



XXXV. Notes on thermometry

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XXXV. *Notes on Thermometry.* By C. CHREE, *Sc.D., F.R.S.*

[Concluded from p. 227.]

- § 20. Lag.
 21. Freezing-point of water.
 22-24. Boiling-point of water.
 25. Calibration.
 26-30. External and internal pressure corrections, standard position for thermometers.
 31, 32. Emergent column.
 33-36. Welsh's method of graduation, and its modern developments.
 37. Method of finding mean coefficient of expansion of mercury in glass.
 38. Comparison of thermometric methods.

Lag.

§ 20. GLASS-MERCURY thermometers, and probably all others, differ from the ideal of our definition in requiring a sensible time to follow a change of temperature. This *lag* in a mercury-thermometer increases with the mass of the mercury and the thickness of the glass. It also depends on the nature of the surrounding medium. A clinical thermometer, for instance, initially at 15° C., will rise to the temperature of the body faster in a moist than in a dry mouth, and much faster in a well-stirred bucket of water than in either. In still air where temperature is altering rapidly, two adjacent thermometers of different sluggishness may differ by degrees.

If T denote time, t the thermometer reading, τ the temperature of its surroundings, the formula usually advanced to represent the phenomena is * :

$$\frac{dt}{dT} + \lambda(t - \tau) = 0,$$

where λ is a constant. When $t - \tau$ is small, this is probably at least a close approximation to the facts. When, however, $t - \tau$ is considerable—as, for instance, when a thermometer initially at 15° C. is suddenly exposed to a temperature of 40° C.—the initial phenomena, in my experience, do not follow so simple a law.

When τ is constant, the solution of the differential equation is

$$t = \tau + (t_0 - \tau)e^{-\lambda T},$$

where t_0 is the value of t when $T = 0$.

* Cf. Guillaume's *Thermométrie*, p. 185.

Thus if the temperature τ be absolutely constant, the difference between t and τ tends ultimately to vanish, and the temperature of the medium may be obtained directly, to any required degree of accuracy, by allowing sufficient time to elapse before reading the thermometer. Usually, however, τ is to some extent variable*, and to arrive at a sufficiently exact interpretation of the readings of the thermometer it may be necessary to carefully observe the rate of variation of t and employ some mathematical analysis. The mere presence of the thermometer tends frequently to introduce variation in the temperature of the medium surrounding its bulb. When, for instance, there is only partial immersion in a liquid bath or in a freezing solution, whose temperature differs appreciably from that of the room, there is a constant transfer of heat along the thermometer-stem, which influences to some extent the temperature recorded. This influence depends largely on the character of the thermometer and the nature of the bath.

The phenomena discussed in this paragraph have an important bearing on the accuracy of ordinary thermometric measurements, which is perhaps somewhat apt to be overlooked by those whose experience has centred in laboratories provided with elaborate thermo-regulators.

Freezing-point of Water.

§ 21. Particulars of the method of determining (depressed zero) freezing-points at the Bureau International will be found in Guillaume's *Thermométrie*, p. 116. The object is to reduce the temperature to 0° C. as rapidly as is consistent with the safety of the thermometer. The ice must be pure, the presence of the least trace of salt being especially objectionable. It should be very finely divided, and sufficiently but not excessively moist. What Guillaume speaks of most favourably is good lake ice or freshly fallen snow moistened with pure distilled water. He quotes the following results so observed by Pernet :—

* The case where τ is a linear function of the time has been treated recently by Grossmann (*Annalen der Hydrographie der maritimen Meteorologie*, vol. x. 1897, p. 483). The same paper contains references to recent work on the subject and also a variety of interesting experimental results.

TABLE VIII.

	Temperature centigrade.
Distilled water-ice, free from air.....	0°·0000
Distilled water-ice	—0·0014
Natural ice mixed with distilled water	—0·0017
Natural ice moist	—0·0003
Natural ice, finely divided, dry.....	—0·0035
Snow, dry	—0·0060
Snow, moistened with distilled water	—0·0004
Snow moistened, after water flowed out	—0·0038

These are to be regarded rather as indications of the order of probable differences under ordinary conditions than as rigid physical data. Thus Marek, whose results Guillaume also quotes *, got for dry artificial ice —0°·0519 or —0°·0919, according as it was finely divided or only pounded.

When ice is dry, its temperature may in reality be very much below the freezing-point. Of course under ordinary experimental conditions the ice is exposed to a temperature over 0°, and its surface is usually moist. The tendency for the ice in the centre of large blocks to be at a lower temperature than 0° C. has been noticed at the Reichsanstalt †, where it is the practice to employ artificial ice from distilled water.

In making artificial ice, the purity of the water seems of some importance, ice from tap or cistern-water giving freezing-points too low by from 0°·002 C. to 0°·003 C. ‡

To obtain results agreeing to 0°·001 C. a very uniform procedure must be followed, and it is quite conceivable that the adoption of different procedures at different places might lead to slightly different scales of temperature §.

One disturbing agent, to which attention may be called, is conduction of heat down the thermometer stem. Though specially to be feared in depressed-zero readings following high temperatures, its absence can hardly, perhaps, be safely assumed in ordinary fixed-zero determinations, especially in a hot room.

In dealing with toluene or other spirit-thermometers, in which the liquid wets the tube, the depressed zero method has been found impracticable at the Bureau International §,

† *Loc. cit.* p. 118.

‡ *Wiss. Abhandl.* vol. i. p. 83.

§ Cf. *Comptes Rendus . . . de la deuxième Conférence général des Poids et Mesures*, Paris, 1896, pp. 125 and 133.

§ See their pamphlet, *Mesures des basses Températures*, Paris, 1893, p. 4.

Phil. Mag. S. 5. Vol. 45. No. 275. April 1898. Y

owing to the slowness with which the spirit drains down the tube after a fall of temperature.

On the fixed zero method, the thermometer ought theoretically to be kept at 0° C. for an indefinite time. In practice, however, prolonged exposure to a temperature of about 15° C. is a much more usual preliminary. Now a *verre dur* thermometer after long exposure to 15° C. would give, even after 10 or 15 minutes immersion in ice, a zero reading about $0^{\circ}015$ C. below the true fixed zero, and this, of course, is a very serious difference in thermometry aiming at accuracy of the order $0^{\circ}001$ C. Even with English glass, where the depression of zero after ordinary atmospheric temperatures is considerably less than with *verre dur*, it is hopeless to think of accuracy above $0^{\circ}01$ C. unless attention is paid to the previous temperature.

For work of the highest accuracy, the pressure to which the ice is exposed may become of importance. According to the theory of Prof. James Thomson, and the experiments of Lord Kelvin, an increase of one atmosphere in the pressure lowers the freezing-point about $0^{\circ}0075$ C.* Thus in vacuum-tubes where the pressure is practically nil, or in laboratory experiments with compressed gases, a very sensible correction might be necessary. Even under natural conditions a correction of the order $0^{\circ}003$ C. might be required at lofty stations.

It must not be overlooked that such variations in external pressure affect the reading in another totally different way, through their direct influence on the glass of the thermometer (see § 26). In fact, care is required to ensure that no external pressure effect is produced by too tight packing of the ice, or by allowing the bulb to rest unsupported on largish ice-crystals.

In determining freezing-points of metals or solutions it is customary to cool down the liquid and observe the stationary point on an immersed thermometer. This method, though not usually adopted, is of course applicable to the freezing-point of water. A discussion of it by Dr. M. Wildermann has appeared recently in the *Phil. Mag.* †, in which will be found references to recent work on the subject.

* Lord Kelvin's 'Mathematical and Physical Papers,' vol. i. pp. 156-169; or Baynes' 'Thermodynamics,' art. lxxxix.

† December, 1897, p. 450.

Boiling-point of Water.

§ 22. In Balfour Stewart's 'Heat' (art. xxi. 1st ed.), 212° F. is defined as the temperature of steam under the pressure of 29·905 inches of mercury reduced to 32° F. at the latitude of London. Balfour Stewart points out a difference between 212° F. and 100° C., defining the latter as the temperature of steam under the barometric pressure of 760 mm. of mercury reduced to the temperature of freezing water in the latitude of Paris. The Bureau International, however, now generally recognized as the central authority, have substituted latitude 45° for that of Paris in the definition *. This has apparently the fortunate result of bringing 212° F. and 100° C. into almost exact agreement.

According to Kater's determination,

$$760 \text{ mm.} = 29\cdot9218 \text{ inches ;}$$

$$\text{and so} \quad 759\cdot573 \text{ mm.} = 29\cdot905 \text{ inches ;}$$

and, according to Guillaume's *Thermométrie*, p. 325,

$$\{\text{Gravity in London (Standards' Office)}\} \div$$

$$\{\text{Gravity in latitude } 45^\circ\} = 1\cdot0005815.$$

These data make the pressure of 29·905 inches of mercury in London equal to that of 760·031 mm. in latitude 45°, and by Regnault's table (*cf.* Guillaume's *Thermométrie*, p. 327) the boiling-point corresponding to the latter pressure is 100°·001 C.

Recent comparisons †, both American and British, give apparently for the ratio (yard/metre) a value something like one part in 40,000 larger than Kater's, and it is uncertain what accuracy is assignable to values of g , and to what extent it presents local fluctuations. Thus the closeness of the above agreement between 212° F. and 100° C. may not be absolutely relied on. If it should prove that g has a daily or annual variation of one part in 25,000, an uncertainty of the order 0°·001 C. would exist in the standard boiling-point.

* Guillaume's *Thermométrie*, p. 4.

† A preliminary discussion of the most recent comparison, initiated by Mr. Chaney, will be found on pp. 37–40 of the *Procès-Verbaux* for 1895 (Paris, 1896) of the International Committee of Weights and Measures. Owing to the absence of a legally defined scale of temperature, there seems some uncertainty as to the interpretation to be put on the standard temperature 62° F. At this point the *verre dur* and hydrogen scales differ by about 0°·075 C.

Also for this degree of accuracy it may prove that specification of the standard latitude, 45° , alone is not sufficient. It is desirable, of course, that the definitions of 212° F. and 100° C. should be absolutely equivalent.

§ 23. When we leave definitions and come to actual determinations of boiling-points, we encounter, as has recently been emphasized by Mr. E. H. Griffiths*, uncertainties in barometric determinations. Differences of $\cdot 002$ inch, or more, in the readings of the same barometer by two skilled observers, due mainly to personal equation in setting the ivory pointer, are not unusual. Mr. Griffiths does not refer to this point, but directs his attention to possible differences between barometers themselves. A comparison of a Tonnelet barometer examined at the Bureau International and an English barometer verified at Kew Observatory leads him to the conclusion † “that a discrepancy of as much as $\cdot 2$ mm. would appear to exist between the Sèvres (Bureau International) and Kew standards.” He adds, however, that the conditions of the comparison “were not entirely satisfactory.”

Several more direct comparisons of the standard barometers at the principal European institutions have been made by skilled observers, using portable barometers as intermediaries. The more important comparisons prior to 1890 are summarized in a table in Waldo's ‘Modern Meteorology,’ from which I have extracted the following results :—

TABLE IX.

Place and Institution.		Type of Barometer.	Corrections in mm. to reduce to normal barometer (Wild's) at St. Petersburg.			
			Chistoni, 1881.	Waldo, 1883.	Sundell, 1886.	Brounow, 1887.
Paris.	Meteorological Bureau.	Regnault.	—·04	—·05	+·16	+·11
„	Bureau International.	Wild-Pernet	—·24		
		Marek.	—·20	+·21	+·10
Berlin.	Normal Standard Commission.	Normal.	—·25	—·05	
Vienna.	Central Meteorological Institute ...	Pistor.	—·17	—·08	+·13	+·11
Hamburg.	Deutsche Seewarte.	Normal.	+·11	—·04	+·14	+·07
Kew.	Observatory	Standard.	—·01	—·10	—·05	

If the discrepancies shown by Table IX. actually existed,

* ‘Camb. Phil. Soc. Proceedings,’ vol. ix. p. 224.

† *Loc. cit.* p. 229.

accuracy even of the order $0^{\circ}005$ C.* in the absolute value of a boiling-point determination would appear highly problematical. It is not unlikely, however, that many of the discrepancies arose in great part from changes in the portable barometers during transit. Barometers, even with contracted tubes, are awkward things to carry by rail, and trustworthy results can only be obtained after the transported instrument has been hung up for a considerable time. Not unlikely Prof. Mohn's † suggestion, viz. to compare barometers by the aid of a travelling hypsometric thermometer, might prove a good one, at least in the case of stations where appliances exist for observing both boiling- and freezing-points with the highest degree of accuracy.

§ 24. There are other sources of uncertainty in boiling-point determinations. Every thermometer has a time-lag, and so probably has every barometer; hence simultaneous readings of thermometer and barometer may not really correspond to simultaneous temperature and pressure phenomena. The columns of wide-tube barometers, in which the lag is least, generally pulsate slightly even to the eye, unless under exceptionally steady conditions of pressure. Under normal conditions, one may expect changes of several thousandths of an inch in barometric pressure during the time occupied by a boiling-point determination.

In the paper already referred to, Mr. Griffiths raises the question whether the temperature of steam accommodates itself immediately to the barometric pressure when the latter changes. On his p. 230 he, however, expresses the opinion that a table on his p. 231 "will . . . convince the most sceptical that the temperature-lag of the steam may be disregarded." I am not sure his argument will be universally conceded, as the data—consisting of observed boiling-points by a platinum thermometer, and boiling-points calculated from simultaneous readings of a barometer in the course of one afternoon—might have been advanced, with perhaps equal force, in support of the contention that barometers and platinum thermometers are instruments free from lag. The uncertainties arising from lag are usually small, but still it is desirable

* Since writing the above, I have observed it stated on p. 102 of the Reichsanstalt's *Wiss. Abhandl.* vol. i. that their determination of the fundamental interval of a certain Tonnelot thermometer differed by $\cdot005$ C. from that of the Bureau International. It is significantly added that $\cdot005$ C. answers to a difference of $\cdot14$ mm. in the barometer-reading at the boiling-point.

† Report of the International Meteorological Conference, Paris, 1896, pp. 76-78.

to select as far as possible times of steady barometric pressure for boiling-point determinations.

Calibration.*

§ 25. This is too technical and mathematical a subject to be profitably discussed here in detail. The methods in use at the Bureau International are described in several of their publications† and in Guillaume's *Thermométrie*, pp. 43 *et seqq.* In calibrating an ordinary thermometer, range 0° to 100° C., use is made of threads of 50, 40, 30, 20, 12, 10, 8, 6, 4, and 2 degrees.

The publications of the Reichsanstalt recommend that the shortest mercury-threads used do not exceed in length one degree-division, and speak of an error as large as $0^{\circ}01$ C.‡ arising in the calibration of a Tonnelot thermometer when the shortest thread equalled $2^{\circ}5$. They also recommend a double calibration with each thread, proceeding from one end of the thermometer to the other and back again. This is to serve as a protection against the error that might arise from gradual heating of the mercury-thread. Alleged "nicht unerheblichen" errors§ in the calibration-data supplied by the Bureau International for one of the Reichsanstalt's *verre dur* thermometers are ascribed to the neglect of this precaution.

The object of calibration at both these institutions is to supply a table of corrections to be applied habitually to every reading of the thermometer. Degree-divisions represent equal lengths, not equal volumes, on the French and German standard thermometers.

Kew standard thermometers, on the other hand, are calibrated before they are subdivided, and the calibration results are used to guide one in dividing the stem into equal volumes, not equal lengths. Most authorities take for granted that this attempt is bound to fail when accuracy of the order $0^{\circ}001$ C. is in question. This is very probably a true conclusion, but it is not always safe to prophecy failure before the attempt is made. The fact is that no attempt has ever been made to construct Kew standards suitable for this degree of accuracy. They are intended to be read easily by the unaided eye, and the width of the divisions is usually a very considerable multiple of the $\frac{1}{1000}$ of a degree interval. The accuracy ordinarily aimed at in the construction of these instruments is really $0^{\circ}05$ F., and if we may judge from

* For a description of the principal methods see a report by Profs. Balfour Stewart, Rücker, and Thorpe, B. A. Report for 1882, p. 145.

† *Travaux et Mémoires*, vol. x. 1894 (*Thermomètres Étalons*, pp. 10-16).

‡ *Wiss. Abhandl.* vol. i. 1894, pp. 39, 40.

§ *Ibid.* vol. ii. 1895, p. 6.

careful calibrations by Dr. Thorpe and Prof. Rücker *, Prof. Rowland, Mr. Waldo †, and others, the accuracy actually attained is usually considerably higher than this. What can be done depends very much on the natural uniformity of the bore. If the preliminary calibration shows the tube destined for a Kew standard to be unduly conical or erratic in bore it is laid aside. When calibration-tables are constructed, a tube of this kind may seem to be as good as another; but I, for one, would be sorry to pin my faith to it even when calibrated with threads of 1° length.

Pressure Coefficients.

§ 26. The application of increased pressure to the outside of a thermometer, whether through rise of barometric pressure or immersion in a liquid, compresses the glass and reduces its internal volume. This makes the mercury rise in the stem, and, supposing the increase in pressure uniform over the outer glass surface, the rise bears to the increase in pressure a constant ratio, known as the "external pressure coefficient." The unit of pressure usually employed is that of a millimetre column of mercury at 0° C., the rise in the stem being measured in degrees. Knowing the external pressure coefficient, one can calculate the correction necessary to reduce the readings taken with the thermometer under any known external pressure to what they would have been under the standard pressure.

Again, when the horizontal position is adopted as the standard one—as is done at the Bureau International and the Reichsanstalt—a correction is necessary when the thermometer is read in the vertical position, to allow for the influence of the increased internal pressure. This pressure expands the bulb and compresses its contents, both effects contributing to lower the reading. If we treat the internal pressure as uniformly distributed, and proportional to the length of the mercury column measured from the centre of the bulb, we can, by observing the difference in the readings of a thermometer when vertical and when horizontal at any one temperature, calculate an "internal pressure coefficient." This may conveniently be the ratio of the observed rise of reading in degrees—when the thermometer is transferred from the vertical to the horizontal position—to the length of the mercury column measured in millimetres. Knowing this internal pressure coefficient, one can calculate a table giving the correction for internal pressure to be applied to any reading taken with the thermometer vertical.

* B. A. Report, 1881, p. 541.

† American Journal of Science, vol. xxi. 1881.

Applying the mathematical theory of elasticity, Guillaume* has deduced a very simple relation between the external and internal pressure coefficients, measured of course in the same units. Calling these coefficients β_e and β_i respectively, as Guillaume does, this relation may be written

$$\beta_i - \beta_e = c,$$

where c is proportional to the difference between the compressibility of mercury and that of the particular glass of which the thermometer is made. Thus in all thermometers of the same glass $\beta_i - \beta_e$ should be constant. Taking the units recommended above, viz. rise of 1° in reading and 1 mm. of mercury pressure, the results deduced at the Bureau International* and the Reichsanstalt† are as follows:—

Glass.	$(\beta_i - \beta_e) \times 10^7$.
<i>Verre dur</i>	154‡
Jena glass 16 ^{III}	143
„ „ 59 ^{III}	138

§ 27. In determining β_e the thermometer, immersed in a liquid, is exposed to various air pressures, from an atmosphere downwards §. The fact that the changes of pressure tend to affect the temperature of the liquid is pointed out in the publications of the Reichsanstalt ||, and it is apparently recommended that water—preferably near 4° C.—should be used rather than glycerine or mercury ¶.

Observations of the thermometer alternately in the vertical and horizontal positions, at any convenient temperature, supply the means of calculating β_i . The Sèvres-pattern hypsometer is specially adapted for this purpose. The experimental determination of β_i , however, is not so easy as it looks. In a horizontal thermometer the end of the mercury column loses its symmetrical shape—especially in a large bore—and the reading becomes uncertain. There is also apt to be trouble through the capillary effect. Capillarity causes a pressure on the mercury and bulb. Unless this is constant, which ceases

* *Thermométrie*, pp. 102–103.

† *Wiss. Abhandl.* vol. i. p. 70 (1894).

‡ Schuster and Gannon, *Phil. Trans.* 1895, p. 436, take 1 cm. of mercury-pressure for the unit instead of 1 mm., giving 154×10^{-6} for the Bureau International's value of $\beta_i - \beta_e$ in *verre dur*. Schuster and Gannon apparently think 141 would be more exact than 154.

§ See Guillaume's *Thermométrie*, p. 103.

|| *Wiss. Abhandl.* vol. ii. 1895, pp. 7–8.

¶ See Lord Kelvin's 'Mathematical and Physical Papers,' vol. iii. pp. 236–239; also Joule, *Phil. Trans.* 1859, pp. 133–136.

to be the case when the meniscus changes shape, its influence cannot be neglected.

Regarding the experimental determination of β_i as somewhat uncertain, Guillaume* recommends that it be deduced from the observed value of β_e , employing the theoretical connexion between the two coefficients.

The capillarity difficulty is dwelt on in the Reichsanstalt's publications †; but, notwithstanding, the plan recommended is to attach equal weights to experimentally determined values β_e' , β_i' of the two coefficients, and to calculate values β_e and β_i by means of these and the theoretical value of $\beta_i - \beta_e$. For instance, in a *verre dur* thermometer we should have

$$\beta_e = \frac{1}{2}(\beta_e' + \beta_i' - \cdot 0000154),$$

$$\beta_i = \frac{1}{2}(\beta_e' + \beta_i' + \cdot 0000154).$$

A table ‡ of observed and calculated values in various thermometers of different patterns shows a pretty close agreement between the *mean* observed difference $\beta_i' - \beta_e'$ and the theoretical; but in individual cases the observed difference varies from 45×10^{-7} to 250×10^{-7} . The explanation favoured is apparently the uncertainty in the capillary effect.

Personally, I am disposed to think that too much reliance is placed on the supposed constancy of $\beta_i - \beta_e$ in all thermometers of the same glass. The supposed relation is exact under two restrictions, absolute homogeneousness in the glass, and uniformity in the distribution of the external and internal pressures over their respective surfaces, inclusive of bulb, stem, and any auxiliary chambers. Under these circumstances a simple proof of the relation has been given by myself §, and independently by Schuster and Gannon ||. The supposed uniformity of pressure is, however, non-existent, in the case more especially of the internal pressure, and the means of allowing for this in the theoretical investigation have not yet been devised.

In applying the internal pressure-correction allowance is made at the Reichsanstalt ¶ for the diminution in the density of the mercury-column as the temperature rises. Guillaume** omits this, on the ground that the elasticity of the glass also

* *Loc. cit.* p. 103.

† *Wiss. Abhandl.* vol. i. 1894, p. 80.

‡ *L. c.* p. 79.

§ *Phil. Mag.* Oct. 1894, p. 371.

|| *Phil. Trans. A.*, 1895, p. 435.

¶ *Wiss. Abhandl.* vol. i. 1894, p. 68.

** *Thermométrie*, p. 107, also *Thermomètres Étalons*, p. 18 (*Bureau's Travaux et Mémoires*, vol. x. 1894).

changes with the temperature, and that, at least in the case of *verre dur*, the two influences nearly neutralize one another. The limits of temperature within which this happy arrangement prevails are, I suspect, somewhat uncertain. At temperatures above 100° C., I suspect the uncertainties proceeding from the above sources and from the capillarity are appreciable. Internal pressure-corrections of the order $0^{\circ}\cdot 1$ C. are by no means unprecedented, so that when accuracy of the order $0^{\circ}\cdot 001$ C. is aimed at, an error of even 2 or 3 per cent. in the value of β_i begins to tell.

§ 28. By adopting the vertical as the standard position, the practice followed at Kew Observatory, the necessity for an internal pressure-correction can usually be avoided, at least for accuracy of the order $0^{\circ}\cdot 01$ C. As already pointed out, when the stem is vertical the reduction of the reading, like its two contributory causes expansion of the bulb and compression of the mercury, is proportional, at least as a first approximation, to the length of the mercury-column measured from the middle of the bulb; but in an ordinary thermometer, increment of stem-length is sufficiently nearly proportional to increment of reading. Hence at any temperature t the depression due to internal pressure is, in stem-divisions,

$$q + t/p,$$

where p and q are constants for the thermometer.

Thus to make a Bureau International thermometer register correctly in the vertical position we need only lower the freezing-point mark q divisions below the position answering to a horizontal reading, and shorten each degree-division by $1/p$ of itself. This obviously comes to the same thing as marking the freezing- and boiling-points with the thermometer vertical and subdividing the fundamental interval in the usual way.

This fact has been pointed out by Prof. Schuster and Mr. Gannon*; it was arrived at independently by myself and embodied in a report on thermometry made some years ago to the Kew Observatory Committee.

When an ordinary Kew thermometer is used in the horizontal position a subtractive correction must be applied to obtain the true temperature. Exceptions to this rule exist in the shape of meteorological maximum and minimum thermometers, which it is the custom to employ in a horizontal position. To prevent possible misconception, the certificates issued state explicitly to which of the two positions the table of corrections applies.

* Phil. Trans. A., 1895, p. 434.

If the stem of a thermometer were of very unequal bore, and consequently its degree-divisions of very unequal length, the adoption of the vertical as the standard position would not eliminate the internal pressure-correction satisfactorily. It is rare, however, even in the commonest commercial tubes to encounter cases where this need be taken into account for accuracy of the order $0^{\circ}\cdot 01$ C.

§ 29. There are, however, thermometers—some of the most approved patterns—for which the ordinary Kew method of avoiding an internal pressure-correction fails. It is becoming increasingly common to have one or more auxiliary chambers blown on stems of thermometers, with the object of including one or both fundamental points in the scale without making the stem inconveniently long. Now the difference between the mercury-pressures in the bulb for two readings, one occurring above, the other below an auxiliary chamber, is no longer proportional to the difference in the temperatures. Thus the determination of the fundamental points in the vertical position, and the subdivision of the stem into equal volumes, each one-hundredth of the fundamental interval, fails to render readings in the vertical position correct measures of temperature. The remedy is easy enough, at least theoretically, as the following investigation shows:—

Let the degree-divisions immediately below and immediately above an auxiliary chamber be respectively t_1 and t_2 , while the interval between the two divisions equals the length of τ degree-divisions, instead of $t_2 - t_1$ divisions as it would in an ordinary thermometer. For shortness let

$$t_2 - t_1 - \tau = \tau' ;$$

in other words let τ' denote the number of degree-divisions by which the stem is shortened through the existence of the chamber.

Let “depression” temporarily signify lowering of reading as the thermometer, its temperature remaining constant, changes from the horizontal to the vertical position; also let Δ represent the depression at the ice-point, and δ the depression due to a column of mercury equal in length to a degree-division.

Above the chamber at a reading t , which may exceed 100° C.,

$$\text{depression} = \Delta + \delta(t - \tau') ;$$

below the chamber at a reading t , which may be below zero,

$$\text{depression} = \Delta + \delta t.$$

The depression, being Δ at the freezing- and $\Delta + \delta(100 - \tau')$ at the boiling-point, would require to be given throughout by

$$\Delta + \delta \frac{t}{100} (100 - \tau')$$

if the adoption of the vertical position alone were to suffice. Thus above the chamber the depression actually found would be too small by

$$\delta \{ (t/100)(100 - \tau') - (t - \tau') \} \equiv \delta \tau' (100 - t)/100 ;$$

while below the chamber it would be too large by

$$\delta \tau' t/100.$$

Consequently we can accomplish our object by simply shortening each degree-division, whether above or below the chamber, by $\delta \tau'/100$ of its natural length (*i. e.* the length of stem whose volume would be one-hundredth of the fundamental interval). This is equivalent to the following rule:—Determine the freezing- and boiling-points in the vertical position; calculate from a calibration where the divisions t_1 and t_2 immediately below and above the chamber would come in the usual way, but mark the former point lower by $\delta t_1 \tau'/100$ and the latter higher by $\delta(100 - t_2) \tau'/100$ than according to the calculation; subdivide the stem between the divisions 0 and t_1 , t_2 and 100, into equal volumes.

This instruction supposes δ to be known. To determine it observe the difference D in the boiling-point readings obtained with the thermometer vertical and horizontal, then

$$\delta = D \div (\tau_0 + 100 - \tau'),$$

where τ' has its previous meaning, while τ_0 is the number of degree-divisions between the centre of the bulb and the zero-point. It must be remembered that the correction is a small one, so that we may safely use in our calculation of δ the values obtained for τ_0 and τ' when the correction is neglected.

In practice, for moderate accuracy, the simplest plan would be to mark off two points one slightly below, the other slightly above the chamber, by comparison with an ordinary standard thermometer of the same kind of glass, both thermometers being vertical. Subdividing the two portions of stem included between these marked points and the respective fundamental points into equal volumes, we attain our object.

If a chamber existed above the 100° C. mark, the only change required would be a raising of all the degree-divisions above the chamber—relative to the positions calculated in

the ordinary way from calibration—by a constant amount $\delta\tau''$, where δ has its previous meaning, while τ'' is the number of degree-lengths by which the stem is shortened through the existence of the chamber.

In all possible combinations of auxiliary chambers the degree-spaces must be equal in volume throughout the entire stem, if the scale is to be that natural to the glass.

§ 30. In all ordinary mercury-thermometers some trace of air* exists, and if this collects in the stem when the thermometer is exposed to a high temperature the mercury-column is apt to break. Partly to prevent this, and partly to increase the range, limited by the boiling-point of mercury, it is becoming common to introduce nitrogen or carbonic-acid gas above the mercury, sometimes under very considerable pressure. In such thermometers the internal pressure from the gas may much exceed that arising from the weight of the mercury-column. If there were a chamber at the top of the tube similar in size to the thermometer's bulb, and if this chamber and its gaseous contents remained at a constant temperature, the internal pressure contributed by the gas would increase very approximately directly as the length of the mercury-column. Thus, supposing the tube calibrated before the introduction of the gas, the experimental determination of two fixed temperatures and the subdivision of the tube into equal volumes would supply, at least theoretically, an independent standard thermometer. In practice, however, the top chamber is usually of restricted size, and its temperature tends to rise with that of the thermometer. For both reasons the gaseous pressure may become dangerously large when the thermometer is recording near the top of its scale.

In practice it is advisable to regard a gas-filled thermometer as deriving its authority from a direct comparison with an ordinary standard. And the conditions of this comparison should approximate as closely as possible to the circumstances under which the thermometer is to be used.

Under ordinary conditions the external pressure-correction is of trifling importance relative to the internal pressure-correction. Variations of barometric pressure at sea-level may give rise to corrections of a few thousandths of a degree, and similar consequences may follow from immersion in a water-bath. In a heavy liquid, however, such as mercury, a comparatively shallow immersion produces effects which cannot be disregarded even for accuracy to $0^{\circ}01$ C.

* In the Reichsanstalt's *Wiss. Abhandl.* vol. i. p. 71, there is a reference to a supposed influence of the small trace of air or vapour in ordinary thermometers on the internal pressure.

The change of external pressure in balloon ascents or in mountain-climbing merits some consideration. At 20,000 feet above the sea, for instance, neglect of this change might lead to an error of $0^{\circ}\cdot 1$ F. in the calculated boiling-point of a thermometer, and thence to an overestimate of some 50 feet in the height.

A more important case arises in the measurement of deep-water temperatures. For this purpose it is customary to use a special pattern of "deep-sea" thermometer, in which complete protection from external pressure is aimed at. Occasionally, however, one meets with instances in which ordinary unprotected thermometers have been used to take readings in deep wells. In one case which came under my notice the consequent error amounted to no less than 15° F. Analogous results would follow the exposure of an ordinary thermometer to high gaseous pressures.

Emergent Column.

§ 31. Elementary theory assumes all the glass and mercury of a thermometer to be at one temperature. Even in well-stirred baths in physical laboratories this is rather an ideal state of matters. In ordinary use there is often an appreciable, sometimes a long, mercury-column exposed to a temperature differing from that of the bulb. This is especially true of thermometers employed to measure the temperature of a liquid, which is largely in excess of the temperature of the surrounding air. In such a case, if a long mercury-column be emergent, the thermometer, if correct, will read considerably below the true temperature of the liquid. A direct mathematical calculation of the correction, supposing even that all the physical data were known, would hardly be feasible, unless one simplified the problem by making jetsom of many of the difficulties. If we suppose the immersed part of the thermometer unaffected by the existence of the emergent part, and assume the whole emergent part, glass and mercury, to be at one temperature, an approximate formula for the correction is easily obtained.

Suppose that the liquid reaches to the division t_1 on the stem, that t is the temperature read and t' the temperature assigned to the emergent column; and use the notation of § 2, x being the correction required. The emergent mercury would at 0° C. have the volume

$$v_0(t - t_1) \frac{1 + a_1 t' + a_2 t'^2 + \dots}{1 + A_1 t' + A_2 t'^2 + \dots},$$

and when it and the corresponding part of the stem are raised in temperature from t' to t the volume increases to

$$v_0(t+x-t_1)(1+a_1t+a_2t^2+\dots).$$

Thus

$$\begin{aligned} v_0(t-t_1) \frac{1+a_1t'+a_2t'^2+\dots}{1+A_1t'+A_2t'^2+\dots} (1+A_1t+A_2t^2+\dots) \\ = v_0(t+x-t_1)(1+a_1t+a_2t^2+\dots); \end{aligned}$$

whence

$$x = \frac{(t-t_1)(t-t')\{A_1-a_1+(A_2-a_2)(t+t')+\dots\}}{(1+a_1t+a_2t^2+\dots)(1+A_1t'+\dots)}. \quad (26)$$

As a first approximation, with the notation of (4),

$$x = e_1(t-t_1)(t-t')^* \dots \dots \dots (27)$$

As already stated, e_1 is approximately the reciprocal of the number of degree volumes included in the bulb up to the zero mark. It is a constant for any particular kind of glass, and in all ordinary kinds of glass it is a little less than 1/6000. Knowing the glass, we should at once know the correction provided we knew t' .

In reality, of course, the emergent column varies in temperature from base to summit, so that t' must be regarded as its mean temperature. To determine t' directly, the Reichsanstalt employ a "Faden-thermometer," placed apparently with its very elongated bulb closely adjacent to the emergent column. Taking the reading of this auxiliary thermometer for t' , and replacing e_1 in (27) by 1/6100, they claim, as already stated, to obtain with thermometers of Jena glass 59^m corrected temperature-readings which are consistent to 0°·1 C. even up to 500° C. This practice of the Reichsanstalt seems a development of an idea which originated with Regnault †.

The theoretical aspects of the case are discussed by Guillaume ‡, who refers to the work of Regnault, Mousson, Wüllner, Holzmann, Thorpe, Mills, and Thiesen.

§ 32. In comparing high-range thermometers at the Kew Observatory I have used a formula

$$x = C(t-t_1)(t-t') \dots \dots \dots (28)$$

* This formula supplies a correction to the reading of a maximum thermometer taken when the surrounding temperature is t' , if $t-t_1$ represent the number of degrees in the detached column. (Cf. Leyst, *Rep. für Meteorologie*, vol. xiv. 1891.)

† *Relation des Expériences*, vol. i. p. 225.

‡ *Thermométrie*, pp. 188-193.

of the same type as (27), but have taken for t' the temperature of the room, and determined C for each thermometer by special experiment. These experiments have been made in two ways:—

1. The lowering in the boiling-point reading caused by increasing the length of mercury-column emergent from the hypsometer has been observed for several lengths of column*.

2. The thermometer has been attached to a platinum resistance pyrometer, so that the centres of the bulb and resistance-coil are at the same level in a well-stirred bath of fusible metal. Then two series of simultaneous readings have been taken of the two instruments, the immersions in the two series being different. If we assume that the difference of emergence has not affected the readings of the platinum pyrometer, whose coil is completely immersed in both series of experiments, one has obviously the means of calculating C.

The first of the above methods enables one to check the adequacy of the formula for a series of values of t_1 at a specified value of t ; the latter enables one to test it readily for a series of values of t . The second method is by no means very difficult in practice when one is content with accuracy of the order $0^{\circ}\cdot 1$ C. in the readings, and some promising results have been obtained by means of it. There are several possible pitfalls, however, which want fuller investigation.

When the resistance-coil of a platinum pyrometer is at a high temperature, there can hardly fail to be an appreciable transfer of heat up the tube, both by air convection currents and conduction along the metal leads, tending of course to depress the reading of the pyrometer, and conceivably this depression may vary sensibly when the length of stem immersed above the platinum spiral is altered. Again at high temperatures there is sometimes difficulty in securing a sufficiently slow steady rise of temperature, so that sensible error may arise from a difference in the lag of instruments so different in type as mercury and platinum resistance thermometers.

The apparatus employed in the second method was kindly lent to the Kew Observatory by Mr. C. T. Heycock and Mr. F. H. Neville, who likewise gave me the benefit of their varied experience in comparing thermometers by means of it.

* Cf. Mills, Trans. R. S. E. vol. xxix, 1880, p. 569.

The values obtained for C in formula (28) at Kew Observatory have varied from one thermometer to another. A good deal seems to depend on the fineness of the bore and the thickness of the enveloping glass. Even with a given thermometer so much depends on the conditions to which its stem is exposed, that corrections, whether from a general formula or from direct experiment under one definite set of conditions, are of uncertain value. If corrections are to be deduced from a particular experiment, the conditions ought to be made as similar as possible to those in which the thermometer will normally be employed. The nature of the ventilation over the surface of the hot liquid seems to have a large influence on the temperature of the emergent column and also, one would suspect, on the height of the point where the emergent column has its mean temperature.

Welsh's Method of Graduation.

§ 33. Forty years ago, when Mr. John Welsh, F.R.S., was Superintendent of Kew Observatory, the accurate pointing of thermometers at temperatures within 40° or 50° F. of the normal boiling-point of water presented serious difficulties to makers. At the same time thermometers with such a range as 172° F. to 212° F. were considerably in demand for estimating heights of mountains &c. by hypsometric observations. It was probably this combination of circumstances that led Mr. Welsh to initiate the method to which his name is here attached. The only description of the method published by Welsh, so far as I know, is a brief note entitled "Instructions for the Graduation of Boiling-point Thermometers, intended for the Measurement of Heights," B.A. Report for 1856, Transactions of Sections, p. 49.

In this he gives only numerical results for a particular case. Briefly stated, the method consists in comparing the thermometer with a standard, not in its final state, but in a preliminary state when it contains a larger quantity of mercury.

In the particular case described by Welsh, the excess of mercury in the preliminary state is such that the thermometer reads 212° F. when the real temperature is 82° F., the extra mercury, filling 130 Fahrenheit degree-divisions, being thrown off between the preliminary and final states. Data applicable to the range 172° to 212° F. in the final state are obtained from a comparison in the initial state between the temperatures 42° and 82° F.

Phil. Mag. S. 5. Vol. 45. No. 275. April 1898. Z

As evidence that Welsh did not confine his attention to this particular case, I may mention that he supplied at least one London optician with data applicable to any length of redundant column, and that since his time a considerable variety of thermometers of the type have been sent to Kew Observatory for verification.

The method has gradually extended to thermometers intended for other than hypsometric purposes. Thus before referring further to Welsh's particular case, it is convenient to discuss the general theory.

§ 34. In the *final* state suppose that at 0° C. the internal volume of the thermometer up to the division 0 (or where that division would come if it existed) is V_0 , while $100 v_0$ is then the volume between the divisions 0 and 100. Suppose also that the mercury thrown over when measured at 0° C. occupied τ scale-divisions.

In the preliminary state suppose the thermometer reads $t + \tau + \bar{x}$ when the hydrogen temperature is t , then in the notation of (1) and (2)

$$(V_0 + v_0\tau)(1 + A_1t + A_2t^2 + \dots) = \{V_0 + v_0\tau + v_0(t + \bar{x})\}(1 + a_1t + a_2t^2 + \dots),$$

where v_0/V_0 is given by (4).

Eliminating v_0/V_0 , and for brevity retaining only constants with suffixes 1 and 2, we get on reduction

$$\bar{x} = x + x', \quad \dots \dots \dots (29)$$

where

$$x = t(100 - t)(1 + a_1t + a_2t^2)^{-1} \left\{ \frac{e_1(a_1 + 100a_2) - e_2}{e_1 + 100e_2} + a_2t \right\},$$

$$x' = \tau t(e_1 + e_2t)/(1 + a_1t + a_2t^2). \quad \dots \dots (30)$$

Referring to (8) we see that x is simply the departure at temperature t of the natural scale of the glass of the thermometer from the hydrogen scale. Thus if we compared the thermometer in its preliminary state with an ordinary standard thermometer of the *same* glass, we should find its reading to exceed that of the standard by $\tau + x'$.

The excess of reading thus consists of a constant part τ which should equal the number of degree-lengths (measured at 0° C.) to be thrown over, and of a variable part x' given by (30).

The data already laid down as to the magnitudes of the constants of expansion point to the conclusion that so long as t and τ , in the centigrade scale, are less than 100, it will

generally suffice for accuracy of the order $0^{\circ}\cdot05$ C. to replace (30) by

$$x' = e\tau t, \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

where e is the mean value between 0° and 100° of the coefficient of expansion of the mercury relative to the glass of the thermometer.

Our calculation assumed the thermometer to read correctly in its *final* state, and our conclusion shows that in order that this may be the case it should in the preliminary state read τ degrees too high in ice, and show an error increasing $e\tau$ per degree as we pass up the scale. In other words, in the preliminary state its scale must be too short for the quantity of mercury then present by $100e\tau$ scale-divisions in 100; the quantity $100e\tau$ may be conveniently called the *percentage contraction*.

For the mean coefficient of expansion of mercury between 0° and 100° C. Stewart and Gee * give $\cdot0001815$. What the mean coefficient of expansion for the glass in English thermometers may be is somewhat uncertain, but it is unlikely to differ much from the value $\cdot000025$, given by Stewart and Gee (*l. c.* p. 120) for the mean from nine different kinds of glass.

Taking these values provisionally we have

$$e = \cdot000156 \dagger, \\ 100 e\tau = \cdot0156 \times \tau.$$

In Welsh's example $\tau = 72\cdot2$, in centigrade degrees, whence

$$100 e\tau = 1\cdot13.$$

The instruction to the optician in this case would thus be: make the scale 1·13 per cent. too contracted for the quantity of mercury originally present; or, more simply, tell him that the following table of relations applies:—

True temperature (Fahrenheit)	42°	52°	62°	72°	82°
Reading of thermometer in preliminary state	171·55	181·66	191·77	201·89	212.

For simplicity it should be noticed that all we have to consider is the absolute value of $e\tau$ in (31), so long as we measure x' and t on the same scale, whether centigrade or Fahrenheit. In other words the suitable contraction of scale depends only on the amount of mercury thrown off. The mistake, however, of supposing that throwing over

* 'Elementary Practical Physics,' vol. i. p. 118.

† For *verre dur* and 59^{III} the values $\cdot000158$ and $\cdot0001645$ respectively are found at the Reichsanstalt (*Wiss. Abhandl.* vol. ii. pp. 9, 17).

100° C. on a centigrade thermometer is equivalent to throwing over 100° F. on a Fahrenheit thermometer is one against which experience shows me a warning is necessary. The contraction of scale required in the former case is of course $\frac{9}{5}$ times that required in the latter.

Welsh's figures, as a matter of fact, differ slightly from those given above, in a systematic way; he has 201·87 for example in place of 201·89, and 171·48 in place of 171·55. The cause of this discrepancy was, I suspect, his neglecting any expansion in the glass, for his figures are deducible from (31) by taking ·000180 as the value of e . The B.A. figures in fact accord with the rule "contract the scale by 1 per cent. for each 100 degrees (Fahrenheit) of mercury to be thrown over," which I know Welsh to have actually laid down.

§ 35. In discussing the theory of Welsh's method we supposed the surplus mercury to be measured at 0° C., as this shows most simply its relation to what remains. In the actual operation it is hardly possible to have mercury and glass at 0° C., and it is usually inconvenient to keep the detached column in the tube and measure it at 0° C. before throwing it off. Welsh's own idea for boiling-point thermometers was simply to put the thermometers in steam and throw off the quantity of mercury required to make them give the correct boiling-point reading. Supposing this not exactly accomplished, he directed that the error observed at 212° F. in the final state should be regarded as a constant zero error, being combined of course with the calibration errors detected by the comparison with the standard thermometer in the preliminary state.

This is certainly satisfactory in short-range thermometers such as Welsh had in view. For example, to introduce an error *varying* by 0°·01 C. throughout a range of 25° C. there would require to be a mistake of about $2\frac{1}{2}$ scale-divisions in the length of the mercury-column thrown over, and this is at least ten times as great as is at all likely in the hands of a competent workman.

§ 36. The modern extension of Welsh's method to high-range thermometers is no doubt exposed to greater uncertainty. Suppose, for instance, we wish the thermometer in its final state to cover the range 200° C. to 350° C. approximately. In the preliminary state the thermometer may cover say from 0° C. to 150° C.—the accuracy of the scale throughout the upper 50° being determined by calibration with a mercury-column whose value is observed at the lower part of the scale, where direct comparison with a standard

thermometer is possible. The simplest way to break off the necessary mercury-column is to heat the bulb with a spirit-lamp so as to force a long column into the tube. After it is broken off and measured it must be run into the top chamber, and a second column must be broken off in the same way, so as to get the complete quantity, 200 divisions long, that is required.

The possible cumulation of errors of measurement may be trifling when the operator is skilled: but the very considerable heating of the bulb and mercury renders it appreciably uncertain what is the true length of the mercury-column at 0° C.

For definiteness and generality let us suppose that the mercury thrown off fills τ stem-divisions when mercury and glass are at a temperature of T (on the hydrogen scale), instead of τ divisions at 0° as was intended. Then in its final state the thermometer when at 0° has a volume of mercury too great by

$$v_0\tau T \frac{e_1 + e_2T + \dots}{1 + A_1T + A_2T^2 + \dots}.$$

Consequently at temperature t the thermometer will read y degrees higher than a standard of its own glass, where

$$y = \tau T \frac{e_1 + e_2T + \dots}{1 + A_1T + A_2T^2 + \dots} \frac{1 + A_1t + A_2t^2 + \dots}{1 + a_1t + a_2t^2 + \dots}. \quad (32)$$

Suppose, for instance, in the case $\tau = 200$, recently selected, that T is 100° C., corresponding to an error of about $1\frac{1}{2}$ per cent. in the quantity of mercury thrown off. Using the mean values of expansion coefficients between 0° and 100° C. instead of the series—which is quite good enough for our present purpose—we get approximately, in degrees centigrade

$$y = 3 \cdot 10(1 + 0\cdot000156 t).$$

This makes the error vary throughout the entire range of 150° C. only between $3^{\circ}\cdot20$ C. and $3^{\circ}\cdot25$ C.

Accuracy to $0^{\circ}\cdot05$ C. is, I suspect, at present a high ideal in mercury thermometers used above 200° C. Thus even in the above case it is doubtful whether we can improve on Welsh's direction to treat the difference observed at some standard point as a constant zero correction.

If, however, the observed error rose to 4° or 5° C. we should certainly expose ourselves to errors of the order $0^{\circ}\cdot1$ C. if we followed this direction.

A direct determination of the error at some one temperature in the final state is all the more necessary because the operation of knocking off the mercury-column, involving severe heating of the bulb, may leave a slight permanent effect.

It is important to notice, in the case of any such high-range thermometer as that just described, that the method assumes the thermometer to be of the same kind of glass as the standard with which it was compared in the preliminary state, and that the readings of the thermometer in its final state—corrected for errors of calibration and of volume of mercury thrown off—are temperatures on the natural scale of a standard thermometer of this particular kind of glass. The readings have no direct relationship to the hydrogen or any other gas-thermometer scale, and temperatures measured near the top of the range must possess an uncertain value unless the natural scale they relate to has been studied.

If the high-range thermometer and the standard it is compared with are of different kinds of glass, and still more if the thermometer has its bulb of one kind of glass and its stem of another, the interpretation of its readings would require further consideration.

§ 37. As related to Welsh's method, I would mention an ingenious way of obtaining e , the mean coefficient of expansion of mercury in glass between 0° and 100° C., described on pp. 102–104 of the Reichsanstalt's *Wiss. Abhandl.* vol. i. It consists in observing the fundamental interval in a thermometer's normal state, and also after a series of threads of mercury of different lengths have been detached and run into a top chamber. For instance, let f, f_1, f_2, f_3 represent the observed fundamental intervals in the original state and after detachment of threads of 50, 100, and 150 degrees respectively. Then, n denoting the number of degree volumes in the bulb up to the zero mark, we have

$$100 e = f/n = f_1/(n-50) = f_2/(n-100) = f_3/(n-150).$$

Thence we can deduce e and n by least squares or otherwise. Obviously the method need not be restricted to the range 0° to 100° C.

Comparison of Thermometric Methods.

§ 38. It remains for us to consider briefly the relative defects and advantages of the ordinary British methods of thermometry and the more refined methods of the Bureau International. In favour of the latter is the strong argument that when a *verre dur* thermometer verified at the Bureau

International is used in a carefully prescribed way, one can deduce the corresponding temperature on the scale of the hydrogen thermometer (at least throughout the range $-20^{\circ}\text{C}.$ to $100^{\circ}\text{C}.$) to a very high degree of accuracy. In a physical laboratory the probable error may be as small as $\pm 0^{\circ}\cdot 002\text{ C}.$, or possibly even $\pm 0^{\circ}\cdot 001\text{ C}.$ It may also be urged that the reduction of the thermometer readings by means of the tables supplied by the Bureau International requires no higher arithmetical powers than every well educated man should possess.

On the other hand, it must be conceded that the existence of lengthy tables is an evil, however necessary, both on account of the very appreciable labour their application entails, and on account of the large increase their calculation makes to the prime cost of the instrument. The fact that for high accuracy an ice reading is desirable after every temperature observation is also a drawback. Very considerable skill is required when the preceding temperature is high, the risk of breaking the thermometer being appreciable, and the frequent preparation of ice is both troublesome and expensive. The conditions under which it is safe to dispense with an ice observation and use the Bureau's table of depressed freezing-points for *verre dur*, are somewhat uncertain. The use of the table is presumably satisfactory when the temperature has been stationary for a long time*, or has been made to rise very slowly. But *how* slow the rise must be, or what is to be done when the temperature falls or oscillates, appears by no means clear. Again, if the thermometer has been at a temperature approaching $100^{\circ}\text{C}.$ one cannot safely plunge it straight into ice. The time required adequately to immerse it varies with the skill and audacity of the operator. Meantime the thermometer is probably changing its condition, with a rapidity which may depend on the time of its exposure to the high temperature and its previous history. At really high temperatures, $150^{\circ}\text{C}.$ and upwards, increasing difficulties can hardly fail to oppose the successful use of the depressed zero method, unless the glass employed be extremely little affected by sudden physical changes.

In favour of the customary methods of using English glass thermometers there are certain advantages. The necessity for an internal pressure correction, as we have seen, is avoided

* Schuster and Gannon (Phil. Trans. for 1895, p. 429) conclude that $2\frac{1}{2}$ hours exposure of a *verre dur* thermometer to a temperature of $22\frac{1}{2}^{\circ}\text{C}.$ was insufficient to produce the full depression of zero. The subsequent ice-reading was, they think, as much as $0^{\circ}\cdot 002\text{ C}.$ too high.

by adopting the vertical as the standard position. The actual readings of a correctly divided thermometer, after allowance is made for any secular change of zero, are likely to give at ordinary atmospheric temperatures—where nearly all very exact absolute measurements are made—results agreeing with those of the hydrogen thermometer to within $0^{\circ}\cdot 1$ C. To those physicists and chemists who are accustomed to record temperatures to $0^{\circ}\cdot 001$ C., or even $0^{\circ}\cdot 0001$ C., this may seem a paltry claim: but it does not seem so to many scientific men whose interests extend beyond the temperature of the thermometer to that of its surroundings. For many purposes, including even some investigations of high scientific importance, the measurement of temperatures to $0^{\circ}\cdot 001$ C. is just as futile as the use of 10 figure logarithms in handling data in which the fourth significant figure is uncertain.

On the other hand, there unquestionably exist physical and chemical investigations, tending to increase in number, in which the absolute determination of temperatures with the highest possible precision is of fundamental importance. For these, in the mean time, English glass thermometers and ordinary British methods are not suitable. However good the workmanship, and however accurate the calibration, there exist the following defects:—

1. Ice readings corresponding to infinitely prolonged exposure to 0° C. are not practically obtainable; and ordinary ice readings being affected to some extent by the previous temperature are not strictly comparable.

2. So long as changes of $3\frac{1}{2}$ inches of mercury in barometric pressure are possible, and occasions arise for the immersion of thermometers in heavy liquids, corrections for external pressure cannot always be avoided.

3. Unless frequently subjected to temperature-cycles of considerable range, an ordinary English glass thermometer is apt to be influenced for days, it may be weeks, by exposure to any temperature much over 120° F.

4. For accuracy of an order higher than $0^{\circ}\cdot 1$ F. it is certainly unsafe to assume the natural scale of an English glass thermometer identical with that of the hydrogen or nitrogen thermometer, even for the restricted range 22° F. to 212° F.

5. Whether through variety in the constitution of the glass or differences in its treatment, the natural scales of ordinary English glass thermometers do not appear sufficiently accordant to render practicable the use of any general table of reductions to a standard scale of temperature.

For leave to publish particulars of experiments made at

Kew Observatory I am indebted to the Kew Observatory Committee. To one of its members, Mr. W. N. Shaw, F.R.S., I am indebted for a variety of suggestions which have added to the lucidity and completeness of the paper. For any expressions of opinion, and for the accuracy of the facts, I am, however, personally exclusively responsible.

XXXVI. *Radiation Phenomena in the Magnetic Field.*

By THOMAS PRESTON, M.A.*

[Plate XXIII.]

IN March 1897, Dr. P. Zeeman announced in the pages of this Magazine the important discovery that the bright lines of the spectra become sensibly modified in appearance and constitution when the source of light is placed in a strong magnetic field. This effect has often been sought for both by men who have published the results of their investigations as well as by many who have not, but it is only now, thanks to the work of Dr. Zeeman, that the scientific world has become convinced of its existence and nature.

Considerable difficulty attends the experimental investigation of this effect owing to its smallness, and it is only when a very strong magnetic field is used in combination with high dispersion that it can be placed distinctly in evidence. Its investigation, however, has been greatly assisted by the present advanced state of electromagnetic theory, which pointed out what the essential features of the phenomenon were which should be looked for. Thus, the theory informed us that each bright line of a line-spectrum should be converted into a doublet, or a triplet, according as the source of light is viewed along, or across, the lines of magnetic force, and further, that each member of a doublet should be circularly polarized, whereas each member of a triplet should be plane polarized, the plane of polarization of the central line being at right angles to that of the two side lines.

This information proved of special importance to the experimental inquiry owing to the fact that a bright spectral line is not a mathematical line, but, on the contrary, possesses a finite width, so that in order to observe the triplet effect the resolution must be great enough to separate the lateral components from each other by a distance greater than the width of the spectral line in question. For resolutions less than this the central member of the triplet overlaps the lateral components, and the effect presented to the eye is

* Communicated by the Author.