

Original Article

Thermoelastic Problem in a Semi-Infinite Hollow Circular Disk

Yogesh U. Naner

Department of Mathematics, Shri Vitthal Rukhmini College, Sawana, Tq. Mahagaon
Dist-Yavatmal

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Correspondence Address:

Yogesh U. Naner
Department of Mathematics, Shri
Vitthal Rukhmini College, Sawana,
Tq. Mahagaon
Dist-Yavatmal
Email: ynaner@gmail.com

Quick Response Code:



Web: <https://rlgjaar.com>

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Abstract:

This paper is concerned with the determination of unknown temperature. The present study considers a semi-infinite hollow circular disk subjected to internal heat generation. Initially, the disk possesses an arbitrary temperature distribution denoted by $F(r, z)$. For time $t > 0$, heat is generated within the domain at a rate $g(r, z, t)$ Btu/hr-ft³. Thermal flux is imposed along both the inner and outer cylindrical boundaries. The lower surface of the disk is maintained at a prescribed temperature $f_1(r, t)$, while the upper surface is kept at zero temperature. The disk extends infinitely in the positive z -direction, starting from $z=0$. The governing equations describing the thermoelastic behavior of the system are solved using the finite Hankel transform in the radial direction and the finite Fourier transform in the axial direction. The resulting temperature distribution is expressed in a series representation involving Bessel functions.

Keywords: Temperature, Thermoelastic, Hollow Circular Disk, Integral Transform.

Introduction:

Research in thermoelasticity and thermoplasticity has gained substantial relevance due to its critical role in evaluating the performance of advanced engineering systems, such as modern steam and gas turbines, jet and rocket propulsion engines, high-speed aerospace vehicles, and nuclear reactors. Recent advancements have significantly expanded the theoretical framework of thermoelasticity, which extends classical elasticity and heat conduction theories to encompass a wide range of physical phenomena. These include steady and unsteady heat transfer in solids, interactions with ambient environments, and scenarios involving internal heat sources.

In the analysis of transient heat conduction problems, the Laplace transform is frequently utilized to eliminate the temporal variable from partial differential equations. However, in certain cases, it is more practical to employ an integral transform that removes the spatial variable instead. This approach is particularly advantageous for both transient and steady-state thermal analyses, as it uniformly treats all spatial variables and avoids the inversion complexities often encountered with the Laplace transform. The integral transform and its corresponding inversion formula are typically defined at the outset, facilitating a more straightforward and consistent solution process. Otao et al. [4] conducted a theoretical investigation of a three-dimensional transient thermal stress problem in a nonhomogeneous hollow circular cylinder subjected to a moving heat source along the axial direction, with heat input applied to both the inner and outer surfaces. Nowinski [5] highlighted the wide range of applications of thermoelasticity, noting its practical relevance in engineering fields such as the design and construction of airplanes, aircraft, and rocket engines. Gaikwad [7] developed a mathematical model and performed simulations for a quasi-static thermoelastic problem in a semi-infinite hollow circular disk with internal heat generation. In the present work, we focus on evaluating the temperature distribution in a semi-infinite hollow circular disk subjected to internal heat generation under unsteady-state thermal conditions.

Formulation of the Problem

The study considers a semi-infinite hollow circular disk subjected to an unsteady temperature field resulting from internal heat generation. Initially, the disk possesses an arbitrary temperature distribution denoted by $F(r, z)$. For $t > 0$, heat is generated within the disk at a rate $g(r, z, t)$ Btu/hr-ft³. Heat flux is applied along both the inner and outer cylindrical boundaries. The lower surface of the disk is maintained at a specified temperature $f_1(r, t)$, while the upper surface is held at zero temperature. The disk extends infinitely in the positive z -direction, beginning from $z=0$. Under the prescribed thermal and boundary conditions, the temperature distribution within the semi-infinite hollow circular disk resulting from internal heat generation is determined. The temperature field $T(r, z, t)$

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in the disk satisfies the transient heat conduction equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{g(r,z,t)}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad \text{in } a \leq r \leq b, 0 \leq z < \infty, \text{ for } t > 0 \quad (1)$$

With the boundary conditions,

$$k \frac{\partial T}{\partial r} = f_1(z, t) \quad \text{at } r = a, \text{ for } t > 0 \quad (2)$$

$$k \frac{\partial T}{\partial r} = f_2(z, t) \quad \text{at } r = b, \text{ for } t > 0 \quad (3)$$

$$T = f_3(r, t) \quad \text{at } z = 0, \text{ for } t > 0 \quad (4)$$

$$T = 0 \quad \text{at } z = \infty, \text{ for } t > 0 \quad (5)$$

and the initial conditions

$$T = F(r, z) \quad a \leq r \leq b, 0 \leq z < \infty, \text{ for } t = 0 \quad (6)$$

Where $T = T(r, z, t)$, k, α are the thermal conductivity and thermal diffusivity of the material of the circular disk.

Solution of the Problem

To derive the temperature function $T = T(r, z, t)$ we begin by applying the finite Fourier transform with respect to the axial coordinate z over the semi-infinite domain $0 \leq z < \infty$, as defined in [1] is

$$T(r, \eta, t) = \int_{z'=0}^{\infty} K(\eta, z') T(r, z', t) dz' \quad (7)$$

$$T(r, z, t) = \int_{\eta=0}^{\infty} K(\eta, z) \bar{T}(r, \eta, t) d\eta \quad (8)$$

where

$$K(\eta, z) = \sqrt{\frac{2}{\pi}} \sin(\eta z)$$

and $\eta_1, \eta_2, \dots$ are the positive roots of the transcendental equation

$$\sin(\eta_p h) = 0, \quad p = 1, 2, \dots$$

i.e. $\eta_p = \frac{p\pi}{h}, \quad p = 1, 2, \dots$

Applying the finite Fourier transform defined in equation (7) to equation (1) and using the conditions (2) to (6),

$$\frac{\partial^2 \bar{T}}{\partial r^2} + \frac{1}{r} \frac{\partial \bar{T}}{\partial r} - \eta_p^2 \bar{T} + \frac{\bar{g}(r, \eta_p, t)}{k} = \frac{1}{\alpha} \frac{\partial \bar{T}}{\partial t} \quad (9)$$

With

$$k \frac{\partial \bar{T}}{\partial r} = \bar{f}_1(\eta_p, t) \quad \text{at } r = a, \text{ for } t > 0 \quad (10)$$

$$k \frac{\partial \bar{T}}{\partial r} = \bar{f}_2(\eta_p, t) \quad \text{at } r = b, \text{ for } t > 0 \quad (11)$$

$$\bar{T} = \bar{F}(r, \eta_p) \quad \text{in } a \leq r \leq b, \text{ for } t = 0 \quad (12)$$

Where $\bar{T} = T(r, \eta_p, t)$.

"Next, we introduce the finite Hankel transform and its corresponding inverse with respect to the variable r over the interval $a \leq r \leq b$, as outlined in [1]."

$$\bar{T}(\beta_m, \eta_p, t) = \int_{r'=a}^b r' K_0(\beta_m, r') T(r', \eta, t) dr' \quad (13)$$

$$\bar{T}(r, \eta, t) = \sum_{m=1}^{\infty} K_0(\beta_m, r) \bar{T}(\beta_m, \eta_p, t) \quad (14)$$

$$\text{Where } K_0(\beta_m, r) = \frac{\pi}{\sqrt{2}} \frac{\beta_m J_0'(\beta_m b) Y_0'(\beta_m a)}{\left[1 - \frac{J_0'^2(\beta_m b)}{Y_0'^2(\beta_m a)}\right]} \left[\frac{J_0(\beta_m r)}{J_0'(\beta_m b)} - \frac{Y_0(\beta_m r)}{Y_0'(\beta_m b)} \right]$$

And $\beta_1, \beta_2, \dots$ are the positive roots of transcendental equation

$$\frac{J_0'(\beta a)}{J_0'(\beta b)} - \frac{Y_0'(\beta a)}{Y_0'(\beta b)} = 0$$

This transform satisfies the relation

$$H \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] = -\beta_m^2 \bar{T}(\beta_m, t)$$

Applying the finite Hankel transform defined in equation (13) to equation (9) and using the conditions (10)-(12), one obtains

$$\left[\frac{\partial \bar{T}(\beta_m, \eta_p, t)}{\partial t} + \alpha(\beta_m^2 + \eta_p^2) \bar{T}(\beta_m, \eta_p, t) \right] = A(\beta_m, \eta_p, t) \quad (15)$$

$$\bar{T}(\beta_m, \eta_p, t) = \bar{F}(\beta_m, \eta_p) \quad \text{for } t = 0 \quad (16)$$

Where

$$A(\beta_m, \eta_p, t) = \frac{\alpha}{k} \bar{g}(\beta_m, \eta_p, t) + \frac{\alpha}{k} \{ a K_0(\beta_m, a) \bar{f}_1(\eta_p, t) - b K_0(\beta_m, b) \bar{f}_2(\eta_p, t) \} + \frac{\alpha}{k} \left\{ \left(\frac{dK_0(\eta_p, z)}{dz} \right)_{z=0} \bar{f}_3(\beta_m, t) \right\} \quad (17)$$

One obtains solution of the equation (15) as

$$\bar{T}(\beta_m, \eta_p, t) = e^{-\alpha(\beta_m^2 + \eta_p^2)t} \left[\bar{F}(\beta_m, \eta_p) + \int_{t'=0}^t e^{\alpha(\beta_m^2 + \eta_p^2)t'} A(\beta_m, \eta_p, t') dt' \right] \quad (18)$$

Now using the finite Hankel transform and inverse finite Fourier transform, one obtains the expression of the temperature $T(r, z, t)$ as

$$T(r, z, t) = \sum_{m=1}^{\infty} K_0(\beta_m, r) \int_{\eta=0}^{\infty} K(\eta, z) e^{-\alpha(\beta_m^2 + \eta_p^2)t} \left\{ \int_{r'=a}^b \int_{z'=0}^{\infty} r' K_0(\beta_m, r') K(\eta, z') F(r', z') dr' dz' + \right. \\ \left. \int_{t'=0}^t e^{-\alpha(\beta_m^2 + \eta_p^2)t'} \left[\frac{\alpha}{k} \int_{r'=a}^b \int_{z'=0}^{\infty} r' K_0(\beta_m, r') K(\eta, z') g(r', z', t') dr' dz' + \frac{\alpha}{k} (a K_0(\beta_m, a) - \right. \right. \\ \left. \left. b K_0(\beta_m, b)) \int_{z'=0}^{\infty} K(\eta, z') f_{1,2}(z', t') dz' + \sqrt{\frac{2}{\pi}} \frac{\alpha}{k} \eta \int_{r'=a}^b r' K_0(\beta_m, r') f_3(r', t') dr' \right] dt' \right\} d\eta \quad (19)$$

Conclusion:

In this research paper, we have calculated the temperature of the heat conduction problem in a semi-infinite hollow circular disk due to internal heat generation under unsteady state temperature distribution. We have obtained temperature in series form in terms of Bessel's function. For calculation of temperature we have used finite Hankel Transform and finite Fourier Transform.

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