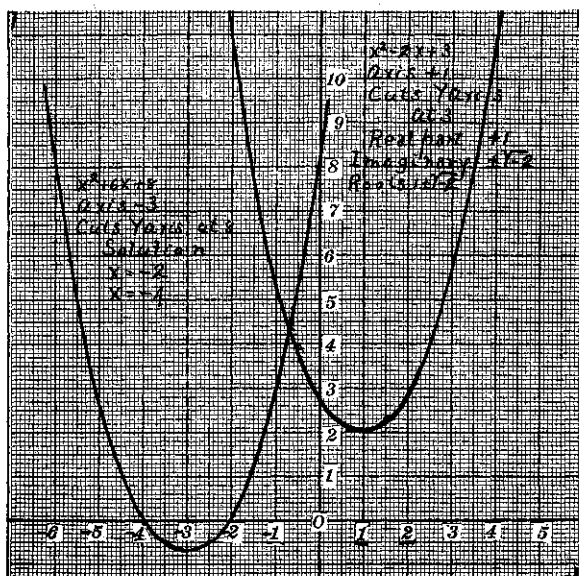


SOME GRAPHICAL METHODS.

BY ROBERT C. COLWELL,

Geneva College, Beaver Falls, Pa.

The general equation of the quadratic written in the form $y = x^2 + bx + c$ (1) represents the parabola $y = x^2$ moved to a certain position on the axes of reference. The position of the axis of the parabola is shown if equation (1) is written $y = \left(x + \frac{b}{2}\right)^2 - \frac{b^2}{4} + c$ (2). From analytics $-\frac{b}{2}$ is the abscissa of the axis of the parabola. Also, from (1) the parabola cuts the y -axis at the point $y = c$, that is, where $x = 0$. Any quadratic in form (1) is solved by moving the parabola $y = x^2$ as indicated. If the roots are real, the parabola cuts the x -axis in two points which give the required roots. If the roots are complex, the real part is given by the abscissa of the axis, and the square root of the ordinate of the vertex of the parabola (in this case measured downward and therefore negative) gives the imaginary part of the root. (See Fig. 1.)



Example I. Solve: $x^2 + 6x + 8 = 0$.

Let $y = x^2 + 6x + 8$.

One-half the coefficient of x with the sign changed is -3 ; hence the axis of the parabola lies along the line $x = -3$. When

$x = 0$, $y = 8$; therefore the curve cuts the y -axis where $y = 8$. Move the parabola $y = x^2$ until its axis lies along the line $x = -3$ and the parabola itself cuts the y -axis at the point $y = 8$; then read the solutions $x = -2$, $x = -4$. (See Fig. 1.)

Example II. Solve $x^2 - 2x + 3 = 0$.

The axis of the parabola lies along the line $x = 1$ and the parabola cuts the y -axis where $y = 3$. When placed in this position, the parabola does not cut the x -axis; therefore the roots are complex. The real part is given by the abscissa of the axis of the parabola (in this case $+1$) and the imaginary part is found by taking the square root of the ordinate of the vertex of the parabola ($\pm\sqrt{-2}$). The complete roots are $+1 \pm \sqrt{-2}$.

The formula for the solution of $x^2 + bx + c = 0$ is usually written $\frac{-b \pm \sqrt{b^2 - 4c}}{2}$ but it may be put in the form

$$-\frac{b}{2} \pm \sqrt{\frac{b^2}{4} - c}.$$

If we take a circle at the origin whose radius is \sqrt{c} (see Fig. 2), lay off an abscissa $= -\frac{b}{2}$ and draw the ordinate PR, then from a well-known proposition in geometry

$$\overline{PR}^2 = MR \cdot RC.$$

But $MR = \sqrt{c} + \frac{b}{2}$

$$RC = \sqrt{c} - \frac{b}{2}$$

$$\therefore PR = \sqrt{c - \frac{b^2}{4}}$$

$$\therefore PR\sqrt{-1} = \sqrt{\frac{b^2}{4} - c}.$$

That is, the ordinate multiplied by $\sqrt{-1}$ represents the imaginary part of the root.

Again in the equilateral hyperbola,

$$x^2 - y^2 = c \quad (3),$$

when the abscissa is $-\frac{b}{2}$ the ordinate is $\sqrt{\frac{b^2}{4} - c}$. This comes at once from substitution in (3)

$$x^2 - y^2 = c,$$

$$\frac{b^2}{4} - y^2 = c,$$

