REVIEWS.

Clive's Mathematical Tables. (University Tutorial Series.) **Four Figure Tables and Constants.** By W. Hall. (Cambridge University Press.)

In the first of these a claim is made in the preface that the tables given have been specially designed to combine speed with accuracy of calculation, and on examination they are found to justify it. Each separate table is printed on two facing pages, and a high degree of accuracy is obtained by the following methods:

(1) The tabulated functions are given to 5 significant figures.

(2) In places where the mean differences change too rapidly for them to be given for a whole line of entries they are given for each half line, and the blank spaces of the entry columns are utilised by the insertion of a few figures indicating the amount of error in the mean difference given.

(3) In places where no reliable mean differences can be given the degree of accuracy to be obtained by ordinary interpolation is indicated.

Thus the cosecant of 49° 27′ is found by adding to the entry for 49° 24′ (1·53663) the 'tabular difference' 155 and the 'correction' 2 obtained by multiplying the small correction supplied for 1 mi. just below the entry for 49° 24′ by the number of extra minutes for which the difference is required. We thus obtain 1·53820, which is the value given in Lodge's Appendix to Bremiker's *Logarithms*. We have checked various other entries by Bremiker. Careful instructions with fully worked examples are given, and sufficient theory for the students to understand the principles on which logarithm work is based.

Mr. Hall's work is intended for nautical as well as for ordinary technical and physical calculations, and therefore contains some tables which are not to be found in ordinary sets. For instance, the 'Haversine' (half versed sine) and its logarithm are given to 4 places at intervals of 6′, and examples are given of the use of this function in calculations, especially in those of Spherical Trigonometry,

$$e.g. \text{ since } \operatorname{Hav}\theta = \tfrac{1}{2}(1-\cos\theta) = \sin^2\frac{\theta}{2},$$

$$\therefore \text{ in Pl. Trig. } \operatorname{Hav} A = \frac{(s-b)\,s-c)}{bc};$$

and in Spherical Trigonometry

$$\begin{aligned} \text{since} \quad \cos a &= \cos b\cos c + \sin b \sin c \cos A \\ &= \cos(b-c) - \sin b \sin c \operatorname{hav} A, \\ \operatorname{hav} a &= \operatorname{hav}(b-c) + \operatorname{hav}\theta, \\ \text{where} \quad \operatorname{hav}\theta &= \sin b \sin c \operatorname{hav} A. \end{aligned}$$

It may interest those readers of the *Gazette* who do not happen to have worked with the Haversine to work out the solution of a problem so as to be able to compare the result obtained by an ingenious graphic method to be mentioned later.

To find a, when $b=75°$, $c=55°$, $A=70°$.

$$\begin{aligned} \log\operatorname{hav}70° &= \bar{1}\cdot5172 & \operatorname{hav}\theta &= \cdot2597 \\ \log\sin75° &= \bar{1}\cdot9849 & \operatorname{hav}20° &= \cdot0302 \\ \log\sin55° &= \bar{1}\cdot9134 & \operatorname{hav}A &= \cdot2899 \\ \log\operatorname{hav}\theta &= \bar{1}\cdot4155 & A &= 65°\ 9' \end{aligned}$$

A traverse table (a storehouse of easy examples for school use) in right-angled triangles, besides a table of chords in addition to those of functions usually given, add to the usefulness of the work. It is moreover beautifully printed in several carefully graduated sizes of type.

Principes et Formules de Trigonométrie Rectiligne et Spherique. Par J. Pionchon. (Gauthier Villars.) 5 francs.

This serviceable work belongs to the Bibliothèque de l'Elève-Ingénieur. It contains all the formulae an engineering student is likely to want in the subjects of which it treats, with sufficient theory to shew how they are derived, and includes the simpler cases of definite and indefinite integration. The treatment of the $\int_0^\pi \sin^2 x\,dx$ is particularly neat. The most interesting paragraph in the work we think to be that on the *Méthode graphique de résolution des triangles sphériques.*

Writing the formula $\cos a = \cos b \cos c + \sin b \sin c \cos A$,

$$2.\cos a - (1 + \cos A)\cos(b - c) - (1 - \cos A)\cos(b + c) = 0,$$

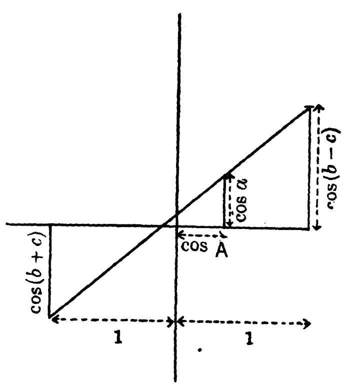
the author points out, after Collignon, that this gives

$$\begin{vmatrix} -1, & \cos(b+c), & 1 \\ 1, & \cos(b-c), & 1 \\ \cos A, & \cos a, & 1 \end{vmatrix} = 0,$$

and hence that the three points $(1, \cos\overline{b-c})$, $(-1, \cos\overline{b+c})$, $(\cos A, \cos a)$ are collinear. Hence marking the points $(1, \cos\overline{b-c})$, $(-1, \cos\overline{b+c})$ and joining them by a straight line, $\cos a$ is the ordinate of the point on it whose abscissa is $\cos A$. If, as the author suggests, we use d'Ocagne's 'abacus,'* constructed by ruling parallels to the coordinate axes through the points of division of these axes into scales of cosines, there is no need to consult tables. Recourse to tables may also be avoided by using Granville's *Plotting Circles for Polar Coordinates* to obtain the requisite cosines graphically and read off the final result. But as the only functions required are natural cosines, these can all be obtained from one opening in a book of 4-figure tables, and we find the method very expeditious on a sheet of Pye's paper ruled in millimetres, the white margin round the 20-centimetre square being distinctly convenient. The final reading for the angle A found by calculation above to illustrate the haversine formula to be 65° 9′ coming out 65° 30′. The method deserves a place by the side of the graphic method derived from R. F. Davis's elegant investigation *in plano* of the same formula (see *Mathematical Gazette*, vol. ii., p. 261). Although derived from the vanishing of a determinant, it seems more simply proved by pointing out that

$$y = \cos b \cos c + x \sin b \sin c$$

is satisfied by $(1, \cos\overline{b-c})$, $(-1, \cos\overline{b+c})$, and that if $x = \cos A$, $y = \cos a$.



Briefwechsel zwischen C. G. J. Jacobi und M. H. Jacobi. Herausgegeben von W. Ahrens.

This part (the 22nd) of the series of 'Abhandlungen zur Geschichte der Mathematischen Wissenschaften' originated by Cantor is of great interest. To mathematicians the name of the younger brother, *Carl Gustav Jacob* (1804-1851),

* D'Ocagne's abacus is of course simply the rectangular network that would be constructed as in Daniell's *Text Book of the Principles of Physics*, pp. 80, 83, 84, for describing a series of ellipse traced by a point having two single harmonic motions at right angles to each other. In graduating the scales of cosines the origin is marked 90° in both cases, the number from 0° to 180° proceeding in the negative direction along each axis.