion calculated.
770	+4000	0	8.6	860	-4000	+1	9.6
740	...	+1	8.2	880	...	-1	9.8
730	...	0	8.1	880	...	+2	9.8
810	...	+4	9.0	870	...	+4	9.7
810	...	0	9.0	850	...	0	9.5
830	...	+2	9.2	860	...	+3	9.6
810	...	-2.5	9.0	840	...	+2	9.4
820	...	+2	9.1	800	...	0	8.9
840	...	-2	9.4				

consisted in overcoming the various causes of fluctuation due to direct electrostatic effects, variations of the earth's field, &c., and once these had been successfully overcome, it was felt that multiplication of the observations could not add anything to the reliability of the results.

Method of measuring the quantity \bar{V} referred to on page 158.

The method adopted was to measure the potential at various points along the line joining the inner point of intersection of the coils with the paraffin wax, then by plotting the potential against the distance from one end of this line, the quantity \bar{V} representing the mean value of the potential could be obtained.

In order to measure the potential at any point along the above line a special method was adopted, since the introduction of anything of the nature of a wire connected to an electrometer of any appreciable capacity would result in such an induced charge on the wire as would completely alter the potential to be measured.

A small proof plane (about 3 mms. in diameter) was placed at the point at which the potential was required, and was then touched by a thin earth-connected wire. The proof plane received an induced charge, and its potential became zero. Now assuming that the induced charge did not appreciably alter the distribution of electricity on the charged plate, and on the coil in its vicinity, which assumption was quite justified to the degree of accuracy to which we were working, we could look upon the potential of the proof plane, which was zero, as being made up of two equal and opposite parts: (1) the positive potential which would have existed at the point at which we desired the potential, if the plane had been absent, (2) the negative potential which the charge on the proof plane contributed. If the earth-connected wire were then removed the potential of the proof plane would still be approximately zero, except in so far as the small amount of charge on the wire itself contributed to the potential of the plane*. If the proof plane were now removed and held away from the apparatus, it would have a potential as much below zero as the potential which we desired to measure was above zero. It only remained to measure the potential of the proof plane.

For this purpose an electrometer of small capacity was constructed. It consisted of a silvered glass fibre, with a

* The error due to this cause became automatically eliminated in practice (see note on page 163).

silvered glass ball, 0.5 mm. in diameter, at each end, which was suspended by an insulating suspension, and provided with a mirror, for obtaining the deflexions. Another silvered glass ball was fastened to a piece of thin platinum wire, which was insulated by sulphur. When this fixed ball was charged it attracted one of the movable balls, and the sensitiveness of the arrangement could be adjusted by moving the fixed ball by a tangent screw.

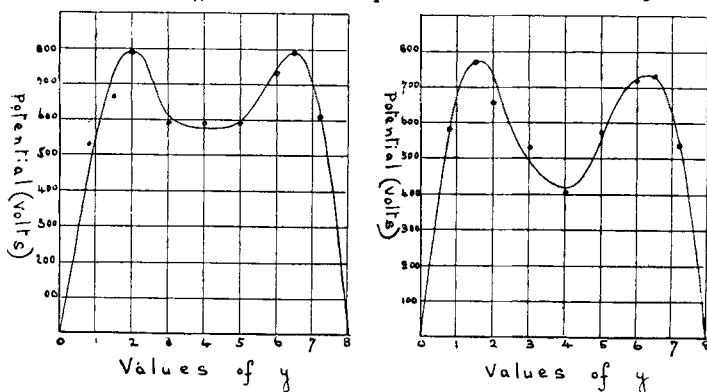
The proof plane whose potential was required was caused to touch the wire connected to the fixed ball, and the deflexion α was observed. The electrometer was standardized by taking a series of readings in which the fixed ball was touched with the proof plane after the latter had been raised to known potentials by allowing it to touch a thin wire connected to the central condenser plate of the apparatus. The potential of this plate, which was of course reduced to potentials varying from 400 to 1000 volts, was measured by the electrostatic voltmeter*.

Tables V. and VI. give for the two blocks the deflexions referred to as α above, corresponding to the potential E of the central charged plate given in the first column. The third column gives the potentials corresponding to these deflexions, and the last columns the distances y of the points at which the potentials were measured, from one of the points of intersection of the coil with the paraffin wax.

The curves (fig. 4) represent the observations plotted

Fig. 4.

Curves showing the variation of potential with the distance y .



* It will be seen that the error due to neglect of the part of the potential of the proof plane contributed by the charge on the wire, just cancels the error referred to in the note on page 162.

graphically, the dips in the middle being due to the brass axle which passed through the centre of the rotating system. From these curves we find for a difference of 4000 volts between the central and external condenser plates, the values $\bar{V}_1 = 588$ volts $\bar{V}_2 = 543$ volts. A slight error in the determination of the positions of the maxima of the curves is not of great importance in view of the sharpness of the maxima. The slight difference in the values of \bar{V} for the two coils is not surprising in view of the large shielding effect of the earthed coils, and consequent rapid variation of the potential in the vicinity of, and in a direction perpendicular to, the surfaces of the dielectrics along which the potentials were measured.

TABLE V.

E.	α .	V Volts.	γ .
4000	22	530	0.8
...	37	665	1.5
...	56	790	2.0
...	28	590	3.0
...	28	590	4.0
...	28	590	5.0
...	47	733	6.0
...	56	790	6.5
...	30.5	610	7.2

TABLE VI.

E.	α .	V Volts.	γ .
4000	27	580	0.8
...	53	770	1.5
...	36	655	2.0
...	22	530	3.0
...	13	405	4.0
...	26	575	5.0
...	45	720	6.0
...	47	733	6.5
...	23	540	7.2

Section 4. *The Nature of Dielectric Action.*

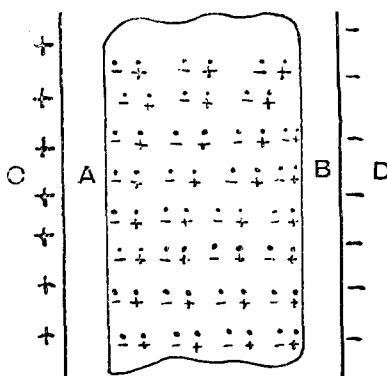
It may at once be remarked that the theorem proved in section 1 shows that if the whole of dielectric action is to be attributed to electric charges in the dielectric, the motion of the system through space cannot produce a magnetic flux through a closed circuit kept at constant potential. It is, I think, nevertheless interesting to verify the conclusion by a rather closer examination of the phenomenon of dielectric action, with a view to a clearer physical coordination of the various quantities involved.

We will first define specific inductive capacity in the following way. Let there be two large parallel condenser plates C, D either moving through space or at rest; let one plate be charged and the other earthed. Let V_1 be the potential difference between two planes A and B situated between and parallel to the two plates. Now fill the space between A and B with a slab of material, and let V_2 be the new potential difference between A and B. Then the specific inductive capacity of the material is $K = \frac{V_1}{V_2}$.

Now there are two distinct classes of action which will explain the existence of the quantity K . We may consider that the electric force causes a separation of positive and negative electricity in each atom or molecule, giving rise to something equivalent to a doublet, or we may imagine the doublets to exist even in the absence of the electric force, in which case they will be orientated at any instant in all sorts of directions, the effect of the electric force being to give them a preference towards an orientation of their axes in one particular direction. We shall first proceed to develop the consequences of the former hypothesis.

Considering the two condenser plates referred to above, with the slab of dielectric between them, we are to look upon the electric force as causing an electrical separation, giving rise to a series of doublets, arranged in the manner shown (fig. 5). Now the tubes of force from C do not pass continuously through the dielectric, most of them, or possibly all,

Fig. 5.



end on some or other of the negative charges on the doublets on the surface of A , and just as many tubes emanating from the positive elements of these doublets are so to speak freed, thus being at liberty to carry on the flux in the direction from A to B . If we were to cut a cylindrical hole in the dielectric with its axis perpendicular to A , and of length small compared with its cross-section, and if we were to measure the electric intensity in this hole, we should get $4\pi c^2$ times the ordinarily defined polarization or number of tubes of force per square centimetre*, we should get the

* The polarization is here in electromagnetic units.

quantity whose line integral across the dielectric is K times the potential difference between A and B .

Viewing the matter from the standpoint we have adopted, we are driven to the conclusion, that though the dielectric acts at all points outside it or at a point in a hole inside it as though the tubes did actually pass continually through it, nevertheless, they do not run continuously in this way; in fact, between the elements of a doublet the tubes actually pass in the opposite direction to the ordinarily defined polarization. At each point inside the material of the dielectric, we must look upon the number of tubes of force per square centimetre as being algebraically made up of two parts: (1) the number ordinarily defined as the polarization, and (2) the number contributed in the opposite direction by the separation of the two kinds of electricity in the doublets. At any point outside the dielectric the second set is of course absent. The first set is equal to $\frac{KF'}{4\pi c^2}$, where F' is the electric intensity as ordinarily understood, at a point within the dielectric. The second set N' is such that

$$\int_R^S \frac{K}{4\pi c^2} F ds - \int_R^S N ds = \frac{1}{4\pi c^2} \left(\begin{array}{c} \text{Potential difference} \\ \text{between R and S} \end{array} \right) = \frac{1}{4\pi c^2} \int_R^S F ds,$$

so that

$$\frac{K-1}{4\pi c^2} \int_R^S F ds = \int_R^S N ds,$$

where F and N are the resolved portions of F' and N' along the path RS , along which the integral is taken, for it is to be remembered that we are admitting no mysterious method of modifying the force in the dielectric other than that due to the charges. The physical effect of the doublets is to reduce the work which we should do in taking a unit of electricity from R to S , since when between the elements of a doublet we should be assisted in our journey, the effect being greater, the greater the moments of the doublets; and to the extent that we may assume the effect produced by the doublets to be proportional to the potential difference V which would exist between the planes of opposite faces of the dielectric if the dielectric were absent and the charges on C and D were the same as at present, we may write ΔV (the lowering of the potential difference between the faces) as $\Delta V = pV$, where p is a constant. The specific inductive capacity will then be $\frac{V}{V-\Delta V}$ or $\frac{1}{1-p}$.

Returning to the argument on page 154 we are now to realize that in taking the flux through the strip we are to take account of the true polarization at each point in the dielectric, and our expression for the flux through the strip AB becomes

$$\text{Flux} = 4\pi v \, dy \int_A^B P \, dx,$$

where P is the resolved portion of the true polarization in the direction dx . Now, as we have seen, P is made up of two parts $\frac{KF}{4\pi c^2}$ and $-N$, so that

$$\begin{aligned} \text{Flux through strip} = \frac{v}{c^2} dy \left[\int_A^C (K_1 F - 4\pi c^2 N) dx + \int_C^D (K_2 F - 4\pi c^2 N) dx \right. \\ \left. + \int_D^B (K_1 F - 4\pi c^2 N) dx \right] \end{aligned}$$

and since as we have seen

$$\frac{(K-1)}{4\pi c^2} \int F \, dx = \int N \, dx$$

over any path, the above equation becomes

$$\text{Flux through strip} = \frac{v dy}{c^2} \int_A^B F dx = \frac{v dy}{c^2} \int_A^B f dx = \frac{v dy}{c^2} \int_A^B (X - v\gamma) dx,$$

where X and γ have the significance accorded them on page 153, and f is the true electric intensity corresponding to F^* at a point in the dielectric.

$$\text{Since} \quad \int_A^B X dx = 0,$$

we have

$$\text{Flux through strip} = -\frac{v^2}{c^2} dy \int \gamma dx = -\frac{v^2}{c^2} dy (\text{Flux}),$$

from which it follows that the flux through the strip is zero.

* f is the x component of the actual force at a point; it is a quantity varying rapidly from point to point, being in fact in opposite directions on the two sides of an element of a doublet. F is the ordinarily defined, smoothed out value of this electric force. It can only be realized as $\frac{\delta V}{\delta s}$ where δV is the alteration of potential in the length δs , δs being a length small compared with ordinary dimensions, but large compared with the distance between two doublets. The line integrals of f and F over any finite path are of course the same.

We thus see that the part of the flux represented by

$$-4\pi v dy \int_A^B N dx$$

is just such as to annul the other portion represented by

$$\frac{v}{c^2} dy \left[\int_A^C K_1 F dx + \int_C^D K_2 F dx + \int_D^B K_1 F dx \right],$$

the equivalent of which we found for the total flux on page 154. If we prefer a rather less exact but more vivid picture of the phenomenon, we may say that the total magnetic flux produced by the motion of the tubes of force between the elements of individual doublets, is just equal and opposite to the magnetic flux produced by the motion of the tubes joining different doublets. Considering the second view in which the doublets exist, but are orientated at a haphazard manner in the absence of the electric force, we must look upon them as being turned on the application of the electric force so that their axes tend to point on the average in one direction, to an extent depending on the electric force. It is easy to see that the effect of this orientation of the doublets is to reduce the potential difference which would exist between the opposite faces of the dielectric for the same distribution of charge on the charged plates, and to the extent that this reduction ΔV is proportional to V , we may write

$$\frac{V}{V - \Delta V} = \frac{V}{V - pV} = \frac{1}{1 - p} = \text{constant} = K.$$

The argument showing the absence of any magnetic flux through a strip joining two points at the same potential follows in a manner exactly similar to that given above. On any conceivable view, in which dielectric action is to be explained entirely by the presence of electric charges in the dielectric, the expression $\int_A^B P ds$ must equal $\frac{1}{4\pi c^2} \int_A^B f ds$,

where the integral is taken along any path between A and B , and the quantities P and f represent the components of the polarization and of the true electric intensity at a point within the dielectric respectively, and as we have seen this latter integral vanishes when A and B are at the same potential, so that on no theory of this kind can we obtain any resultant magnetic flux through the closed circuit referred to on page 154, due to the motion through space of a system of charged bodies along with the circuit.