and oxygen is further oxidised, with separation of yellow sulphur, yielding (2) hyposulphite of trimethyl-sulphine.

The action is thus similar to that of sulphuretted hydrogen on potash or carbonate of potash, but takes place with much greater rapidity.

The examination of the sulphide and polysulphide of trimethyl-sulphine will form the subject of the next part of the paper.

2. On Links. By Professor Tait.

(*Abstract.*)

Though in my former papers on knots I have made but little allusion to cases in which two or more closed curves are linked together, the method I have employed is easily and directly applicable to them. I stated to the British Association that the number of intersections passed through in going continuously along a curve, from any intersection to the same again, is always even—whether it be linked with other curves or not Hence, even when a number of closed curves are linked together, the intersections may be so arranged as to be alternately over and under along each of the curves.

When this is done, each of the meshes has all its angles right or left handed ; so that Listing's type-symbols may be employed, just as for a single knotted curve. The scheme, however, consists of as many parts as there are intersecting curves—each part containing, along with each of its own crossings *twice*, each of its intersections with other curves *once*.

Thus A B | A A B | A

or

$$\left.\begin{matrix} 2r^2 \\ 2l^2 \end{matrix}\right\}$$

represents a couple of ovals linked together.

When three ovals are joined, so as to form an endless chain, we have

A B C D | A D C E F | D F E B A | F

or

$$\left.\begin{matrix} 2r^3 + 3r^2 \\ 3l_4 \end{matrix}\right\}$$

Of course such figures can be transformed or deformed according to the methods given in my first paper—the scheme and the type-symbol alike remaining unaltered. And alterations of both scheme and symbol are, in various classes of cases, producible by the processes of my last paper without any change of links or linking.

The genesis of the scheme of a link may be most easily studied by forming a knot into a link. This is done by cutting both turns of the wire at any junction, and joining them again so as to make two closed curves instead of one. No intersections are lost by this process, except that which was cut across, provided, of course, that the original knot had no nugatory intersections, and that none are rendered nugatory by the operation of cutting the whole across.

Any crossing with four adjacent crossings when the turns of the coil pass alternately over and under one another will appear in a scheme as follows :—

$$\begin{array}{ccccccccccccccc} . & . & . & . & A & X & B & . & . & . & . & . & C & X & D & . & . & . & . \\ & & & & - & + & - & & & & & & + & - & + & & & & \end{array}$$

implying that from X through B and C back to X forms one continuous circuit; similarly from X through D and A back to X.

There are but two ways in which continuity can be kept up if we cut the cord twice at X, and reunite the ends in a different arrangement from the original one.

It is obvious that if we pass from C to B, by way of X (abolished), and similarly with the rest, we divide the continuous closed curve into two separate (but generally inter-linked) closed curves. If we pass from A, by way of X (abolished) to C, we pass thence in time to B, and finally by way of D to A. Thus the curve remains continuous, but with one intersection less than at first. And, in either case, the alternate order of the signs of the crossings will be maintained throughout.

In the former of these modes we take the part containing C and B (and we may, if we please, also take the rest) in the same order as before the change. The scheme is therefore, without any other change, simply divided into two parts, which are separated from one another by the (abolished) junction X in its two positions.

In the second mode, it is obvious that the letters in one of the two parts separated from one another by the mark X in its two places are simply to be inverted in order without change.

The process presents no difficulties, so that I shall give only two simple examples. Thus the scheme of the pentacle, viz. :—

$$ADBECADBEC \mid A$$

is divided at A (in this case it does not matter which junction we take) into the two superposed non-autotomic ovals

$$DBEC \mid D, \qquad DBEC \mid D,$$

by the first mode :—, and is simplified into

$$DBECCEBD \mid D$$

(*i.e.*, a wholly nugatory scheme) by the second.

The type-symbols in the original state, and in the two altered states, are, respectively,

$$\left.\begin{matrix} 2r^5 \\ 5l^2 \end{matrix}\right\}$$

$$\left.\begin{matrix} 2r^4 \\ 4l^2 \end{matrix}\right\}$$

$$\left.\begin{matrix} r^3 \\ 3l^2 + 2l \end{matrix}\right\}$$

The last of these is virtually nothing. In fact, terms in r or l to the first power are rejected by Listing. And, when these loops are taken off by untwisting or by opening up, the scheme becomes

$$\left.\begin{matrix} r^4 \\ l^2 + 2l \end{matrix}\right\}$$

and a second application of the process removes the whole.

Operating in a similar way upon the only other figure with five non-nugatory intersections—viz. :—

$$A_4D_4B_2E_2C_2A_4D_4C_2E_2B_2 \mid A$$

or

$$\left.\begin{matrix} 2r^4 + r^2 \\ 2l^3 + l^2 \end{matrix}\right\}$$

we find three classes of cases, according to the particular intersection operated on.

[I may here introduce, though it involves a slight digression, a method which I have found very convenient as an assistance in finding which intersections have similar properties as regards the figures which will be obtained when they are made in turn the point of section. In the scheme above written the suffixes express the numbers of letters which intervene, in the scheme, between the two appearances of the same letter. If n be the whole number of letters, the suffix may of course be either $2r$ or $2n-2r-2$. It is convenient to write always that one of these two numbers which is not greater than the other. When a particular suffix occurs only once, the corresponding crossing has evidently different properties from the others; if twice, we find in general that the corresponding crossings have similar properties. If three times, two of them have usually like properties, but the third not—and so on. This method is useful, but it is in certain cases misleading. In fact, we must look not only at the suffix itself, but at the place which it occupies relatively to the whole group of suffixes, in order to obtain absolutely definite information. Something similar to this is obviously hinted at in Listing's paper, where he seems to determine the number of possible transformations of the figure representing a symbol, by treating the numerical coefficients much as I have here treated the suffixes. But this is mere conjecture on my part.]

By this method then, or by examining the diagram, we see that A and D are similar, so are B and C, while E may possibly possess distinct properties of its own. We need, therefore, take only three cases, A, B, and E.

a.) Divide at A. Then we have either

$$\mathrm{D\,B\,E\,C} \mid \mathrm{D} \quad \mathrm{D\,C\,E\,B} \mid \mathrm{D}$$

$$\left.\begin{matrix} 2r^4 \\ 4l^2 \end{matrix}\right\}$$

two ovals crossing one another, one taken right-handed, the other left; or

$$\mathrm{D\,B\,E\,C\,B\,E\,C\,D} \mid \mathrm{D} = \mathrm{B\,E\,C\,B\,E\,C} \mid \mathrm{B}$$

$$\left.\begin{matrix} 2r^3 \\ 3l^2 \end{matrix}\right\}$$

the trefoil knot; for D becomes nugatory.

b.) Divide at B. We have either

$$\mathrm{A\,D} \mid \mathrm{A} \qquad \mathrm{E\,C\,A\,D\,C\,E} \mid \mathrm{E} = \mathrm{D\,A} \mid \mathrm{D}$$

$$\left.\begin{matrix} 2r^2 \\ 2l^2 \end{matrix}\right\}$$

two linked ovals, C and E having become nugatory; or

E C A D C E D A | E

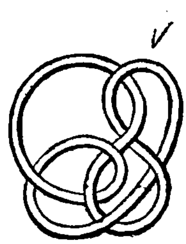
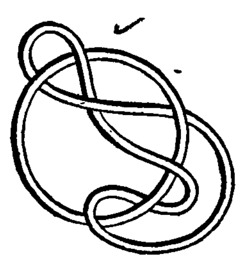
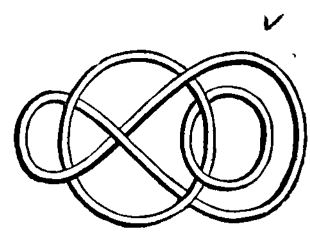
$$\left.\begin{matrix} 2r^3+r^2 \\ 2l^3+\ l^2 \end{matrix}\right\}$$

an amphicheiral knot, the only knot with 4 intersections.

c.) Dividing at E we find the same results as for B and C.

From the rules just given for removing an intersection, it is of course easy to pass to those required for the introduction of a new intersection.

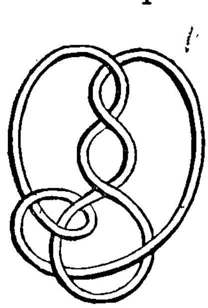
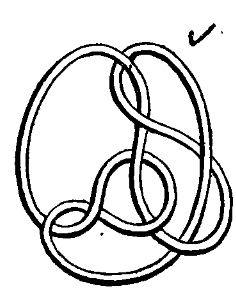
In endeavouring to frame a general method of determining whether a particular type-symbol necessarily denotes one continuous curve, or a superposition of two or more curves, I was completely unsuccessful. But, as indicated in a note to my last paper, I found the reason to be that *no such distinction necessarily exists.* And by the application of the methods of adding or removing intersections given, I found a number of instances in which the same type-symbol may represent many entirely different kinds of figures. Thus the following

are all represented alike by the symbol

$$\left.\begin{matrix} r^5+r^4+r^3+r^2 \\ l^4+2l^3+2l^2 \end{matrix}\right\}$$

But I have since succeeded in obtaining cases in which the same type-symbol represents two perfectly distinct single closed curves. One instructive example is the following

The common type-symbol is

$$\left.\begin{array}{l} r^5+r^4+r^3+2r^2 \\ l^5+l^4+l^3+2l^2 \end{array}\right\}$$

But the schemes are

$$A_6E_6B_4G_6C_6H_6D_6B_4E_6A_6F_2C_6G_6F_2H_6D_6 \mid A$$

and

$$A_6D_4B_2H_4C_6F_4D_4A_6E_4G_2F_4C_6G_2E_4A_4B_2 \mid A$$

Now no change in lettering can affect the suffixes, so that the two schemes are essentially different. In fact the sum of the suffixes is 84 in the first scheme, but only 64 in the second. The first has only one degree of beknottedness, the second has two. The first is not amphicheiral, the second is.

There is no connection between the type-symbol, as Listing gives it, and the singleness or complexity of the curve represented, but it is possible to make analogous symbols capable of expressing everything of this kind. Only we must now adopt something very much resembling Crum Brown's Graphical Formulæ for chemical composition. Some very remarkable relations follow from this process, but I can only allude to a few of the simpler of them in this abstract.

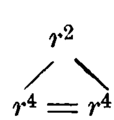
The only necessary relations among the numbers forming the right or left part of a symbol are satisfied if no one is greater than the sum of the others, and if the sum of all is even. With any set of numbers subject to these conditions, we can form the right or left-hand side of a symbol—and from that we can form the other when we know the grouping.

An example or two will make this clear. Take, for instance, the symbol

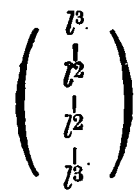
$$\left.\begin{array}{l} 2r^4+r^2 \\ 2l^3+l^2 \end{array}\right\}$$

which represents the five-crossing knot of p. 242 above.

A glance at the figure shows that the following is the arrangement of the right-handed meshes.

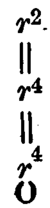


the single mesh with two corners having one of these corners in common with each of the two four-sided meshes, which again have three corners in common. Hence in this notation *the joining lines represent the crossings.* Hence also the characters of the left-hand meshes are obvious from the figure. Outer space has the three external lines for corners—inside there is one triangle and two spaces bounded by two lines each (*i.e.*, with two corners). Thus we reproduce the left-hand part of Listing's symbol. But the figure also shows us which lines (corners) each pair of these has in common, and enables us at once to draw the annexed figure



which gives us exactly the same information as the first, only from a different point of view.

The connections in the former figure cannot be varied, so that, in this particular case, Listing's symbol for the right-handed meshes alone suffices to draw the figure; at least if nugatory crossings be rejected. Such would arise, for instance, if we tried to draw the symbol in the form

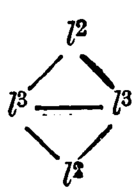


which would give three ovals joined like the links of a chain—the last having an internal nugatory loop. In this case the second part of the symbol would be

$$l^5 + 2l^2 + l$$

where the nugatory character of one intersection is clearly exhibited.

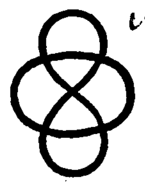
But, if we had merely the left-hand part of the symbol given us, we might adjust it thus



which would correspond to

$$r^4 + 2r^3$$

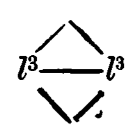
for the right-handed part, and would give us the form



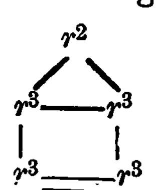
or one of its deformations.

The criterion by which to distinguish at once whether such symbolic representations as those just given represent knots or links is easy to find. If we remember that each of the (even number of) crossings lying on a closed curve is a corner of one black and of one white mesh (contained within the curve)—while each of the crossings lying within it is a corner of each of two white and of two black meshes—we see that unless we can enclose a part of the graphic symbol in such a way that the sum of the exponents within the enclosure, and that formed by the doubling of the number of the joining lines which are wholly within the enclosure, and adding it to the number of those which cut the boundary, are *equal even numbers*—the figure is necessarily a knot. But if we can enclose such a part, it requires to be farther examined to test whether the figure consists of links or is a single knot.

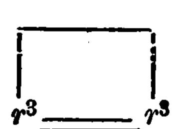
Thus, in the example just given, the part



is a simple oval divided by two intersecting chords into three-cornered meshes—but in the following formula



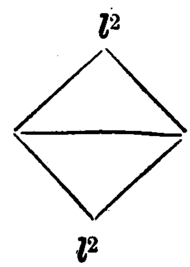
although the par



seems to fulfil the conditions above, it does not represent a separate closed curve. In fact, the upper line represents a crossing *on the boundary*, at which there is (internally) only a left-handed mesh, which is impossible if the boundary were a closed curve.

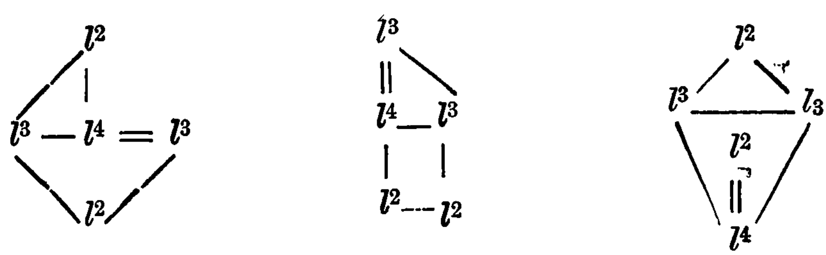
And the lowest line in the figure is a point in the boundary which forms a common vertex of *three* (internal) meshes, two right and one left-handed. This, also, is inconsistent with the boundary's being a closed curve.

There is only one other case which may cause a little trouble. It can easily be seen by the fig. of last page. For we may take out the following part of the symbol

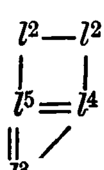


which must obviously represent the lemniscate in the figure. Its exponents and lines do not satisfy our condition: but they will do so if we remove the diagonal line—which corresponds to what is (in the lemniscate when alone) a nugatory intersection.

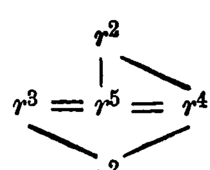
I conclude by giving the representations, according to the method just explained, of some of the preceding figures. Thus the three first figs. of p. 325 are, respectively,



while the pair of common-symbol knots on the same page are



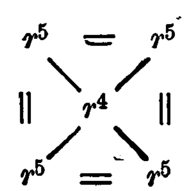
and

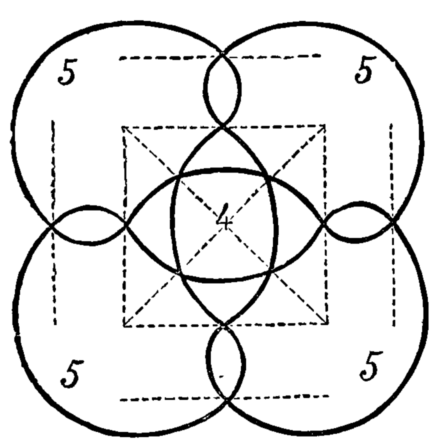


It may be observed that the present method gives great facilities for the study of cases in which knots are reduced, or are changed into links, by the removal of an intersection. For, to take off an intersection is easily seen to be equivalent simply to rubbing out one connecting line in the figure, and simultaneously diminishing by unity each of the exponents at its ends. If it be the only line connecting these exponents, they are (after reduction by unit each), to be added together. And this consideration enables us to obtain, even more simply than before, the rules for distinguishing a knot from a link. I propose, when I have sufficient leisure, to re-investigate the whole subject from this point of view.

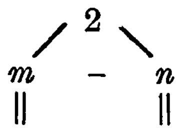
Meanwhile I may notice that it is exceedingly easy to draw the outline of any knot or link by this method. All that is necessary is to select a point in each of the lines in the figure, and join (two and two) all these points which are in the boundary of each closed area. The four lines which will thus be drawn to each of the chosen points must be treated as pairs of continuous lines *intersecting* at these points, and at these only. When there are only two sides—and, therefore, only two points—in an area, two separate lines must be drawn between them, and these must *cross* one another at each of the two points.

The annexed diagram shows the result of this process as applied to the following symbol

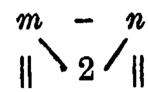




This method also clears up in a remarkable manner the whole subject of change of scheme of a given knotting which was dis cussed in my last paper. To give only a very simple instance, notice that the first of the changes there mentioned is merely that from



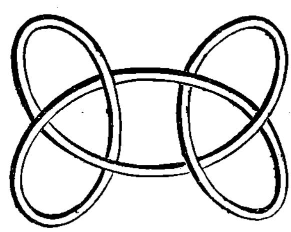
to



where the double lines may stand for any numbers of connection whatever.

I conclude by stating, in illustration of the remarks made at the end of my last paper, that I have hastily (though I hope correctly) investigated the nature of all the valid combinations among 720 which are possible in the even places of a scheme corresponding to 6 intersections (only 80 of these are not obviously nugatory) —and that I find *only four really distinct forms.* They are

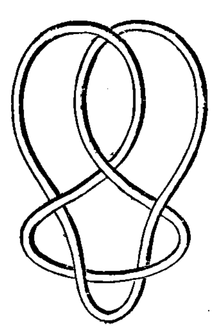
1. Two separate trefoil knots. Here there are two degrees of beknottedness.



2. The amphicheiral form. (Figured on p. 295 of my *Note on Beknottedness.* Also in a clear form in the last cut of my first paper.)

3. Fig. p. 297 of the same paper. These two forms are essentially made up of a trefoil knot and a loop intersecting it.

4. The following knot, which belongs to a species found with every possible number of crossings from 3 upwards. This species furnishes the unique knots with 3 and with 4 crossings, and one of the only two kinds possible with 5.



Its symbol is always of the form

$$\left.\begin{array}{l}2r^3+(n-3)r^2\\ 2l^{n-1}+l^2\end{array}\right\}\text{ or more fully } (n-1)\overset{2}{\equiv}(n-1),$$

there being n—2 lines in the lower group.

The three last forms have each essentially only one degree of beknottedness. In certain cases (see the foot note *ante* p. 296) we may give two degrees of beknottedness by altering some of the signs —but the knot has then one nugatory intersection, and falls into the class with five crossings.

A number of curious problems are suggested by the process which I employed in the investigation of these six-crossing forms. I give the following as an instance.

Take any arrangement whatever of the first n letters:—Say, for instance,

C N D A . . . L E.

change each to the next in (cyclical order, so that A becomes B, B becomes C, ... , N becomes A) and bring the last of the row to the beginning. The result is

F D A E B . . . . M.

After performing this operation n times we obviously get back the arrangement from which we started. [Thus in seeking all the different forms of knots of a given number of crossings, *one alone* of this set of n need be kept.] The problem is to find sets such that the original combination is repeated after m operations like that above. It is obvious that if m is to be less than n it must be an aliquot part of it, and thus n must be a composite number.

[*April* 11.—The references to Listing's type-symbol here given must be taken in connection with the extracts from his letter, *ante*, p. 316.]

3. Laboratory Notes. By Professor Tait.

(*a*.) Measurement of the Potential, required to produce Sparks of various lengths, in Air at different pressures, by a Holtz machine. By Messrs Macfarlane and Paton.

The general result of these strictly preliminary experiments appears to show that for sparks not exceeding a decimetre in length