



XXVIII. On the acceleration of secondary electromagnetic waves

Fred. T. Trouton

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XXVIII. *On the Acceleration of Secondary Electromagnetic Waves.* By FRED. T. TROUTON*.

[Plate VI.]

IT is well known that if, after the manner of Huygens's construction, the effect of a wave-surface at any point be determined by summing up the individual effects of the secondary waves obtained, by supposing the surface divided up into elementary portions, each element of surface acting as an independent source, it becomes necessary to assume that an elementary portion produces its effect at the point with an acceleration of phase of $\frac{1}{4}$ period in advance of the effect produced at the point by the general wave-surface, otherwise the sum of the secondary effects would possess a phase $\frac{1}{4}$ period in error.

As ordinarily considered †, it is somewhat surprising that there should be this change of phase in a wave coming from an element of surface. But the consideration of this question, in the light of Hertz's investigations on the radiation emitted from an electromagnetic "vibrator" or discharging condenser, seems to be particularly suggestive.

At first the magnetic component of the wave will be taken, as it can be more simply dealt with. Considering, for the moment, points situated along a line at right angles to, and starting from, the centre of a "vibrator," Hertz's expression ‡ for the magnetic component of the electromagnetic wave at a distance r is

$$P = \frac{a}{r} \left\{ \sin 2\pi \left(\frac{r}{\lambda} - \frac{t}{\tau} \right) + \frac{\lambda}{2\pi r} \cos 2\pi \left(\frac{r}{\lambda} - \frac{t}{\tau} \right) \right\}$$

At points near the "vibrator" the vibration is easily seen to depend almost entirely on the latter term, or

$$P = \frac{a\lambda}{2\pi r} \cos 2\pi \left(\frac{r}{\lambda} - \frac{t}{\tau} \right);$$

while at a distance it depends in like manner on the first term, or, as it may be written,

$$P = \frac{a}{r} \cos 2\pi \left\{ \frac{r}{\lambda} - \left(\frac{t}{\tau} + \frac{1}{4} \right) \right\}.$$

* Communicated by the Author.

† The complete Elastic Solid Theory contains this acceleration. See *Enc. Brit.*, art. "Wave Motion," p. 453.

‡ Wiedemann's *Annalen*, January 1889; also 'Nature,' February 21, 1889.

This at once points to an acceleration of phase of $\frac{1}{4}$ period at distances remote from the "vibrator."

The rate at which the magnetic component of the disturbance is propagated can be determined by taking a definite position on the wave and finding how v varies with respect to t on travelling out with the wave in this phase. If $P=0$, then

$$\tan (mr-nt)=-1/mr.$$

Here m stands for $2\pi/\lambda$, and n for $2\pi/\tau$. From this we get for the velocity of the wave,

$$v=\frac{dr}{dt}=V\frac{m^2r^2+1}{m^2r^2},$$

where V is the normal velocity, or that at a great distance from the "vibrator." When $r=0$, v has an infinite value, but on going outwards rapidly approximates to V . Hertz has pointed out that the true interpretation of this great value for the velocity at points near the origin involves the idea of the energy radiated really coming from points in the surrounding medium, the true seat of the oscillations.

Thus, from the consideration of velocity an acceleration in phase is observed to occur. To find its amount, the time the disturbance takes to reach a point at distance $N\lambda$ from the origin can be easily found and compared with the normal time. Thus

$$\int_0^{N\lambda} \frac{dr}{v} = \frac{1}{V} \left\{ N\lambda + \frac{\lambda}{2\pi} \tan^{-1} \frac{1}{2\pi N} - \frac{\lambda}{4} \right\}.$$

When N is a large number this approximates to the value $N\tau-\tau/4$, and is an acceleration corresponding to one quarter period.

If the position of maximum or minimum value of P (i. e. $\frac{dP}{dt}=0$ *) be taken on the wave instead of the zero value,

we shall have $\tan (mr-nt)=mr$, and the same expression for the velocity is found. The circular electric-line of force in Hertz's diagrams of the disturbance, here reproduced for convenience of reference, also spreads out with this velocity.

* It is to be particularly observed that in the neighbourhood of the conductor, unlike in normal wave motion, the position on the wave where $\frac{dP}{dr}=0$ is not the position on the wave of maximum displacement; this state of the wave will be found like the electric force to originate at $\lambda/4$, having a very similar expression for the rate of its propagation. On plotting the curve P at a series of stated epochs this is well shown.

The electric component of the wave*

$$z = \frac{a}{r} \left\{ -\sin(mr - nt) - \frac{\cos(mr - nt)}{mr} + \frac{\sin(mr - nt)}{m^2 r^2} \right\}$$

may in like manner be considered. Similarly to before, if $z=0$, it is necessary everywhere along the line considered that

$$\tan(mr - nt) = \frac{mr}{1 - m^2 r^2}.$$

From this we get for the velocity,

$$v = V \frac{m^4 r^4 - m^2 r^2 + 1}{m^2 r^2 (m^2 r^2 - 2)}.$$

In this case v has an infinite value both at the centre and at distance $\lambda/\pi\sqrt{2}$ —roughly $\lambda/4.4$. At intermediate points it is negative or towards the centre. As the wave passes outwards it is at this point $\lambda/4.4$ that the zero value of z is first reached (at about $.12\tau$ before it becomes zero at the centre, which occurs at $\tau/2$), whence it spreads both outwards and inwards. On the diagrams this appearance of the zero value of z is easily traced, and its subsequent outward movement at the centre of the small circles; these of course are the cross sections of a vortex-ring, as it were, thrown off by the “vibrator.”

For determining the time to reach any point, we have

$$\int \frac{dr}{v} = \frac{1}{V} \left\{ r - \frac{1}{m} \tan^{-1} \frac{mr}{1 - m^2 r^2} + \text{const.} \right\}.$$

This between $N\lambda$ and $\lambda/4.4$, when N is a large number, gives as the time taken,

$$\tau(N - 1/4.4) - \tau/2\pi \cdot \tan^{-1} \sqrt{2},$$

the first part being the normal time.

If we wish from this to determine the period elapsing between the arrival of the wave in this phase at the point $N\lambda$ and the time of zero value of the electric force at the centre which occurs at the epoch of fig. 1, Plate VI., we must add the epoch of the appearance at $\lambda/4.4$. This will be found to afford NT , the normal time. But if we reckon from the time the zero value, having appeared at $\lambda/4.4$, travelling inwards reaches the centre, we obtain $NT - \tau/2$, an acceleration of one half-period. In neither case, it is to be observed, does it represent the time of passage between two places. The zero value of z at the centre in fig. 1 ought really to be referred

* Wiedemann's *Annalen* for 1889.

to the previous swing. This will be better understood by recollecting that at the conductor, when the electric force is at its greatest development, the current is zero, and consequently so, too, the magnetic force, while in radiation the electric force and the magnetic force simultaneously reach their maximum value. This is beautifully shown in Hertz's diagrams. In fig. 1, at the centre the electric force is zero, while the magnetic force is about at its greatest development. The latter travels outwards with such a velocity at each point (the same as that of the circular line of force in the diagrams) that there is an acceleration of $\frac{1}{4}$ period; while the former vanishes at the centre but reappears at $\lambda/4$, as is seen in fig. 4, so as to be now one quarter wave-length behind. In this way the necessary readjustment of the relative phases of the electric and magnetic components is effected.

If the position on the wave of maximum or minimum value of z (that is to say, the "crest") be taken instead of the zero value, the condition at any point is that

$$\tan (mr - nt) = \frac{m^2 r^2 - 1}{mr},$$

and the same expression for the velocity as before is obtained, so that the position at distance $\lambda/4$ may be considered the point from which the electric disturbance originates, and not the centre. As the energy in the neighbourhood of the conductor turns from its magnetic to its electric form*, it is at this point that the maximum value is first reached, and is in advance of the "normal epoch" $\tau/4$, or that of the "crest" at the centre by about $\frac{1}{8}\tau$. The maximum value spreads both outwards and inwards from $\lambda/4$, reaching the centre at $\tau/4$, the epoch of fig. 3.

As in the case of the zero value of z , so here also, if the period elapsing between the arrival at any point and the time of greatest electric development at the centre, which occurs at the epoch of fig. 3, be calculated, an acceleration of $\tau/2$ on the normal time is found. This virtual acceleration of $\tau/2$ is, however, due to the combined effect of the "crest" or maximum value of z , (1) starting from the point $\lambda/4$ instead of from the centre, (2) at a time about $\frac{1}{8}\tau$ previous to the normal epoch $\tau/4$ or that at the centre, (3) as well as to the

* In the curves obtained by plotting z for a series of epochs the formation of the wave at $\lambda/4$ is clearly observed. The wave after springing here into existence above the zero-line, lengthens out in both directions, and then beginning to sag in the centre, finally splits into two at this point, as the force reaches zero, again to pass over to the other side of the zero-line.

increased velocity. As mentioned before, the acceleration of the rate of propagation of the electric component of the disturbance must differ by $\tau/4$ from that of the magnetic component in order to provide the necessary readjustment of their relative phases.

The question now arises if, on the whole, the elementary disturbance to be supposed in Huygens' construction* may not fairly be considered as being of the same character as that produced by a Hertzian "vibrator." Such an elementary disturbance as is there assumed, in the absence of neighbouring ones, would have apparently uncompensated ends or poles equivalent to the opposite electrifications on a "vibrator." Of course the only way we know of producing an elementary disturbance is by the presence of matter, so that it seems natural to take the disturbance assumed in Huygens' construction to be similar in its effects. It might perhaps be objected to this that a "vibrator" is a conductor, and that there are conduction-currents to be dealt with; but as against this the fact is to be considered that, as we shall see, a non-conductor such as glass can act by reflexion so as to be the source of such an elementary disturbance.

Some experiments were described in 'Nature' (August 22, 1889) in which this acceleration in phase was actually observed to take place in the reflexion from a small surface. It was there described how, in Hertz's experiment of "loop and nodes," if a *small* sized reflector be employed, the magnetic node, instead of being at $\frac{1}{4}\lambda$ from the reflector, is found to be nearly $\frac{1}{8}\lambda$ further out. This evidently corresponds to a change of phase of $\frac{1}{4}$ period in the reflected wave, for the distance $\frac{1}{8}\lambda$ has to be twice traversed.

The way in which this change in phase is brought about is perhaps most conveniently considered by supposing the small reflector to be itself resonant, or equivalent to a second vibrator having the requisite phase with reference to the incident wave. And indeed that something such as this occurs may even be noticed by touching the mirror with a small piece of metal; for bright sparks can be drawn from the two edges running at right angles to the electric force, while little or no effect is obtained from the central parts.

Let the magnetic component of the wave incident vertically on the reflector, and supposed a parallel beam, be taken as

$$P = A \cos 2\pi \left(\frac{t}{\tau} + \frac{r}{\lambda} \right);$$

* In the case of sound, the divergency of the direction of motion of the particles from an element of surface affords the analogous reason for the acceleration.

where r is the distance to the reflector, and A is the amplitude of the magnetic force. The magnetic component of the reflected wave, supposed as above to be from a second "vibrator," will be given by *

$$P' = \frac{4\pi^2 E l}{\lambda^2 r} \left\{ \sin 2\pi \left(\frac{r'}{\lambda} - \frac{t}{\tau} \right) + \frac{\lambda}{2\pi r} \cos 2\pi \left(\frac{r}{\lambda} - \frac{t}{\tau} \right) \right\},$$

as this has the same phase at the reflector as the incident beam. Here $\pm E$ is the maximum value of the electrification induced by the incident wave on the second "vibrator," and l is the equivalent distance apart of the positive and negative charges, which, taken roughly, is the length in the direction of the electric displacement of the reflector supposed rectangular. An approximation for E may be easily obtained from the value for the induced current arrived at, by assuming that the magnetic force parallel to the reflector due to the induced current is such as to neutralize the effect of the direct radiation immediately behind the reflector†. If c is the amount of the current per centimetre cross section in the reflector, the magnetic force due thereto parallel to the reflector and close up to it is $2\pi c$; that is to say, it is assumed equal to the force due to an infinite current-sheet of intensity c per centimetre cross section. (See Maxwell, chap. xii.) This is to be put equal to the magnetic force of the direct radiation at the reflector $A \cos 2\pi \frac{t}{\tau}$. Recollecting that c is in electromagnetic units, we have for the amplitude of the total charge along the edge k of the reflector,

$$E = kV \int_0^{\tau/4} c \, dt = kV \int_0^{\tau/4} \frac{A}{2\pi} \cos 2\pi \frac{t}{\tau} \, dt = \frac{Ak\lambda}{4\pi^2}.$$

This neglects the end irregularities in the electrification.

At any point in front of the reflector the disturbance is the sum of P and P' , and we have a series of stationary waves, complicated, however, by the fact that the velocity and the amplitude of one of the components are functions of the distance.

$$P + P' = B \cos 2\pi \left(\frac{t}{\tau} + \alpha \right)$$

represents the stationary waves when

$$B^2 = A^2 \left\{ 1 + 2 \cdot \frac{kl}{r\lambda} \left(\sin 4\pi \frac{r}{\lambda} + \frac{\lambda}{2\pi r} \cos 4\pi \frac{r}{\lambda} \right) + \left(\frac{kl}{r\lambda} \right)^2 \left[1 + \left(\frac{\lambda}{2\pi r} \right)^2 \right] \right\}.$$

* Wiedemann's *Annalen*, January 1889.

† This, as a method of evaluating the electrification, was pointed out to the author by Prof. Fitzgerald.

Practically the quickest method for determining the positions of maxima and minima for B is to plot the curve represented by this equation in each particular case. The curve shown in fig. 5, Pl. VI. is that where $k=30c$, $l=22c$, and $\lambda=72c$. The abscissa represents the distance, measured in centimetres, from the reflector to the point, while the ordinate multiplied by A^2 represents the intensity or square of the amplitude of the magnetic force of the stationary wave at the point. The dotted line indicates the intensity of the direct radiation, to which the curve is seen to approximate as r increases in value. The curve is seen to be going off the limits of the paper on the side next the reflector; but it must be remembered that in this region the formulæ used have but an indirect interpretation as the finite size of the reflector becomes important, and that there is involved the idea of the æther here being in special commotion, so that reflexion may in a certain sense be looked upon as taking place before really reaching the mirror.

On examining the curve the first minimum will be seen to occur at about $24c$ from the reflector, which agrees sufficiently well for an approximate method of calculation with the result of experiment, which for a wave-length of $72c$ gives the first minimum at about $25c$.* And in fact we might expect, on looking at the curve, that the experiment as made would always tend to give the minimum a little beyond its true position; for if the spark-gap be set to just observe a spark of, say, the direct waves' intensity, and if the distance be bisected between the positions to right and left of the minimum where sparking is observed to commence on moving along the "receiver," this point will be too far out, owing to the portion of the curve to the left being steeper than that to the right. This is well marked when working with the spark-gap comparatively open; that is to say, various positions are then obtained for the minimum according to the width of the spark-gap. This was experimentally noticed before the curve was plotted.

The curve indicates at once the reason why it is difficult to

* In this Table the results of experiments with different wave-lengths are shown in the third column.

λ .	$\frac{2}{3}\lambda$.	Min.
72	27	25
68	25.5	24
60	22.5	21
56	21	19

observe even the second minimum. It will also be noticed from the curve that the second minimum is at a distance from the first slightly greater than the half wave-length; so that if the curve were continued further out we might very well expect the minima to be shifted out the complete $\frac{1}{2}\lambda$, as in fact may be seen to be the case from the equation of the curve. This in ordinary wave theory would be referred to the necessity for the point to be at a proportionate distance before a given surface can be considered as truly sending out a *secondary* wave. Some experiments were made which certainly tended towards showing that this lengthening of the distance between the first and second minimum exists; but the experiments were unsatisfactory, for the second minimum is very slightly marked, and experimentally indeed seems to be even less so than one would expect from the curve. However, it is most likely that the Hertzian stationary waves are never so well marked as they would ordinarily be calculated; for it seems most probable that primary "vibrators" do not send out strictly "monochromatic light," but send out a number of wave-lengths of nearly the same period—a "band-spectrum," so to speak, the centre of which no doubt may be by far the strongest, and correspond with the "period" as would be calculated for the "vibrator;" also that "secondary vibrators" or resonators may in like manner be forced so as to take up, in some degree depending on the discrepancy, any member of a "band-spectrum" special to each resonator, its "period" corresponding to the centre*. Thus the marking of a node belonging to the central wave-length will probably be weakened, owing to the presence of the other members of the "band" whose nodes on the whole will occur elsewhere.

An observation which has proved of use in the course of these experiments on secondary waves was made during the autumn, namely that glass absorbed Hertzian radiations comparatively rapidly; in fact on account of this property it was found quite impracticable to determine the velocity of "light" in glass by the method† of interposing a sheet of the substance in front of a reflector affording stationary waves, because the front surface of the glass itself gave a well-marked series of loops and nodes. A comparatively thin piece of glass, say of one centimetre, will afford an observable reflexion. This must be due chiefly to the reflexion from the second surface being weakened by absorption, so that it is insufficient to

* The author hopes shortly to publish an account of experiments made during the autumn which have led to these conclusions.

† 'Nature,' August 22, 1889.

seriously interfere with that from the front, but also no doubt to reflexions from points situated in the substance of the glass. A sheet of glass two centimetres thick gives fairly good reflexion, while a sheet of paraffin of that thickness would give almost no effect.

Thus by using glass it was comparatively easy to obtain stationary waves by reflexion from a small surface of a non-conducting substance, in order to compare the effect thus produced on the position of the stationary waves with that produced by employing a small metallic reflector. The first node was found to be shifted out nearly $\frac{1}{2}\lambda$, as in the case of metallic reflexion. That reflexion from glass is not of the metallic sort was proved by obtaining polarized reflexion. In this case the two opposite edges of the nonconductor may be looked upon as undergoing variations of apparent electrification.

XXIX. *On the Time-integral of a Transient Electromagnetically Induced Current.* By Sir WILLIAM THOMSON, F.R.S.*

IT has hitherto been generally supposed that, in ordinary apparatus for electromagnetic induction, with or without soft iron, the oppositely directed transient currents, in the secondary circuit, induced by startings and stoppings of current in the primary circuit, have equal time-integrals.

I have recently perceived [been wrongly led to imagine] that this may be far from being practically the case by the following considerations. The starting and stopping of the current in the primary circuit was, in Faraday's original discovery of this kind of electromagnetic-induction (Exp. Res. Series I. Nov. 1831), and is generally in elementary illustrative experiments, produced by making and breaking a circuit consisting of a voltaic element or battery and the inductor-wire. In this arrangement the starting of the inductor-current is generally much less sudden than the stopping. Hence a thicker shell of the secondary wire (or portion inwards from the outward boundary) is utilized for conducting the secondary current, on the make, than on the break†. Hence the effective ohmic resistance in

* Communicated by the Author.

† [In reality the whole cross sectional area of the secondary conductor is utilized, equally in all its parts, in conducting the secondary current. See Postscript of February 23.]